

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

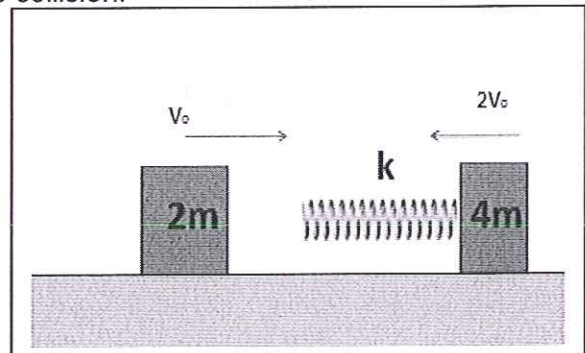
Name:

Student ID:

Signature:

A block of mass $2m$ moving to the right with a speed of V_0 on a frictionless surface collides with a light spring attached to a second block of mass $4m$ initially moving to the left with a speed of $2V_0$ as shown in Figure. The spring constant is k .

- Find the velocities of the two blocks when the spring reaches its maximum compression.
- Calculate the maximum compression of the spring.
- Find the velocities of the blocks after the collision.



(a) In maximum compression blocks A and B move together:

$$(2m)V_0 - (4m)(2V_0) = 6m v_{cm}$$

$$-6V_0 = 6v_{cm} \Rightarrow v_{cm} = -V_0$$

So A and B move with v_0 to the left at maximum compression.

(b) Collision is elastic and energy is conserved:

$$\frac{1}{2}(2m)V_0^2 + \frac{1}{2}(4m)(2V_0)^2 = \frac{1}{2}6m(-V_0)^2 + \frac{1}{2}kx^2$$

$$12mV_0^2 = kx^2 \Rightarrow x = 2V_0 \sqrt{\frac{3k}{m}}$$

(c) Here we write energy and momentum conservation before and after collision:

Momentum Conservation:

$$2mV_0 - (2V_0)(4m) = (2m)v_1 + (4m)v_2$$

$$-6V_0 = 2v_1 + 4v_2 \Rightarrow -3V_0 = v_1 + 2v_2$$

$$v_1 = -3V_0 - 2v_2 \quad \text{①}$$

(c) Energy Conservation:

$$\frac{1}{2} (m) v_0^2 + \frac{1}{2} (4m) (2v_0)^2 = \frac{1}{2} (m) v_1^2 + \frac{1}{2} (4m) v_2^2$$

$$\boxed{9v_0^2 = v_1^2 + 2v_2^2} \quad \textcircled{\text{II}}$$

From $\textcircled{\text{I}}$, $\textcircled{\text{II}}$:

$$9v_0^2 = (-3v_0 - 2v_2)^2 + 2v_2^2$$

$$9v_0^2 = \cancel{9v_0^2} + 12v_0v_2 + 4v_2^2 + 2v_2^2$$

$$12v_0v_2 + 6v_2^2 = 0 \Rightarrow 2v_2(2v_0 + v_2) = 0 \Rightarrow \begin{cases} v_2 = 0 \\ v_2 = -2v_0 \end{cases}$$

if $v_2 = 0 \Rightarrow v_1 = -3v_0$ This is acceptable.

if $v_2 = -2v_0 \Rightarrow v_1 = -3v_0 - 2(-2v_0) = v_0$ this result is not acceptable.

$$\text{So } \boxed{v_1 = -3v_0, v_2 = 0}$$

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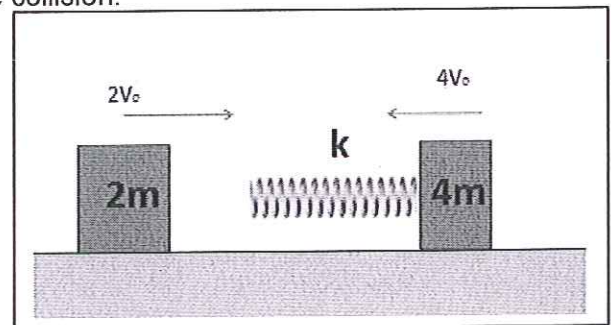
Name:

Student ID:

Signature:

A block of mass $2m$ moving to the right with a speed of $2V_0$ on a frictionless surface collides with a light spring attached to a second block of mass $4m$ initially moving to the left with a speed of $4V_0$ as shown in Figure. The spring constant is k .

- Find the velocities of the two blocks when the spring reaches its maximum compression.
- Calculate the maximum compression of the spring.
- Find the velocities of the blocks after the collision.



(a) When blocks A and B compress the spring to the maximum point they also come to rest relative to each other but still they have

identical velocity equal to the velocity of their center of mass, so the conservation law for momentum before collision and during the moment of maximum compression is:

$$(2m)(2v_0) - (4v_0)(4m) = (6m)(v_{cm}) \Rightarrow v_{cm} = \frac{-12}{6}v_0 = -2v_0$$

So both block A and B go to left with $-2v_0$ in maximum compression.

(b) From the energy conservation law we have:

$$\frac{1}{2}(2m)(2v_0)^2 + \frac{1}{2}(4m)(4v_0)^2 = \frac{1}{2}(6m)v_{cm}^2 + \frac{1}{2}kx^2$$

$$8mv_0^2 + 64mv_0^2 - 24mv_0^2 = kx^2$$

$$x = v_0 \sqrt{\frac{48m}{k}}$$

(C) From the momentum and energy conservation before and after collision we have:

Momentum Conservation:

$$(2m)(2v_0) - (4m)(4v_0) = 2mv_1 + (4m)v_2$$

$$6v_0 = v_1 + 2v_2 \Rightarrow \boxed{v_1 = 6v_0 - 2v_2} \quad \text{(I)}$$

Energy Conservation:

$$\frac{1}{2}(2m)(2v_0)^2 + \frac{1}{2}(4m)(4v_0)^2 \Rightarrow v_1^2 + 2v_2^2 = 4v_0^2 + 32v_0^2 = 36v_0^2$$

$$\boxed{v_1^2 + 2v_2^2 = 36v_0^2} \quad \text{(II)}$$

$$\text{From I, II: } (6v_0 - 2v_2)^2 + 2v_2^2 = 36v_0^2$$

$$\Rightarrow 36v_0^2 - 24v_0v_2 + 4v_2^2 + 2v_2^2 = 36v_0^2$$

$$v_2(-24v_0 + 6v_2) = 0 \Rightarrow \begin{cases} v_2 = 0 \\ v_2 = 4v_0 \end{cases}$$

if $v_2 = 0 \Rightarrow v_1 = 6v_0$ unacceptable solution

if $v_2 = 4v_0 \Rightarrow v_1 = -2v_0$ acceptable solution.

$$\boxed{\text{So } v_1 = -2v_0, v_2 = 4v_0}$$

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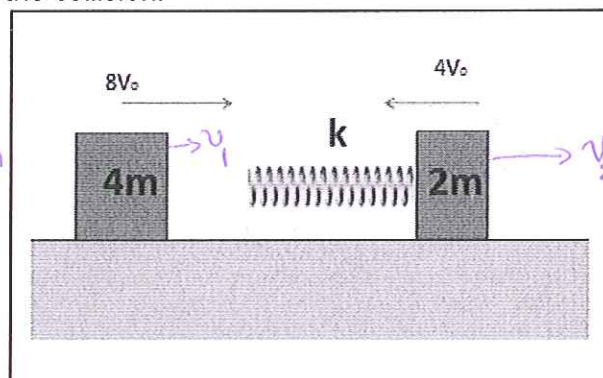
Name:

Student ID:

Signature:

A block of mass $4m$ moving to the right with a speed of $8V_0$ on a frictionless surface collides with a light spring attached to a second block of mass $2m$ initially moving to the left with a speed of $4V_0$ as shown in Figure. The spring constant is k .

- Find the velocities of the two blocks when the spring reaches its maximum compression.
- Calculate the maximum compression of the spring.
- Find the velocities of the blocks after the collision.



(a) At maximum compression the velocities for blocks A and B are equal so conservation of momentum results to:

$$(8V_0)(4m) - (4V_0)(2m) = 6m v_{cm}$$

$$v_{cm} = \frac{1}{6} 24V_0 = 4V_0$$

So A and B move to right with v_{cm} at maximum compression.

(b) Energy is conserved by elastic collision so we have:

$$\frac{4m}{2}(8V_0)^2 + \frac{1}{2}(2m)(4V_0)^2 = \frac{1}{2}6m(4V_0)^2 + \frac{1}{2}kx^2$$

$$x = 8V_0 \sqrt{\frac{3m}{k}} = \sqrt{\frac{192V_0^2 m}{k}}$$

(c) The Momentum is conserved before and after collision so:

$$(8V_0)(4m) - (4V_0)(2m) = 4m v_1 + 2m v_2$$

$$\Rightarrow 24V_0 m = 4m v_1 + 2m v_2 \Rightarrow v_2 + 2v_1 = 12V_0$$

$$\boxed{v_2 = 12V_0 - 2v_1} \quad \text{I}$$

(c) Energy conservation gives:

$$\frac{1}{2} (8v_0)^2 (4m) + \frac{1}{2} (7m)(4v_0)^2 = \frac{1}{2} (4m)v_1^2 + \frac{1}{2} (7m)v_2^2$$

$$(12v_0)^2 = 2v_1^2 + v_2^2 \quad \textcircled{\text{II}}$$

$$\text{From } \textcircled{\text{I}} \text{ and } \textcircled{\text{II}} \Rightarrow (12v_0)^2 = 2v_1^2 + (12v_0 - 2v_1)^2$$

$$\Rightarrow \cancel{(12v_0)^2} = 2v_1^2 + \cancel{(12v_0)^2} - 48v_1v_0 + 4v_1^2 \Rightarrow$$

$$v_1(-48v_0 + 6v_1) = 0 \Rightarrow \begin{cases} v_1 = 0 \\ +48v_0 = 6v_1 \Rightarrow v_1 = 8v_0 \end{cases}$$

if $v_1 = 0 \Rightarrow v_2 = 12v_0 - 2(0) = 12v_0$ feasible.

if $v_1 = 8v_0 \Rightarrow v_2 = 12v_0 - 2(8v_0) = -4v_0$ infeasible

$$\text{So } \boxed{v_1 = 0, v_2 = 12v_0}$$