

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

As seen in figure, **varying horizontal force F** is pushing the object very slowly from $\alpha = 0$ to $\pi/2$ and in a swing, it remains in equilibrium through the process.

- a) Calculate F as a function of angle α ✓
- b) What is the total work done by the force F ✓
- c) What is the total work done by the gravitational force (mg) ✓
- d) What is the total work done by the tension T in the rope

a) applying Newton's Law
 $\sum \vec{F} = m\vec{a}$
 projection on x-axis gives

$$F_x + T \sin \alpha = 0 \quad (1)$$

$$\Rightarrow \left[T = -\frac{F_x}{\sin \alpha} \right] \quad (1')$$

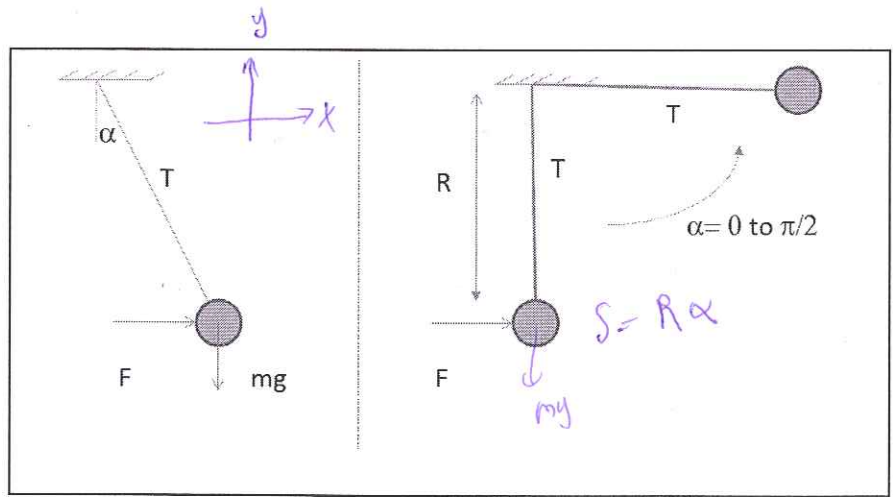
on y-axis: $T \cos \alpha - mg = 0 \quad (2)$
 from (1), (2) when find

$$\boxed{F_x = F = mg \tan \alpha = w \tan \alpha} ; \quad w = mg$$

$$(b) W = \int_0^{\pi/2} \vec{F} \cdot d\vec{l} = \int_0^{\pi/2} F \cos \alpha ds = \int_0^{\pi/2} w \sin \alpha R d\alpha = wR \int_0^{\pi/2} \sin \alpha d\alpha$$

$$= wR (1 - \cos \alpha) \Big|_0^{\pi/2}$$

$$\boxed{W_{(0 \rightarrow \pi/2)} = mg R} \checkmark$$



$$\textcircled{c} W = \int \vec{w} \cdot d\vec{l} = \int (-mg \sin \alpha) R d\alpha = -mgR \int_0^\alpha \sin \alpha d\alpha = -mgR (1 - \cos \alpha)$$

$$W(0 \rightarrow \pi/2) = -mgR \quad (\text{For gravity})$$

$$\textcircled{d} \vec{F} \cdot d\vec{l} = (-T \sin \alpha) (ds \cos \alpha) + (T \cos \alpha) (ds \sin \alpha) = 0$$

there is no obvious work done by tension.

~~guy~~

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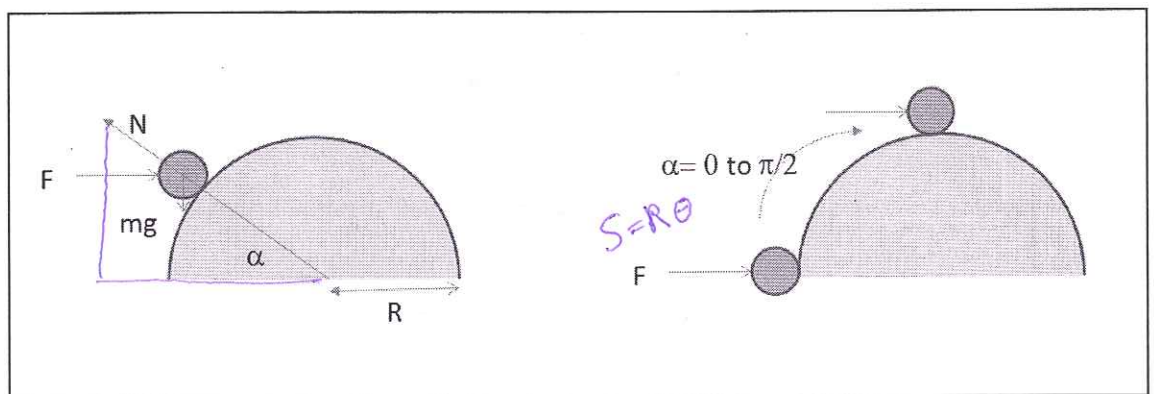
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As seen in figure, varying horizontal force F is pushing the object very slowly from $\alpha = 0$ to $\pi/2$ and it remains in equilibrium through the process.)

- Calculate F as a function of angle α
- What is the total work done by the force F
- What is the total work done by the gravitational force (mg)
- What is the total work done by the normal force N



a) using Newton's Second Law $\sum \vec{F} = m\vec{a} = 0$; $ds = R d\theta$

projection on x-axis $F_x + N \cos \alpha = 0$ (1)

$$N = -\frac{F_x}{\cos \alpha} \quad (1)$$

projection on y-axis : $N \sin \alpha - mg = 0$ (2)

solving (1) and (2) we find

$$F_x = F = mg \cot \alpha$$

$$b) W = \int_0^{\frac{\pi}{2}} \vec{F} \cdot d\vec{l} = \int_0^{\frac{\pi}{2}} mg \cos \alpha R d\alpha = mgR$$

$$c) \text{ Work done by gravity } w = - \int_0^{\frac{\pi}{2}} mg \cos \alpha R d\alpha = -mgR \sin \alpha \Big|_0^{\frac{\pi}{2}} \\ = \underline{-mgR}$$

d) the work done by normal force \vec{N} is zero.

$$\vec{N} \cdot d\vec{l} = (-N \sin \alpha)(R \cos \alpha d\alpha) + (N \cos \alpha) R \sin \alpha d\alpha = \underline{0}$$

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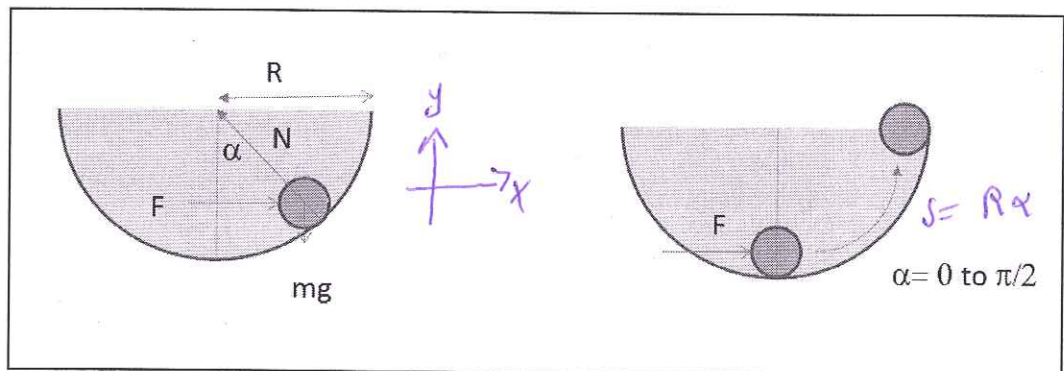
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a) using Newton's second law $\sum \vec{F} = m\vec{a}$

projection on x -axis: $F_x + N \sin \alpha = 0$ (1)

$$N = \frac{-F_x}{\sin \alpha} \quad (1')$$

projection on y -axis: $N \cos \alpha - mg = 0$ (2)

From (1)', (2) we find the force $F_x = mg \tan \alpha$

since $F_y = 0 \Rightarrow$

$$\boxed{F = F_x = mg \tan \alpha}$$

$$(b) W = \int_0^{\alpha} \vec{F} \cdot d\vec{l} = \int_0^{\alpha} F \cos \alpha \, ds = \int_0^{\alpha} mg \sin \alpha \, R \, d\alpha = mgR (1 - \cos \alpha)$$

The Total Work in the interval $[0, \frac{\pi}{2}]$ is $\boxed{W = mgR}$

$$(c) W_{mg} = \int_0^{\frac{\pi}{2}} (mg \sin \alpha) R \, d\alpha = \overset{+}{-} mgR$$

Minus because mg towards the ground and y -axis upward

$$\boxed{ds = R \, d\alpha}$$

since R is constant

$$(d) \vec{N} \cdot d\vec{l} = (-N \sin \alpha) (ds \cos \alpha) + (N \cos \alpha) (ds \sin \alpha) = 0$$

No work is done by the normal force.