

Section

Quiz 1-1

February 2014

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:

Student ID:

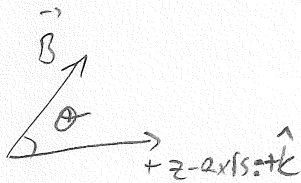
Signature:

Given two vectors $\vec{A} = 2\hat{i} - \hat{j}$, and $\vec{B} = -\hat{i} + b\hat{j} + c\hat{k}$, where b and c are some constants. If $|\vec{B}| = 3$ and \vec{A} and \vec{B} are perpendicular, find the angle the vector \vec{B} makes with the positive z-axis (you can give your result in terms of inverse trigonometric functions)

$$|\vec{B}| = 3 \Rightarrow |\vec{B}|^2 = \vec{B} \cdot \vec{B} = 1 + b^2 + c^2 = 9 \Rightarrow b^2 + c^2 = 8$$

$$\vec{A} \cdot \vec{B} = -2 - b = 0 \Rightarrow b = -2 \Rightarrow 4 + c^2 = 8 \Rightarrow c^2 = 4$$

$$c = \pm 2$$



$$\vec{B} \cdot \hat{k} = |\vec{B}| \cdot |\hat{k}| \cos \theta$$

$$c = 3 \cdot 1 \cdot \cos \theta \Rightarrow \cos \theta = \pm \frac{2}{3}$$

$$\theta = \arccos\left(\pm \frac{2}{3}\right)$$

Section

Quiz 1-2

February 2015

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Two perpendicular unit vectors \hat{u} and \hat{v} satisfy $\hat{u} \times \hat{v} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$. If \hat{u} has no x-component, find \hat{v} . (Hint: write \hat{u} and \hat{v} as three dimensional vectors unknown components and determine the components of \hat{v})

$$\hat{u} = u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k}$$

$$\hat{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}$$

$$u_1 = 0 \quad (\text{no x-component})$$

$$\hat{u} \cdot \hat{v} = 0 \quad (\text{perpendicular})$$

$$u_2 v_2 + v_3 u_3 = 0$$

$$\hat{u} \times \hat{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2 v_3 - u_3 v_2) \hat{i} + (u_3 v_1) \hat{j} + (u_2 v_1) \hat{k}$$

$$\hat{u} \times \hat{v} = \frac{1}{\sqrt{2}} \hat{i} - \frac{1}{\sqrt{2}} \hat{j}$$

$$u_3 v_3 = 0$$

\Downarrow

$$v_3 = 0$$

$$\Rightarrow u_2 v_1 = 0$$

$$u_3 v_1 = \frac{-1}{\sqrt{2}}$$

$$u_2 v_3 - u_3 v_2 = \frac{1}{\sqrt{2}}$$

$$u_2 = 0$$

$$u_3 \neq 0$$

$$v_1 \neq 0$$

$$u_3 v_2 = -\frac{1}{\sqrt{2}}$$

$$u_3 v_1 = \frac{-1}{\sqrt{2}}$$

$$|\hat{u}| = 1 \Rightarrow u_3 = \pm 1$$

\Downarrow

$$v_1 = -v_2 = \pm \frac{1}{\sqrt{2}} \quad v_3 = 0$$

$$\hat{v} = \pm \frac{1}{\sqrt{2}} (\hat{i} + \hat{j})$$

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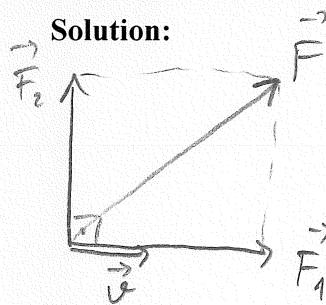
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At an instant the velocity of a particle is given as $\vec{v} = 3\hat{i} - \hat{j}$ (m/s). The force acting on the particle is $\vec{F} = \hat{i} + \hat{j} - \hat{k}$ (N). If we consider the force vector as the sum of two vectors $\vec{F} = \vec{F}_1 + \vec{F}_2$ such that \vec{F}_1 is parallel to \vec{v} and \vec{F}_2 is perpendicular to \vec{v} , determine the magnitude of \vec{F}_2 . (Hint: Consider the geometrical interpretation of the scalar product of two vectors).



$$\vec{F} \cdot \vec{v} = (\vec{F}_1 + \vec{F}_2) \cdot \vec{v} = \vec{F}_1 \cdot \vec{v} + \underbrace{\vec{F}_2 \cdot \vec{v}}_{=0}$$

$$\vec{F} \cdot \vec{v} = 3 - 1 = 2 = \vec{F}_1 \cdot \vec{v} = F_1 v \cos\theta$$

$= 1$ as $\vec{F}_1 \parallel \vec{v}$

$$v = \sqrt{9+1} = \sqrt{10}$$

$$\Rightarrow F_1 = \frac{2}{\sqrt{10}} \quad \vec{F}_1 = F_1 \hat{v} = F_1 \frac{1}{\sqrt{10}} (3\hat{i} - \hat{j})$$

$$\vec{F}_1 = \frac{3}{5}\hat{i} - \frac{1}{5}\hat{j}$$

$$\vec{F}_2 = \vec{F} - \vec{F}_1 = \frac{2}{5}\hat{i} + \frac{6}{5}\hat{j} - \hat{k}$$