PHYS 102:General PhysicsI **KOÇ UNIVERSITY** **Spring Semester 2016**

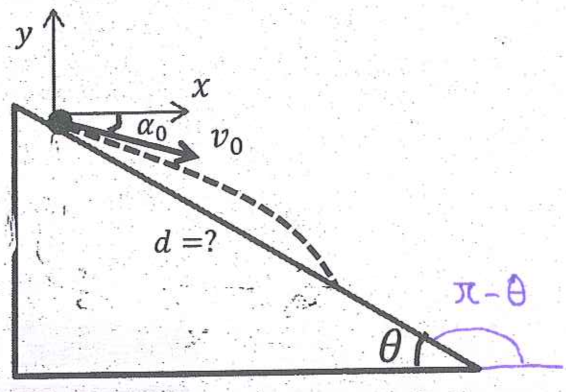
College of Sciences

Section 1 **Quiz 3** **26 February 2016**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A projectile is launched from the origin with initial speed v_0 on an inclined surface. The launch angle with the horizontal is $\alpha_0 = \frac{-\pi}{6}$. The inclination angle of the surface is $\theta = \frac{\pi}{4}$. Determine the distance of the landing point from the launch point in terms of the given parameters. The gravitational acceleration is g. ($\sin\frac{-\pi}{6} = \frac{-1}{2}$, $\cos\frac{-\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$)



Let the starting point of the projectile be the origin and final position of the projectile be (x_f, y_f).

$$x(t) = v_0 \cos\alpha_0 t$$
$$y(t) = v_0 \sin\alpha_0 t - \frac{1}{2} g t^2$$

$$\frac{y_f}{x_f} = \tan(\pi - \theta)$$
$$= -\tan\theta = -\tan\frac{\pi}{4} = -1$$

$$\frac{v_0 \sin\alpha_0 t - \frac{1}{2} g t^2}{v_0 \cos\alpha_0 t} = -1$$

$$v_0 \sin\alpha_0 t - \frac{1}{2} g t^2 = -v_0 \cos\alpha_0 t$$

$$v_0(\sin\alpha_0 + \cos\alpha_0) = \frac{1}{2} g t$$

$$t = \frac{2 v_0 (\sin\alpha_0 + \cos\alpha_0)}{g}$$

$$= \frac{2 v_0 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right)}{g} = \frac{v_0}{g}(\sqrt{3} - 1)$$

$$x_f = v_0 \cos\alpha_0 \frac{v_0}{g}(\sqrt{3} - 1)$$
$$= \frac{v_0^2}{2g}(3 - \sqrt{3})$$

$$y_f = -x_f = -\frac{v_0^2}{2g}(3 - \sqrt{3})$$

$$d = \sqrt{x_f^2 + y_f^2} = \sqrt{x_f^2 + (-x_f)^2} = x_f\sqrt{2}$$
$$= \frac{v_0^2}{g}\,\frac{3\sqrt{2} - \sqrt{6}}{2}$$

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Section 2 | Quiz 3 | 26 February 2016

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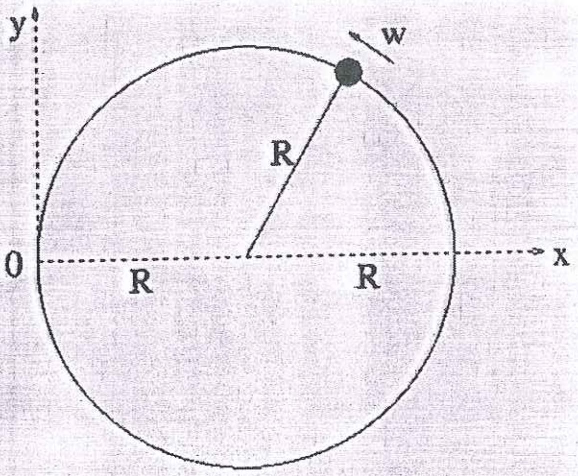
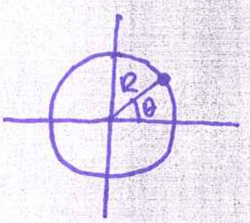
Name: **Student ID:** **Signature:**

Consider the uniform circular motion shown in the figure, where an object is rotating along the circular track with a constant angular frequency $w = 2\pi/T$. Here T is the period of the motion. Using the x-y coordinate system indicated in the figure, find the position and acceleration of the object as a function of time t, assuming the object passes $(x = 0, y = 0)$ point at time $t = 0s$.



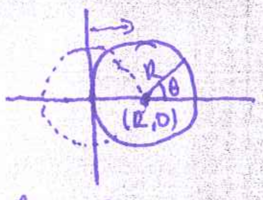


For a circle located at the origin, x-y coordinates of an object located at the angle θ would be:

$x = R\cos\theta$

$y = R\sin\theta$

The center of our motion is at $(R,0)$, so we should displace our coordinates by $(R,0)$.



$x = R + R\cos\theta$

$y = 0 + R\sin\theta$

• For the motion given,

$\theta = wt + \phi$

where ϕ is a constant phase shift. At time $t=0$:

$x(t=0) = R + R\cos(\phi) = 0$

$\cos(\phi) = -1 \Rightarrow \phi = \pi$

$x(t) = R + R\cos(wt+\pi)$

$v_x(t) = \frac{d}{dt}x(t) = -Rw\sin(wt+\pi)$

$a_x(t) = \frac{d}{dt}v_x(t) = -Rw^2\cos(wt+\pi)$

$y(t) = R\sin(wt+\pi)$

$v_y(t) = +Rw\cos(wt+\pi)$

$\cancel{a_y(t) = -Rw^2}$

$a_y(t) = \frac{d}{dt}v_y(t) = -Rw^2\sin(wt+\pi)$

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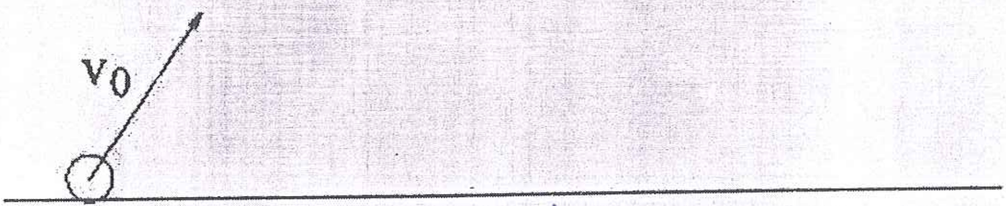
Name: **Student ID:** **Signature:**

Consider the motion shown in the figure, where a ball starts moving at time $t = 0$ with speed v_0 and making an angle of $\pi/4$ radians with the horizontal axis. If the horizontal and vertical accelerations of the ball are given by $a_x = A$ and $a_y = -B$, respectively, find the maximum displacements in the vertical and horizontal directions. Here A and B are positive constants. You may ignore the effects of gravity.



$x(t) = V_0 \cos(\pi/4)t + \frac{1}{2}At^2$

$y(t) = V_0 \sin(\pi/4)t - \frac{1}{2}Bt^2$

V_y should be zero when the ball is at maximum height.

$V_0 \sin(\pi/4) - Bt = 0$

$t = \frac{V_0 \sin(\pi/4)}{B} = \frac{V_0\sqrt{2}}{2B}$

$y_{max} = \frac{V_0\sqrt{2}}{2} \cdot \frac{V_0\sqrt{2}}{2B} - \frac{1}{2}B\left(\frac{V_0\sqrt{2}}{2B}\right)^2$

$= \frac{V_0^2}{2B} - \frac{1}{2}B\frac{V_0^2}{2B^2}$

$= \frac{V_0^2}{2B} - \frac{V_0^2}{4B} = \frac{V_0^2}{4B}$

is the maximum vertical displacement

Since both initial velocity and acceleration are positive in x direction, the balls displacement is maximum when it hits the ground. $(y=0)$

$y(t) = V_0 \sin(\pi/4)t - \frac{1}{2}Bt^2 = 0$

$V_0 \sin(\pi/4) = \frac{1}{2}Bt$

$t = \frac{2V_0 \sin(\pi/4)}{B} = \frac{V_0\sqrt{2}}{B}$

This is twice the time it takes for the ball to reach maximum height as expected.

$x_{max} = \frac{V_0\sqrt{2}}{2} \frac{V_0\sqrt{2}}{B} + \frac{1}{2}A\left(\frac{V_0\sqrt{2}}{B}\right)^2$

$= \frac{V_0^2}{B} + \frac{1}{2}A\frac{2V_0^2}{B^2}$

$= \frac{BV_0^2}{B^2} + \frac{AV_0^2}{B^2} = V_0^2\frac{A}{B^2}$ is the maximum horizontal displacement.