PHYS 101: General Physics 1 **KOÇ UNIVERSITY** **Spring Semester 2015**

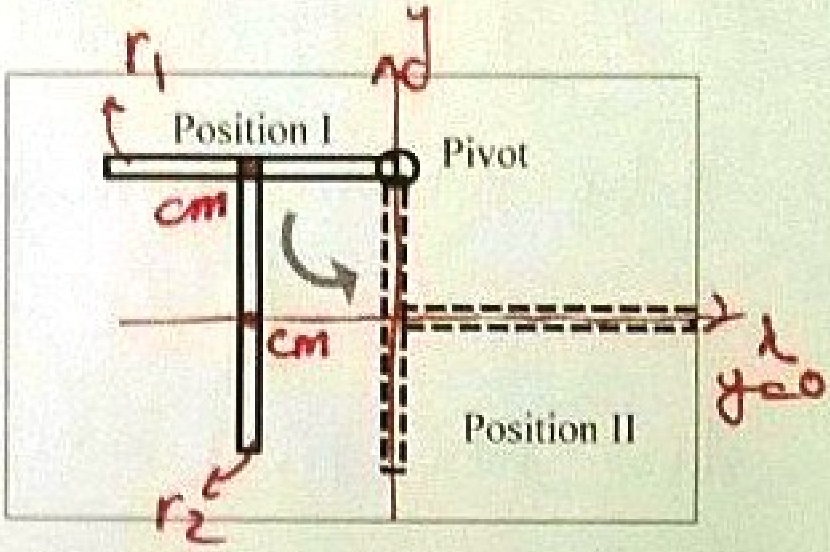
College of Arts and Sciences

Section **Quiz 9-1** **May 8, 2015**

Closed book. Duration:

Name: **Student ID:** **Signature:**

A "T" shape rigid body is formed by two identical uniform rods each of mass m, length L as shown in the figure. The rigid body can rotate freely about a pivot at the end of one rod. The rigid body is released from position (I). Take gravitational acceleration as (g).



a) The moment of inertia of the rigid body about the pivot axis can be written as $I = \beta mL^2$, where β is a constant. Find β. (Hint: The moment of inertia of a uniform rod of mass m and length L about *an axis through its center of mass is* $I_{cm} = \frac{mL^2}{12}$. Also recall the parallel axis theorem)

b) Using conservation of mechanical energy, determine the angular speed of the rigid body when it is in position II. (Note: If you did not solve part (a), you can still calculate part (b) by taking the moment of inertia of the rigid body as $I_p = \beta mL^2$.

c) Is the angular acceleration of the rigid body during the motion between position (I) and (II) constant? Explain.

(a) $I_{r1} = I_{cm} + md^2_{cm1-piv} \rightarrow I_{r1} = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2$

$I_{r2} = I_{cm} + md^2_{cm2-piv} \rightarrow I_{r2} = \frac{mL^2}{12} + m\left[\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2\right]$

$I_{total-piv} = I_{r1} + I_{r2} = \frac{11}{12} mL^2 = 0.92\, mL^2 = \beta mL^2$

$\rightarrow \beta = 0.92$

(b) conservation of mechanical Energy:

$$U_1 + K_1 + \cancel{W_{other}}^{0} = U_2 + K_2$$

$U_1 = U_{1r1} + U_{1r2}$

$U_{1r1} = mgL/2$; $U_{1r2} = 0$

Since rigid body is released from rest, $K_1 = 0$

$U_2 = U_{2r1} + U_{2r2}$

$U_{2r1} = 0$; $U_{2r2} = 0$

$$K_2 = \frac{1}{2} I_{Piv} \omega^2$$

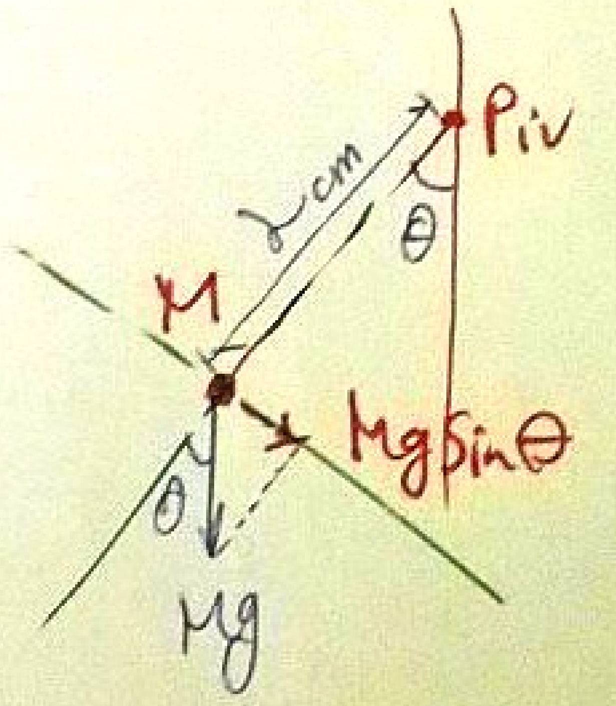
$$\rightarrow \quad \frac{1}{2} mgL = \frac{1}{2} I_{Piv} \omega^2 \rightarrow \omega^2 = \frac{mgL}{I_{Piv}}$$

$$= \frac{mgL}{0.92 mL^2}$$

$$\rightarrow \omega = 1.04 \sqrt{g/L}$$

(c) M represents the mass of the whole rigid body system, concenterted at a distance L_{cm} from the pivot axis.

The net force being applied on the system is $Mg \sin\theta$, so torque can be calculated as follows:



$$\begin{cases} \tau = L_{cm} Mg \sin\theta \\ \tau = I_{Piv} \alpha \end{cases} \rightarrow \alpha = \frac{L_{cm} Mg \sin\theta}{I_{Piv}} \quad (***)$$

As it is evident from (***), α is dependent on θ, so it is not constant.

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Section **Quiz 9-2** **May 8, 2015**

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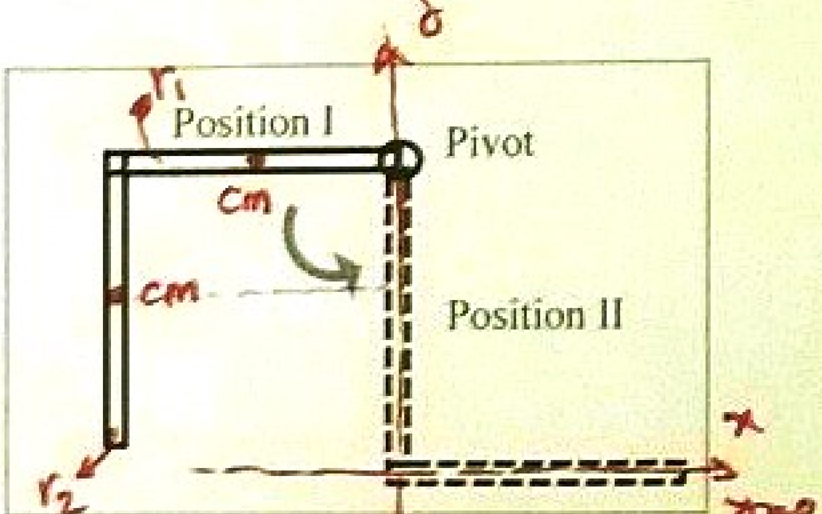
A rigid body is formed by two identical uniform rods each of mass m, length L which are attached at one end perpendicular to each other as shown in the figure. The rigid body can rotate freely about a pivot at the end of one rod. The rigid body is released from position (I). Take gravitational acceleration as (g).



a) The moment of inertia of the rigid body about the pivot axis can be written as $I = \beta m L^2$, where β is a constant. Find β. (Hint: The moment of inertia of a uniform rod of mass m and length L about *an axis through its center of mass is* $I_{cm} = \frac{mL^2}{12}$. Also recall the parallel axis theorem)

b) Using conservation of mechanical energy, determine the angular speed of the rigid body when it is in position II. (Note: If you did not solve part (a), you can still calculate part (b) by taking the moment of inertia of the rigid body as $I_p = \beta m L^2$.

c) Is the angular acceleration of the rigid body during the motion between position (I) and (II) constant? Explain.

(a)
$$I_{r1} = I_{cm} + m d^2_{cm_1 - Piv} \Rightarrow I_{r1} = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2$$

$$I_{r2} = I_{cm} + m d^2_{cm_2 - Piv} \rightarrow I_{r2} = \frac{mL^2}{12} + m\left[\left[\left(\frac{L}{2}\right)^2 + L^2\right]^{\frac{1}{2}}\right]^2$$

$$\rightarrow I_{r2} = \frac{mL^2}{12} + mL^2\left(\frac{5}{4}\right)$$

$$I_{total-piv} = I_{r1} + I_{r2} = \frac{20}{12} mL^2 = 1.66\, mL^2 = \beta m L^2$$

$$\Rightarrow \beta = 1.66$$

(b) Conservation of mechanical Energy:

$$U_1 + K_1 + \cancel{W_{other}} = U_2 + K_2$$

$$U_1 = U_{1r1} + U_{1r2}$$

$$U_{r1} = mgL \; ; \; U_{r2} = mgL/2 \Rightarrow U_1 = \frac{3mgL}{2}$$

Since rigid body is released from rest $K_1 = 0$

$$U_2 = U_{2r1} + U_{2r2}$$

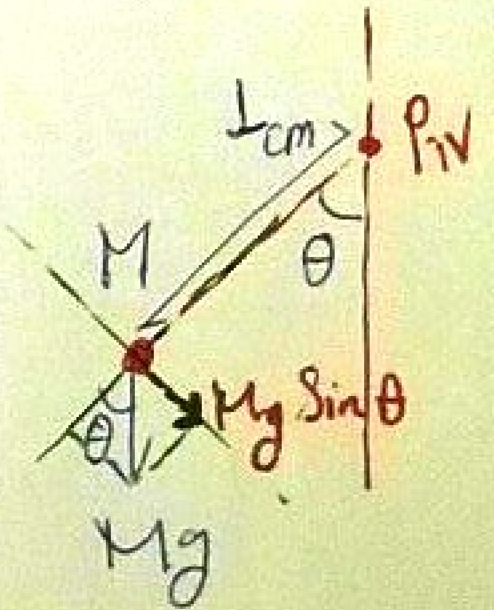
$$U_{2r1} = mgL/2 \; ; \; U_{2r2} = 0 \Rightarrow U_2 = mgL/2$$

$$K_2 = \frac{1}{2} I_{Piv} \omega^2$$

$$\rightarrow \frac{3}{2} mgL = \frac{1}{2} mgL + \frac{1}{2}(1.66) mL^2 \omega^2$$

$$\omega^2 = \frac{2}{1.66} \frac{g}{L} \rightarrow \omega = 1.10 \sqrt{\frac{g}{L}}$$

(c) M represents the mass of the whole System being concentrated at the cm of the whole rigid body System which is located at distance L_{cm} from the pivot (axis).



The net force exerting on the System is $Mg\sin\theta$, So the relevant torque can be calculated as follows:

$$\begin{cases} T = L_{cm} Mg \sin\theta \\ T = I_{Piv} \alpha \end{cases} \rightarrow \alpha = \frac{L_{cm} Mg \sin\theta}{I_{Piv}} \quad (***)$$

From (***), it can be seen that α is dependent on θ, So it is not constant.

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Section Quiz 9-3 May 8, 2015

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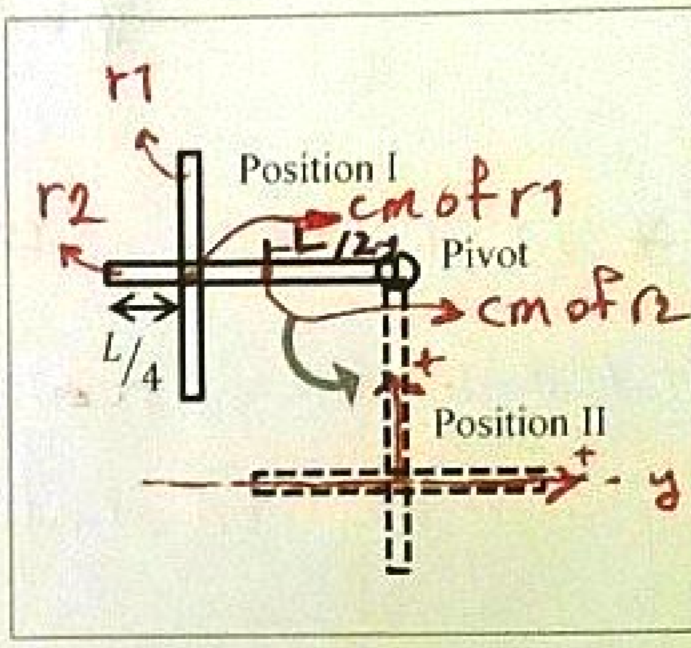
A rigid body is formed by two identical uniform rods each of mass m, length L, where one rod is attached from its center to the other rod at a distance $L/4$ from its end as shown in the figure. The rigid body can rotate freely about a pivot at the end of one rod. The rigid body is released from position (I). Take gravitational acceleration as (g).

a) The moment of inertia of the rigid body about the pivot axis can be written as $I = \beta m L^2$, where β is a constant. Find β. (Hint: The moment of inertia of a uniform rod of mass m and length L about *an axis through its center of mass* is $I_{cm} = \frac{mL^2}{12}$. Also recall the parallel axis theorem)

b) Using conservation of mechanical energy, determine the angular speed of the rigid body when it is in position II. (Note: If you did not solve part (a), you can still calculate part (b) by taking the moment of inertia of the rigid body as $I_p = \beta m L^2$.

c) Is the angular acceleration of the rigid body during the motion between position (I) and (II) constant? Explain.



(a)

moment of inertia around Pivot for r1

$$I_{r1p} = I_{cm} + m d^2_{r1p-cm}$$

center of gravity of ~~both bars~~ each bar is located on the center of ~~the bar~~ that bar

$$\rightarrow I_{r1p} = \frac{mL^2}{12} + m\left(\frac{3L}{4}\right)^2$$

$$\rightarrow I_{r2p} = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2$$

total moment of inertia $\leftarrow I_{tp} = I_{r1p} + I_{r2p} = \left(\frac{47}{48}\right) mL^2 = \beta m L^2 \rightarrow \beta = \frac{47}{48} = 0.98$

(b) $U_1 + K_1 + \cancel{W_{other}}^{0} = U_2 + K_2$ → Position 2

Position I

In position I, System is at rest $\rightarrow K_1 = 0$

$$U_1 = U_{1r1} + U_{1r2} \quad \begin{cases} U_{1r1} = mg\left(\frac{3L}{4}\right) \\ U_{1r2} = mg\left(\frac{3L}{4}\right) \end{cases} \rightarrow U_1 = \frac{3}{2} mgL$$

$$U_2 = \cancel{U_{2r1}}^{\,0} + U_{2r2}$$

$$U_{2r2} = mg\left(\frac{L}{4}\right)$$

$$K_2 = \frac{1}{2} I_p \omega^2$$

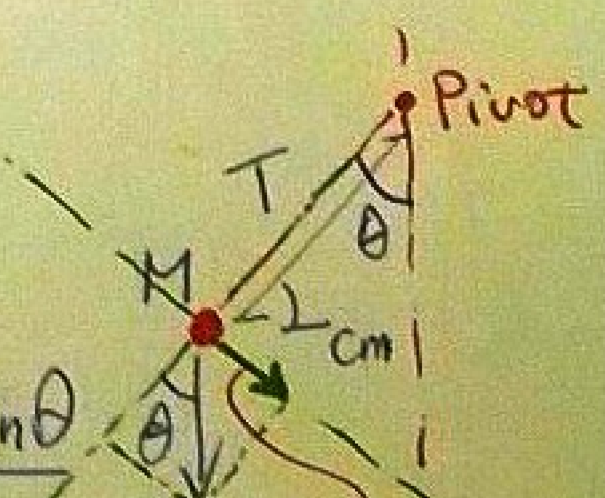
(*) → $\frac{3}{2} mgL = \frac{1}{4} mgL + \frac{1}{2} I_p \omega^2 \rightarrow \omega^2 = \left(\frac{10}{4} mgL\right) / I_p$

$$\omega^2 = \frac{2.5\, mgL}{0.98\, mL^2} \Rightarrow \omega = 1.58\sqrt{g/L}$$

** It is also possible that the (coordinate of the) center of mass for the whole system (cross) be firs calculated, then U_1 and U_2 be calculated for the cross in positions I&II.

(c) The whole system can be considered as a concenterated mass in the cm of the cross. In that case the free mass diagram of the system will be as follows:

The only net force which is acting on the system is $Mg\sin\theta$



$$\left.\begin{array}{l} T = (Mg\sin\theta) L_{cm} \\ T = I_p \alpha \end{array}\right\} \rightarrow \overset{(***)}{\alpha = \frac{L_{cm} Mg \sin\theta}{I_p}}$$

Since all the parameters in equation (***) are constant, except for θ; So α is dependen on a varying parameter θ, and it is <u>not</u> constant.