

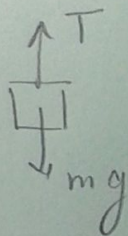
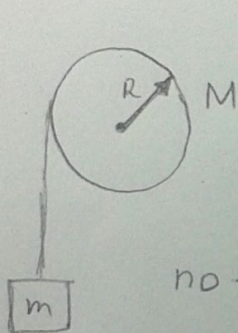
Closed book. Duration: 10 minutes

Name:

Student ID:

Signature:

We wrap a light, nonstretching cable around a solid cylinder with mass M and radius R . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. What are the acceleration of the falling block and the tension in the cable?



$$\Rightarrow mg - T = ma \Rightarrow T = m(g - a) //$$

$$T \cdot R = I \alpha$$

no-slip condition: $\alpha = \frac{a}{R}$; $I = \frac{1}{2} MR^2$

$$\hookrightarrow T \cdot R = \frac{1}{2} MR^2 \frac{a}{R} \Rightarrow T = \frac{Ma}{2} //$$

$$\hookrightarrow T = m(g - a) = \frac{Ma}{2} \Rightarrow 2mg - 2ma = Ma$$

$$\hookrightarrow a = \frac{2mg}{2m + M} = \frac{1}{1 + \frac{M}{2m}} g //$$

$$T = \frac{Ma}{2} = \frac{M}{2} \cdot \frac{g}{1 + \frac{M}{2m}} = \frac{Mg}{2 + \frac{M}{m}} //$$

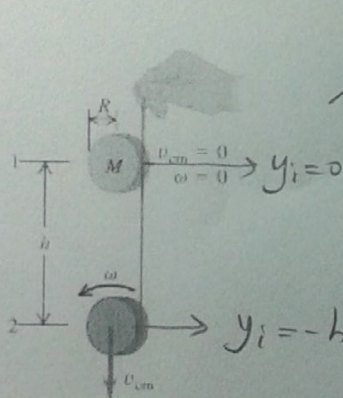
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You make a primitive yo-yo by wrapping a massless string around a solid cylinder with mass M and radius R as shown in the figure. You hold the free end of the string stationary and release the cylinder from rest. The string unwinds but does not slip or stretch as the cylinder descends and rotates. Using energy considerations, find the speed v_{cm} of the center of mass of the cylinder after it has descended a distance h .



$$\overset{=0}{K_i} + \overset{=0}{U_i} = K_f + U_f$$

$$\left(\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2 \right) + m g y_f = 0$$

where $y_f = -h$ and $\omega = \frac{v_f}{R}$ from

$$I = \frac{1}{2} m R^2$$

no-slip condition

$$\Rightarrow \frac{1}{2} m v_f^2 + \frac{1}{2} \left(\frac{1}{2} m R^2 \right) \left(\frac{v_f}{R} \right)^2 = m g h$$

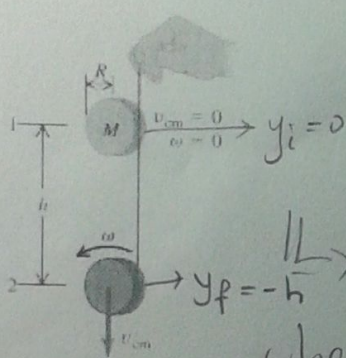
$$\Rightarrow \frac{3}{4} m v_f^2 = m g h \Rightarrow v_f = \sqrt{\frac{4 g h}{3}} //$$

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You make a primitive yo-yo by wrapping a massless string around a solid cylinder with mass M and radius R as shown in the figure. You hold the free end of the string stationary and release the cylinder from rest. The string unwinds but does not slip or stretch as the cylinder descends and rotates. Find the downward acceleration of the cylinder and the tension in the string.



$$I \alpha = T \cdot R \quad ; \quad \alpha R = a$$

$$K_i + U_i = K_f + U_f$$

$$\left(\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2 \right) + m g y_f = 0$$

where $y_f = -h$ and $\omega = \frac{v_f}{R}$ from no-slip condition

$$I = \frac{1}{2} m R^2$$

no-slip condition

$$\frac{1}{2} m v_f^2 + \frac{1}{2} \left(\frac{1}{2} m R^2 \right) \left(\frac{v_f}{R} \right)^2 = m g h$$

$$\frac{3}{4} m v_f^2 = m g h \Rightarrow v_f = \sqrt{\frac{4 g h}{3}}$$

$$v_f^2 - v_i^2 = 2 a h \Rightarrow a = \frac{v_f^2}{2 h} = \frac{2 g h}{3} \cdot \frac{1}{2 h} = \frac{2 g}{3}$$

$$T = \frac{I \alpha}{R} = \frac{\frac{1}{2} m R^2 \frac{a}{R}}{R} = \frac{1}{2} m a = \frac{1}{2} m \left(\frac{2 g}{3} \right) = \frac{m g}{3}$$