

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

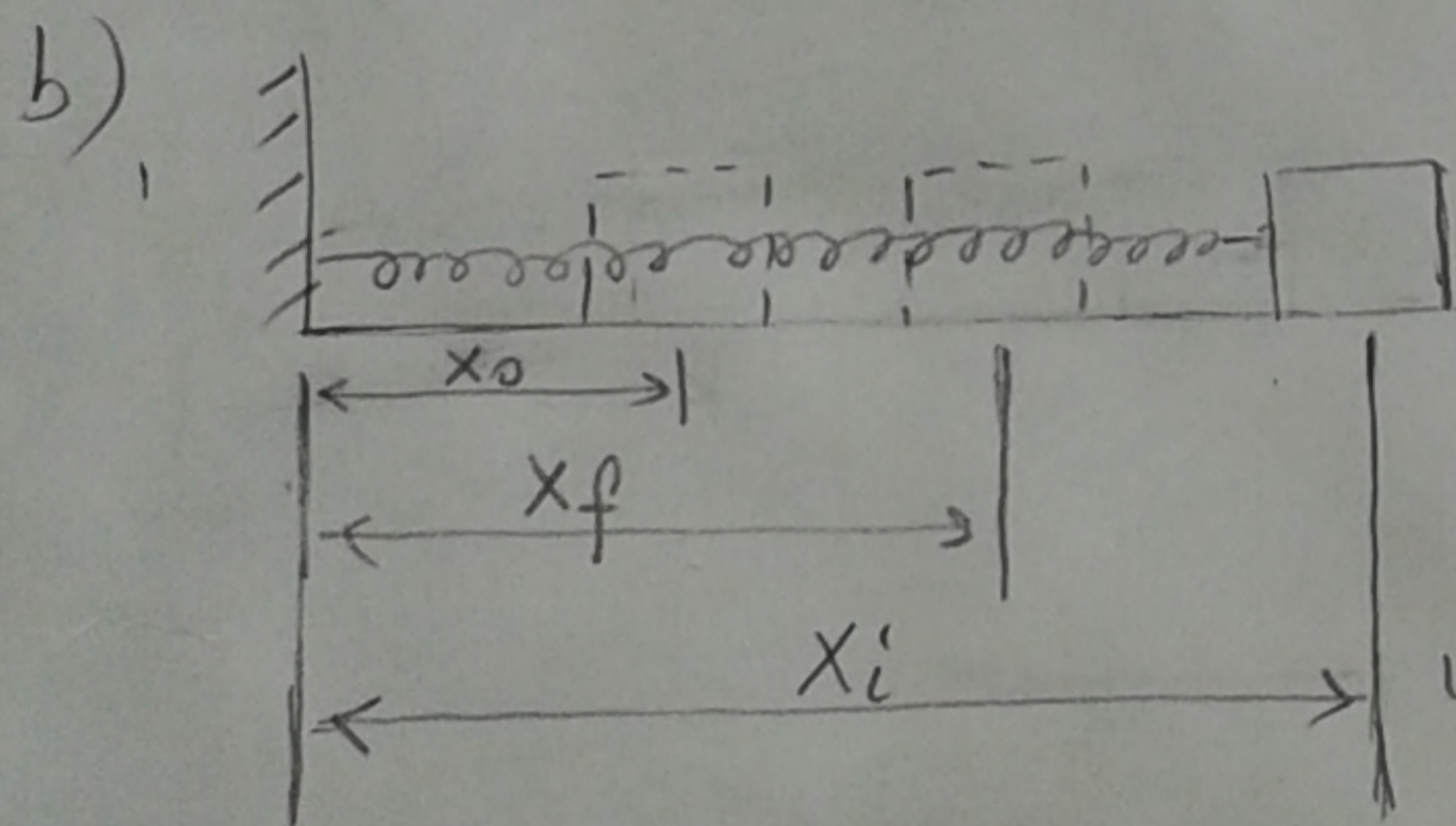
A certain spring is found not to obey Hooke's Law; it exerts a restoring force

$F_x(x) = -\alpha x - \beta x^2$ if it is stretched or compressed, where $\alpha = 60 \text{ N/m}$ and $\beta = 18 \text{ N/m}^2$. The mass of the spring is negligible.

- a) Calculate the potential-energy function $U(x)$ for this spring. Let $U=0$ when $x=0$.
 b) An object with mass 9 kg on a frictionless, horizontal surface is attached to this spring, pulled a distance 2 m to the right (the $+x$ -direction) to stretch the spring, and released. What is the speed of the object when it is 1 m to the right of the $x=0$ equilibrium position?

$$a) U(x) = - \int_0^x F_x(x) dx = \int_0^x [\alpha x + \beta x^2] dx = \frac{\alpha x^2}{2} + \frac{\beta x^3}{3} //$$

where $U(x=0) = 0$



$$\Delta x_i = x_i - x_o = 2 \text{ m} \Rightarrow v_i = 0$$

$$\Delta x_f = x_f - x_o = 1 \text{ m} \Rightarrow v_f \neq 0$$

In general,

$$K_i + U_i + W_{\text{others}} = K_f + U_f$$

where $W_{\text{others}} = 0$ (no friction)

$$\Rightarrow \cancel{K_i} + U_i = K_f + U_f \text{ and } U_i = \left[\frac{\alpha x^2}{2} + \frac{\beta x^3}{3} \right]_{x=2 \text{ m}}$$

$$\Rightarrow U_i(x=2 \text{ m}) = K_f + U_f(x=1 \text{ m})$$

$$\frac{60 \cdot 2^2}{2} + \frac{18 \cdot 2^3}{3} = K_f + \left(\frac{60 \cdot 1^2}{2} + \frac{18 \cdot 1^3}{3} \right) \Rightarrow K_f = 168 - 78 = 90 \text{ J}$$

$$\Rightarrow v_f = \sqrt{\frac{2 K_f}{m}} = \sqrt{\frac{2 \cdot 90}{9}} = \boxed{\sqrt{20} \text{ m/s}} //$$

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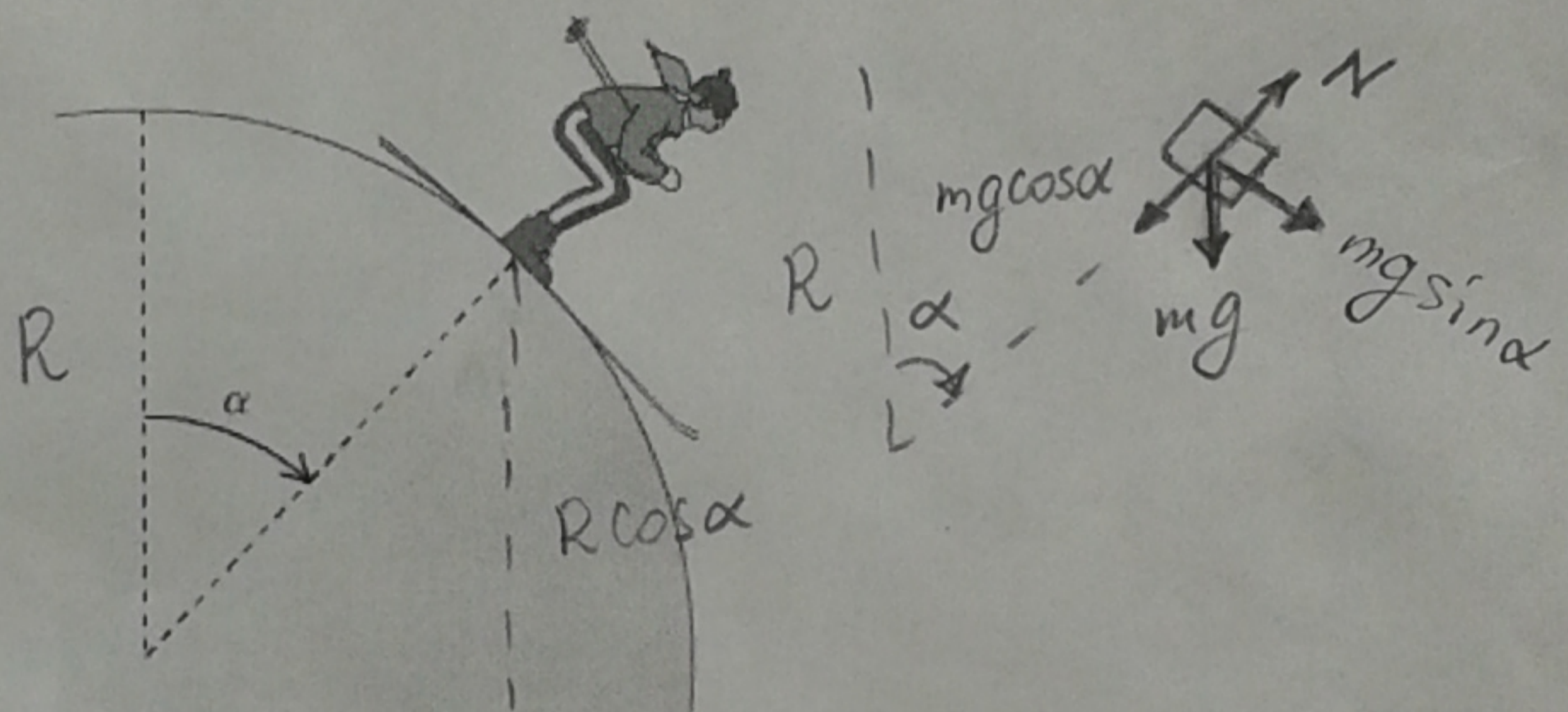
A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side as shown in the figure below. At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle α does a radial line from the center of the snowball to the skier make with vertical?

Lost contact:

$$mg \cos \alpha - N = ma_r = 0$$

$$\text{where } a_r = \frac{v^2}{R}$$

$$\Rightarrow mg \cos \alpha = \frac{mv^2}{R}$$



From the conservation of energy:

$$mgR = mgR \cos \alpha + \frac{mv^2}{2} \Rightarrow v = \sqrt{2gR(1 - \cos \alpha)}$$

$$\Rightarrow \frac{mg \cos \alpha}{R} = \frac{2gR(1 - \cos \alpha)}{2R} \Rightarrow \cos \alpha = 2 - 2 \cos \alpha$$

$$\Rightarrow \cos \alpha = \frac{2}{3} \Rightarrow \alpha = \cos^{-1}\left(\frac{2}{3}\right)$$

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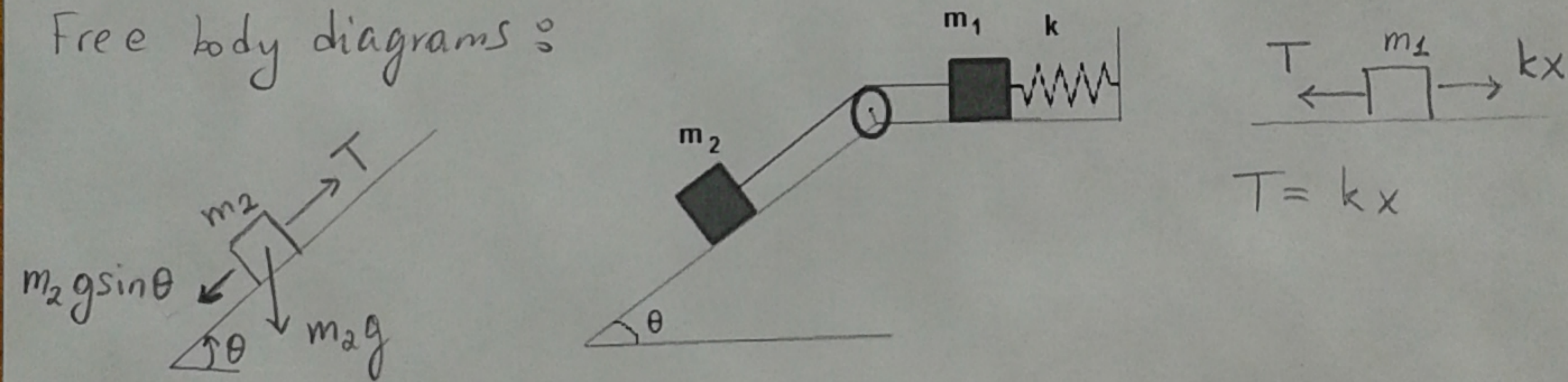
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In the figure below, the block of mass m_1 is connected by a spring to a wall on the right side and to block of mass m_2 by a string through a pulley on the left side. The block m_2 is suspended on an inclined plane. The string is always tight. The gravitational acceleration is g . The whole system is frictionless. Answer the following questions. Express your answer only in terms of g , k and θ .

Suppose that the system is released from the position, when the spring is unstretched/uncompressed and masses are not moving. What will be the maximum potential energy stored in the spring during the first forward motion?

Free body diagrams:



$$m_2 g \sin \theta = T \Rightarrow kx = m_2 g \sin \theta \Rightarrow x = \frac{m_2 g \sin \theta}{k}$$

$$U_{\text{spring}} = \frac{1}{2} kx^2 = \frac{1}{2} k \left(\frac{m_2 g \sin \theta}{k} \right)^2 = \frac{m_2^2 g^2 \sin^2 \theta}{2k} \cdot k =$$

$$= \boxed{\frac{m_2^2 g^2 \sin^2 \theta}{2k}}$$