PHYS 101: General Physics 1 **KOÇ UNIVERSITY** **Spring Semester 2016**

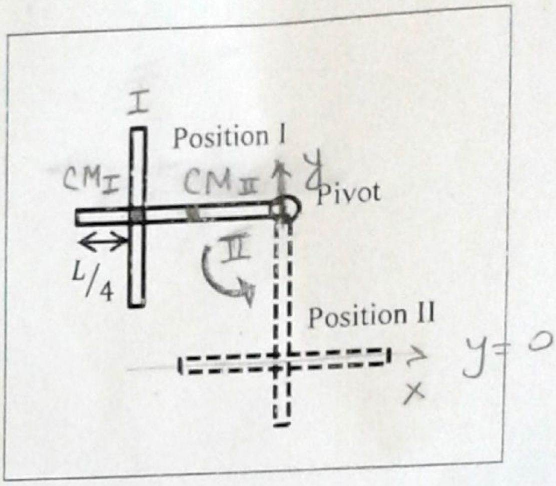
College of Arts and Sciences

Section 1 **Quiz 9** **April 8, 2016**

Closed book. Duration:10 minutes

Name: **Student ID:** **Signature:**

A rigid body is formed by two identical uniform rods each of mass m, length L, where one rod is attached from its center to the other rod at a distance $L/4$ from its end as shown in the figure. The rigid body can rotate freely about a pivot at the end of one rod. The rigid body is released from position (I). Take gravitational acceleration as (g).



a) The moment of inertia of the rigid body about the pivot axis can be written as $I = \beta mL^2$, where β is a constant. Find β. (Hint: The moment of inertia of a uniform rod of mass m and length L about *an axis through its center of mass* is $I_{cm} = \frac{mL^2}{12}$. Also recall the parallel axis theorem)

b) Using conservation of mechanical energy, determine the angular speed of the rigid body when it is in position II. (Note: If you did not solve part (a), you can still calculate part (b) by taking the moment of inertia of the rigid body as $I_p = \beta mL^2$.

c) Is the angular acceleration of the rigid body during the motion between position (I) and (II) constant? Explain.

a) $I_I = I_{CM_I} + m d_{CM_I - piv.} \Rightarrow I_I = \frac{mL^2}{12} + m\left(\frac{3L}{4}\right)^2 = \frac{31 mL^2}{48}$

$I_{II} = I_{CM_{II}} + m d_{CM_{II} - piv.} \Rightarrow I_{II} = \frac{mL^2}{2} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3}$

$\Rightarrow I_{tot.-piv.} = \frac{31 mL^2}{48} + \frac{mL^2}{3} = \frac{47 mL^2}{48} = \beta mL^2$

$\Rightarrow \beta = 47/48$

b) $U_i + \cancel{K_i}^{=0} + \cancel{W_{others}}^{=0} = U_f + K_f$

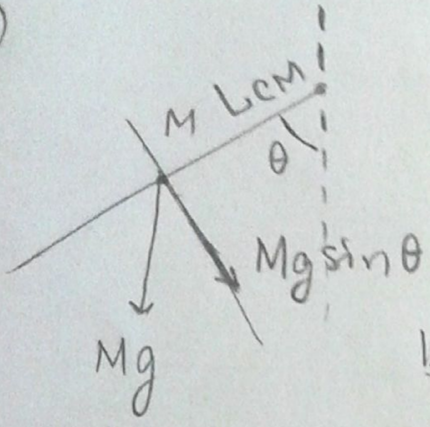
$U_i = U_{i,I} + U_{i,II} = mg \cdot \left(\frac{3L}{4}\right) + mg\left(\frac{3L}{4}\right) = 3mgL/2$

$U_f = \cancel{U_{f,I}}^{=0} + U_{f,II} = mg\left(\frac{3L}{4} - \frac{L}{2}\right) = \frac{mgL}{4}$

$\Rightarrow U_i = U_f + K_f \Rightarrow \frac{3mgL}{\cancel{2}} - \frac{mgL}{2\cancel{4}} = \frac{I_{tot.-piv.}\,\omega^2}{\cancel{2}}$

$$\Rightarrow \frac{5\cancel{m}g\cancel{L}}{\cancel{2}} = \frac{47\cancel{m}L^{\cancel{2}}}{\cancel{48}\,24}\omega^2 \Rightarrow \omega = \sqrt{\frac{120g}{47L}}\ \frac{rad}{s}\ //$$

c)



where $M = 2m$ and $\sum_i \tau_i = I\alpha$

$$\Downarrow\ Mg\sin\theta \cdot L_{CM} = I\alpha$$

$$L_{CM} = \left(\frac{L}{2} + \frac{3L}{4}\right)/2 = \frac{5L}{8}$$

$$\Downarrow\ \cancel{2}\cancel{m}g\sin\theta \cdot \frac{5\cancel{L}}{\cancel{8}\,4} = \frac{47\cancel{m}L^{\cancel{2}}}{\cancel{48}\,12}\alpha$$

$$\Downarrow\ \alpha = \frac{60\,g\sin\theta}{47L}\ rad/s^2\ //$$

α depends on "θ", not const.

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Section 2 **Quiz 9** **April 8, 2016**

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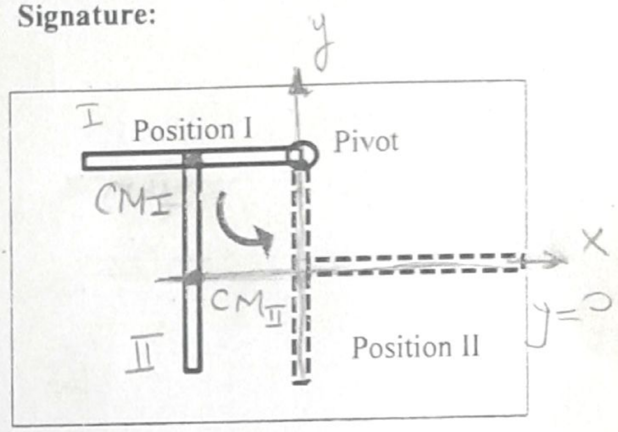
A "T" shape rigid body is formed by two identical uniform rods each of mass m, length L as shown in the figure. The rigid body can rotate freely about a pivot at the end of one rod. The rigid body is released from position (I). Take gravitational acceleration as (g).



a) The moment of inertia of the rigid body about the pivot axis can be written as $I = \beta mL^2$, where β is a constant. Find β. (Hint: The moment of inertia of a uniform rod of mass m and length L about *an axis through its center of mass* is $I_{cm} = \frac{mL^2}{12}$. Also recall the parallel axis theorem)

b) Using conservation of mechanical energy, determine the angular speed of the rigid body when it is in position II. (Note: If you did not solve part (a), you can still calculate part (b) by taking the moment of inertia of the rigid body as $I_p = \beta mL^2$.

c) Is the angular acceleration of the rigid body during the motion between position (I) and (II) constant? Explain.

a) $I_I = I_{CM_I} + m d^2_{CM_I - piv.} \Rightarrow I_I = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3}$ "

$I_{II} = I_{CM_{II}} + m d^2_{CM_{II} - piv} \Rightarrow I_{II} = \frac{mL^2}{12} + m\left[\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2\right] =$

$= \frac{7mL^2}{12}$ "

$I_{tot.-piv} = I_I + I_{II} = \frac{mL^2}{3} + \frac{7mL^2}{12} = \frac{11\, mL^2}{12} = \beta m L^2$

$\Rightarrow \beta = \frac{11}{12}$

(no non-conservative force)

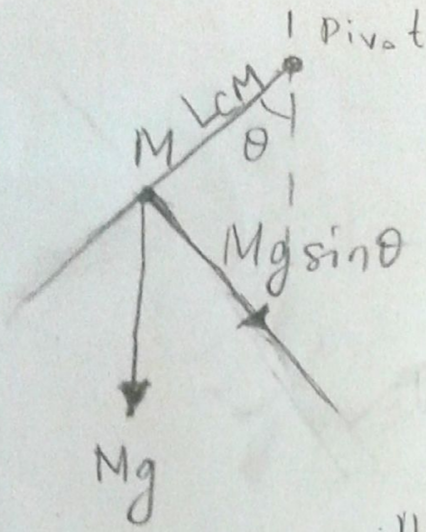
b) Conservation of Energy: $U_i + \underbrace{\cancel{K_i}}_{=0} + \underbrace{\cancel{W_{others}}}_{=0} = U_f + K_f$

$U_i = U_{i,I} + U_{i,II} \Rightarrow U_{i,I} = mgL/2 \,,\; U_{i,II} = 0$

$U_f = \underbrace{\cancel{U_{f,I}}}_{=0} + \underbrace{\cancel{U_{f,II}}}_{=0} = 0$ and $K_f = \frac{I_{piv}\,\omega^2}{2}$

$\Rightarrow \frac{mgL}{\cancel{2}} = \frac{I_{piv}\,\omega^2}{\cancel{2}} \Rightarrow \cancel{m}g\cancel{L} = \frac{7\cancel{m}L^{\cancel{2}}}{12}\omega^2 \Rightarrow \omega = \sqrt{\frac{12g}{7L}}\ \frac{rad}{s}$

c)



where $M = m_I + m_{II} = 2m$

$$\sum_i \tau_i = I \cdot \alpha$$

$\Rightarrow \; Mg \sin\theta \cdot L_{CM} = I\alpha$

$$L_{CM} = \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{4}\right)^2} = \frac{\sqrt{5}}{4} L \;//$$

$\Rightarrow \; 2\cancel{m} g \sin\theta \cdot \frac{\sqrt{5}}{\cancel{4}} \cancel{L} = \frac{11}{\cancel{12}_3} \cancel{m} L^{\cancel{2}} \alpha$

$\Rightarrow \; \alpha = \dfrac{6\sqrt{5}\, g \sin\theta}{11 L}$ rad/s² //

α depends on "θ", not constant.

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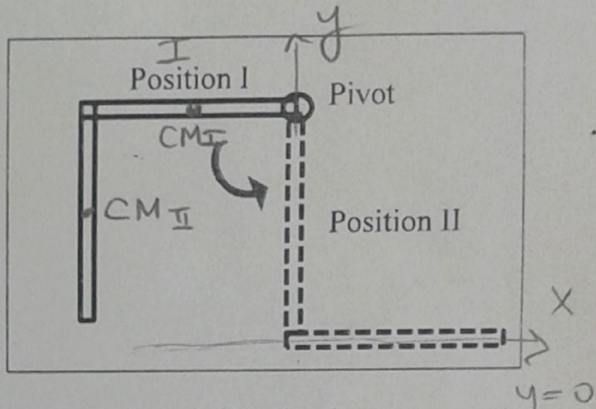


A rigid body is formed by two identical uniform rods each of mass m, length L which are attached at one end perpendicular to each other as shown in the figure. The rigid body can rotate freely about a pivot at the end of one rod. The rigid body is released from position (I). Take gravitational acceleration as (g).

a) The moment of inertia of the rigid body about the pivot axis can be written as $I = \beta m L^2$, where β is a constant. Find β. (Hint: The moment of inertia of a uniform rod of mass m and length L about *an axis through its center of mass is* $I_{cm} = \frac{mL^2}{12}$. Also recall the parallel axis theorem)

b) Using conservation of mechanical energy, determine the angular speed of the rigid body when it is in position II. (Note: If you did not solve part (a), you can still calculate part (b) by taking the moment of inertia of the rigid body as $I_p = \beta m L^2$.

c) Is the angular acceleration of the rigid body during the motion between position (I) and (II) constant? Explain.

a) $I_{I} = I_{CM_I} + m d^2_{CM_I - piv} \Rightarrow I_I = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3}$

$I_{II} = I_{CM_{II}} + m d_{CM_{II} - piv} \Rightarrow I_{II} = \frac{mL^2}{12} + m\left[L^2 + \left(\frac{L}{2}\right)^2\right] = \frac{mL^2}{12} + \frac{5mL^2}{4} = \frac{16mL^2}{12}$

$I_{tot.-piv.} = \frac{\cancel{16}^{4} mL^2}{\cancel{12}_{3}} + \frac{mL^2}{3} = \frac{5mL^2}{3}$

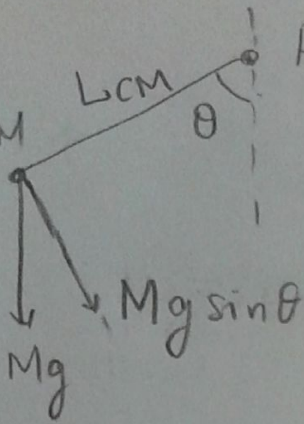
$I_{tot.-piv.} = \beta m L^2 \Rightarrow \beta = 5/3$ //

b) $U_i + \cancel{K_i}^{=0} + \cancel{W_{others}}^{=0} = U_f + K_f \Rightarrow K_f = U_i - U_f$

$$\left.\begin{array}{l} U_i = mgL + mgL = 2mgL \\ U_f = mg\frac{L}{2} + 0 = \frac{mgL}{2} \end{array}\right\} \Rightarrow \frac{I_{tot.}\omega^2}{\cancel{2}} = \frac{3mgL}{\cancel{2}}$$

$\frac{5\cancel{m}L^{\cancel{2}}}{3}\omega^2 = 3\cancel{m}g\cancel{L} \Rightarrow \omega = \frac{3}{\sqrt{5}}\sqrt{\frac{g}{L}}$ rad/s //

c)



Piv. where $M = 2m$ and $\sum_i \tau_i = I\alpha$

$\Rightarrow Mg\sin\theta \cdot L_{CM} = I\alpha$

$$L_{CM} = \sqrt{\left(\left(L + \frac{L}{2}\right)/2\right)^2 + \left(\frac{L}{2}\right)^2} =$$

$$= \sqrt{\left(\frac{3L}{4}\right)^2 + \left(\frac{L}{2}\right)^2} = \frac{\sqrt{13}\,L}{4} \text{ "}$$

$$\Rightarrow \cancel{2}\cancel{m}g\sin\theta \cdot \left(\frac{\sqrt{13}\,\cancel{L}}{\cancel{4}\;2}\right) = \frac{5\cancel{m}L^{\cancel{2}}}{3}\cdot\alpha \Rightarrow \alpha = \frac{3\sqrt{13}\,g\sin\theta}{10L}\ \frac{rad}{s^2}$$