

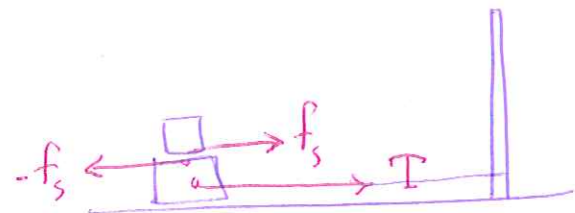
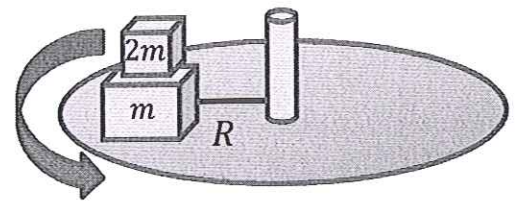
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A box of mass  $2m$  is placed on another box of mass  $m$  on a frictionless table. The box  $m$  is attached by a massless string of length  $R$  to a pivot and the boxes are in uniform circular motion together. The static friction coefficient between the boxes is  $\mu_s = 0.75$ . Find the tension in the string just before the box at the top of the other starts to slide.



For the mass  $2m$  we have just

one force that holds it at circular motion that is  $f_s$ . We know that maximum of  $f_s$  is  $f_{smax} = \mu_s n = \mu_s 2mg$ . In radial direction

we have:  $f_s = 2m a_r \Rightarrow f_{smax} = 2m a_{rmax} \Rightarrow \mu_s 2mg = 2m a_{rmax}$

$$\Rightarrow a_{rmax} = \mu_s g$$

For the other mass with mass  $m$  we can write:

$$T - f_s = m a_r \Rightarrow T_{max} - f_{smax} = m a_{rmax} \Rightarrow$$

$$\Rightarrow T_{max} = m a_{rmax} + 2m a_{rmax} = 3m a_{rmax} = 3m \mu_s g$$

$$\Rightarrow \boxed{T_{max} = 3m g \mu_s} \approx 22 m$$

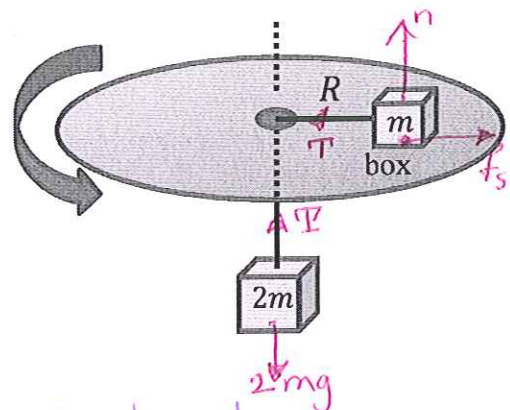
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A circular plate can rotate around an axis about its center as shown in the figure. A box of mass  $m$  is placed at a distance  $R$  from the center axis and it is connected by a massless rope through a hole at the center of the plate to another object of mass  $2m$ . The static friction coefficient between the plate and the box is  $\mu_s = 0.75$ . Assume that the rope is free to move (without friction) through the hole. Determine the minimum linear speed of the box during uniform circular motion so that the box does not slide.



When the box does not slide the hung mass also does not move and we have:  $T = 2mg$ .

For the box when the linear speed goes to minimum, it tends to slide toward the axis of rotation thus the friction will be outward the center of rotation. we have:  $f_s \leq \mu_s n = \mu_s mg \Rightarrow f_{s \max} = \mu_s mg$

In radial direction the sum of  $T$  and  $f_s$  provides the centripetal force

$$T - f_s = ma_r \Rightarrow T - f_{s \max} = ma_{r \min} \Rightarrow 2mg - \mu_s mg = ma_{r \min}$$

Knowing  $a_r = \frac{v^2}{R} \Rightarrow a_{r \min} = \frac{v_{\min}^2}{R}$  we have:  $(2 - \mu_s)g = \frac{v_{\min}^2}{R}$

$$\Rightarrow v_{\min} = \sqrt{Rg(2 - \mu_s)} = \sqrt{1.25gR}$$

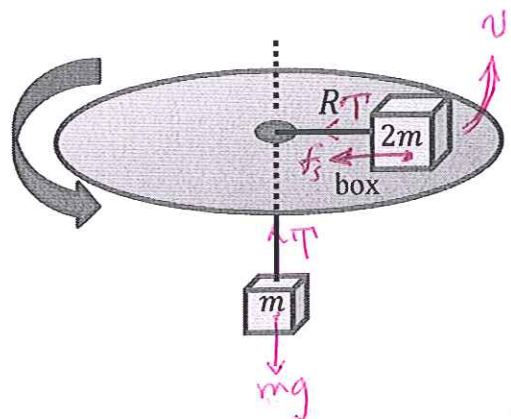
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A circular plate can rotate around an axis about its center as shown in the figure. A box of mass  $2m$  is placed at a distance  $R$  from the center axis and it is connected by a massless rope through a hole at the center of the plate to another object of mass  $m$ . The static friction coefficient between the plate and the box is  $\mu_s = 0.75$ . Assume that the rope is free to move (without friction) through the hole. Determine the maximum linear speed of the box during uniform circular motion so that the box does not slide.



When the box does not slide the hung mass also doesn't move and we have:  $T = mg$ ,

For the box when linear speed goes to maximum, it tends to scape from the center of rotation so the friction force will act toward the center of rotation. The maximum for static friction is:

$$f_{s\max} = \mu_s n = \mu_s 2mg$$

Now in the radial direction the sum of  $T$  and  $f_s$  provides centripetal force:

$$T + f_s = 2ma_r \Rightarrow T + f_{s\max} = 2ma_{r\max} \Rightarrow$$

$$mg + \mu_s 2mg = 2ma_{r\max} \Rightarrow \text{knowing } a_r = \frac{v^2}{R} \Rightarrow a_{r\max} = \frac{v_{\max}^2}{R}$$

$$(2\mu_s + 1)g = 2 \frac{v_{\max}^2}{R} \Rightarrow v_{\max} = \sqrt{\frac{Rg}{2} (2\mu_s + 1)} \approx 3.5\sqrt{R}$$