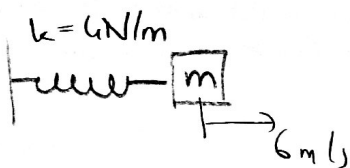


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A body of mass $m = 1$ kg is attached to a wall by a spring of constant $k = 4$ N/m. It is given an initial velocity, at $x = 0$ cm (its equilibrium position), of 6 m/s. Plot the time evolution of the displacement $x(t)$, velocity $v(t)$ and acceleration $a(t)$ for one period.



to be determined

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -A\omega \sin(\omega t + \phi)$$

$$a(t) = -A\omega^2 \cos(\omega t + \phi)$$

$$x(0) = 0 \quad \phi = -\frac{\pi}{2}$$

$$v(0) = 6 \text{ m/s} = -A\omega \sin\left(-\frac{\pi}{2}\right) = A\omega$$

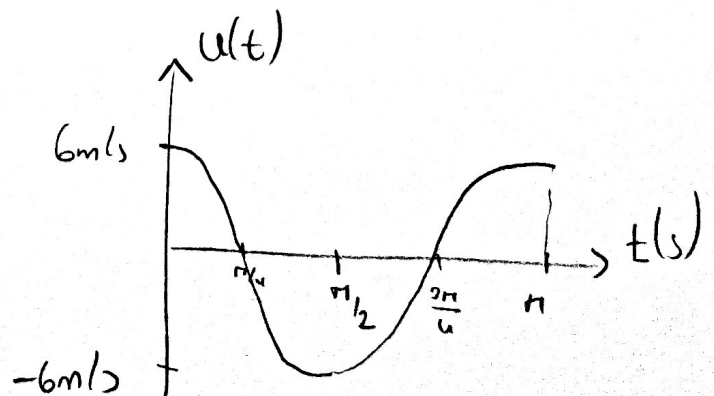
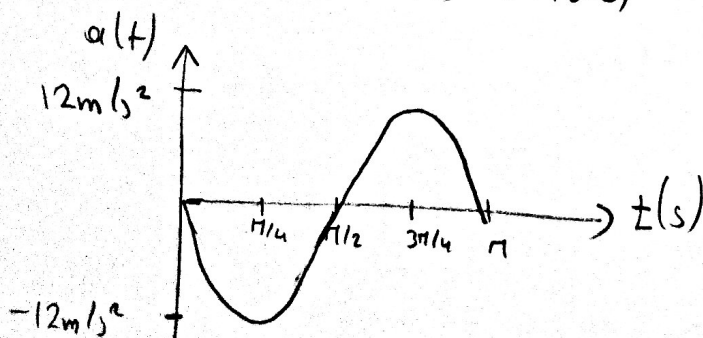
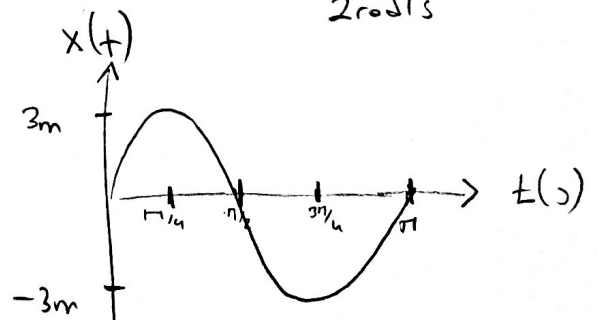
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4 \text{ N/m}}{1 \text{ kg}}} = 2 \text{ rad/s} \quad A = \frac{6 \text{ m/s}}{2 \text{ rad/s}} = 3 \text{ m}$$

$$T = 2\pi/\omega = \pi \text{ seconds}$$

$$x(t) = 3 \text{ m} \sin(2t)$$

$$v(t) = 6 \text{ m/s} \cos(2t)$$

$$a(t) = -12 \text{ m/s}^2 \sin(2t)$$



Closed book. Duration: 10 minutes

Name: Ongun ARISEV Student ID:

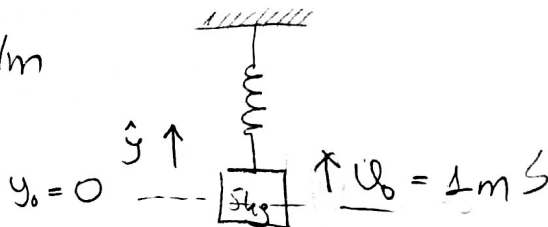
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A spring stretches 10 cm when its tension is 50 N. A body of mass 5 kg is hung from the spring. When at rest the body is given an initial upward velocity of 1 m/s. (Take $g=10\text{m/s}^2$) (i) Find the amplitude and the frequency of the motion. (ii) Find the acceleration of the mass when it is 5 cm above its equilibrium position. (iii) Find the force of tension in the spring at this point.

$$k = \frac{50\text{N}}{10\text{cm}} = 500\text{N/m}$$

As always



$$y(t) = A \cos(\omega t + \phi)$$

$$v(t) = -A\omega \sin(\omega t + \phi) \Rightarrow \phi = -\frac{\pi}{2} \text{ (initial vel. is positive)} \Rightarrow v(t) = A\omega \cos(\omega t)$$

$$a(t) = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 y(t)$$

$$(i) \quad \omega = \sqrt{\frac{k}{m}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{500\text{N/m}}{5\text{kg}}} = \frac{5}{\pi} \text{ Hz}$$

$$A\omega = 1\text{ m/s}$$

$$\omega = 10 \text{ rad/s} \Rightarrow A = 0.1\text{ m}$$

$$(ii) \quad a(5\text{cm}) = -\omega^2 5\text{cm} = -100 \frac{\text{rad}^2}{\text{s}^2} \cdot 0.05\text{m} = -5\text{m/s}^2$$

$$(iii) \quad T_{\text{spring}} - mg = ma \Rightarrow T_{\text{spring}} - 50\text{N} = -25\text{N}$$

$$T_{\text{spring}} = 25\text{N}$$

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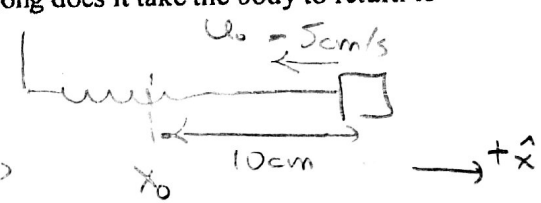
A body in simple harmonic motion with angular frequency $\omega = 0.5 \text{ rad/s}$ is initially 10 cm away from its equilibrium position and moving back toward the equilibrium position with a velocity 5 cm/s. (i) Find the period of the motion. (ii) Find the coordinate and velocity of the body as a function of time. (iii) How long does it take the body to return to its equilibrium position?

$$x(t) = A \cos(\omega t + \phi) \quad x(0) = 10 \text{ cm}$$

$$v(t) = -A\omega \sin(\omega t + \phi) \quad v(0) = -5 \text{ cm/s}$$

$$a(t) = -A\omega^2 \cos(\omega t + \phi)$$

i) $\omega = 0.5 \text{ rad/s} \quad T = \frac{2\pi}{\omega} = 4\pi \text{ seconds}$



$$+10 \text{ cm} = A \cos(\phi)$$

$$-5 \text{ cm/s} = -A\omega \sin(\phi)$$

$$10 \text{ cm} = A \sin(\phi)$$

$$\tan(\phi) = 1 \quad \phi = \frac{\pi}{4}$$

$$10 \text{ cm} = A \frac{\sqrt{2}}{2} \Rightarrow A = 10\sqrt{2} \text{ cm}$$

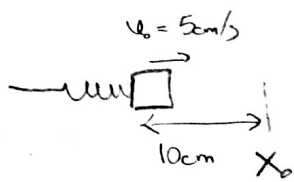
(ii) $x(t) = (10\sqrt{2} \text{ cm}) \cos(\omega t + \frac{\pi}{4}) \quad \omega = 0.5 \text{ rad/s}$

$$v(t) = (-5\sqrt{2} \text{ cm/s}) \sin(\omega t + \frac{\pi}{4})$$

(iii) $x(t_{\text{return}}) = 0 = (10\sqrt{2} \text{ cm}) \cos(\omega t_{\text{return}} + \frac{\pi}{4})$

$$\omega t_{\text{return}} = \frac{\pi}{4} \Rightarrow t_{\text{return}} = \frac{\pi}{2} \text{ seconds} \Rightarrow$$

Alternative solution



$$-10 \text{ cm} = A \cos(\phi)$$

$$+5 \text{ cm/s} = -A\omega \sin(\phi)$$

$$(i) \quad T = \frac{2\pi}{\omega} = 4\pi \text{ seconds}$$

$$-10 \text{ cm} = A \cos(\phi)$$

$$-10 \text{ cm} = A \sin(\phi) \quad \tan(\phi) = 1$$

$$\phi = \frac{\pi}{4} + k\pi \quad \phi = \frac{5\pi}{4} = 225^\circ$$

k is integer

Should lie in the third quadrant

since $x < 0, y < 0$

$$-10 \text{ cm} = A \cos\left(\frac{5\pi}{4}\right) \Rightarrow A = 10\sqrt{2}$$

(ii)

$$x(t) = 10\sqrt{2} \text{ cm} \cos\left(\omega t + \frac{5\pi}{4}\right)$$

$$v(t) = -5\sqrt{2} \text{ cm/s} \sin\left(\omega t + \frac{5\pi}{4}\right)$$

(iii)

$$x(t_{\text{return}}) = 0 = \cos\left(\omega t_{\text{return}} + \frac{5\pi}{4}\right)$$

$$\omega t_{\text{return}} + \frac{5\pi}{4} = \frac{3\pi}{2}$$

$$\omega t_{\text{return}} = \frac{\pi}{4}$$

$$t_{\text{return}} = \frac{\pi}{2}$$