Spring 2019

Name: «Name_»	Signature:
Surname: «Surname_»	Student ID Number: «Student_ID»

Solutions

PHYS 101 General Physics I – Midterm 2

April 18, 2019 Friday 19:00 -21:00

Please read!

- Count to make sure that there are 5 pages in the question booklet
- Check your name and surname on front page, and student ID number on each page, and sign each page.
- This examination is conducted with closed books and notes.
- Put all your personal belongings underneath your seat and make sure that pages of books or notebooks are not open.
- Absolutely no talking or exchanging anything (like rulers, erasers) during the exam.
- You must show all your work to get credit; you will not be given any points unless you show the details of your work (this applies even if your final answer is correct!).
- Write neatly and clearly; unreadable answers will not be given any credit.
- If you need more writing space, use the backs of the question pages and put down the appropriate pointer marks.
- Make sure that you include units in your results.
- Make sure that you label the axis and have units in your plots.
- You are not allowed to use calculators during this exam.

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1	2	3	4	5	Total

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1) a) (10 Points) A 3.00-kg block is connected to two ideal horizontal springs having force constants $k_1 = 30\ N/cm$ and $k_1 = 10\ N/cm$. The system is initially in equilibrium on a horizontal, frictionless surface. The block is now pushed 10.0 cm to the right and released from rest. (a) What is the maximum speed of the block? Where in the motion does the maximum speed occur? (b) What is the maximum compression of spring 1?

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$W_{other} = 0$ & $K_1 = 0$ (rest)

$k_1 = 30\ N/cm = 3000\ N/m$
$k_2 = 1000\ N/m$
$x_1 = -x_2 = 10\,cm = 0.1\,m$

a) $U_1 = K_2 + U_2$ for the maximum velocity $\longrightarrow U_2 = 0$

$U_1 = K_2 \longrightarrow \cancel{\frac{1}{2}} k_1 x_1^2 + \cancel{\frac{1}{2}} k_2 x_2^2 = \cancel{\frac{1}{2}} m v^2$

$3000\,(0.1)^2 + 1000\,(0.1)^2 = 3 \cdot v^2 \rightarrow \boxed{V = 3.65\ m/s}$

b) $U_1 = K_2 + U_2$ for the maximum compression $\rightarrow K_2 = 0$

$U_1 = U_2 \rightsquigarrow \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 = \frac{1}{2} k_1 (x_1')^2 + \frac{1}{2} k_2 (x_2')^2$ since $x_1' = -x_2'$

$\frac{1}{2}(k_1 + k_2) x_1^2 = \frac{1}{2}(k_1 + k_2)(x_1')^2$

$\boxed{x_1' = -10}$

b) (10 Points) A conservative force $\vec{F}$ is in the +x-direction and has magnitude $F(x) = \alpha/(x + x_0)^2$, where $\alpha = 0.1\ Nm^2$ and $x_0 = 0.2\ m$. (a) What is the potential energy function $U(x)$ for this force? Let $U(x) \to 0$ as $x \to \infty$. (b) An object with mass $m = 0.4\ kg$ is released from rest at $x = 0$ and moves in the + x-direction. If $\vec{F}$ is the only force acting on the object, what is the object's speed when it reaches $x = 0.5\ m$?

a) $U(x) - \cancel{U_\infty}^{\,0} = \int_\infty^x -F(x')dx' = -\int_\infty^x \frac{\alpha}{(x'+x_0)^2} dx' = \frac{\alpha}{x'+x_0}\Big|_\infty^x = \frac{\alpha}{x + x_0}$

$$\boxed{U(x) = \frac{\alpha}{x + x_0}}$$

b) $U_1 + K_1 + W_{other} = U_2 + K_2$ $\qquad W_{other} = 0$ & $K_1 = 0$

$U_1 = U_2 + K_2$ $\qquad U_1 = U(x=0)$ & $U_2 = U(x=0.5)$

$U(x=0) = U(x=0.5\,m) + \frac{1}{2} m v^2$

$\frac{\alpha}{x_0} = \frac{\alpha}{0.5 + x_0} + \frac{1}{2} m v^2$

$\frac{0.1\ Nm^2}{0.2\ m} = \frac{0.1\ Nm^2}{0.7\ m} + \frac{1}{2} \cdot 0.4 \cdot v^2$

$V = 1.336\ m/s$

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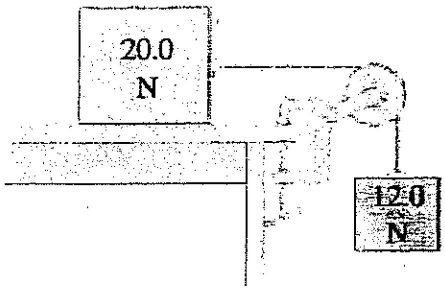
2) (20 Points) Two blocks are connected by a very light string passing over a massless and frictionless pulley. Consider $\mu_s = \mu_k = 0.2$ between the table and the 20.0-N block. The 20.0-N block moves 75.0 cm to the right and the 12.0-N block moves 75.0 cm downward. How much work is done (a) on the 12.0-N block by (i) gravity and (ii) the tension in the string? (b) How much work is done on the 20.0-N block by (i) gravity, (ii) the tension in the string, (iii) friction, and (iv) the normal force? (c) Find the total work done on each block.



$F_{net} = \sum F = m.a$

$W = F.\ x'\cos\theta$

$f_s = 20 \times 0.2 = 4N$

$F_{net,12\text{-}N} = 12N - T$

$F_{net,20\text{-}N} = T - f_s$

$\dfrac{F_{net,12\text{-}N}}{12N} = \dfrac{F_{net,20\text{-}N}}{20N}$

$\dfrac{12 - T}{\cancel{12}_3} = \dfrac{T - 4}{\cancel{20}_5}$

$60 - 5T = 3T - 12$

$72 = 8T \longrightarrow T = 9N$

a) on 12-N block

i) $W_{G,12\text{-}N} = F.\ x.\cos\theta = 12 \times 75 \times 10^{-2} = 9J$

ii) $W_{T,12\text{-}N} = F.\ x.\cos\theta = 9 \times 75 \times 10^{-2} \times \cos\pi = -6.75J$

b) on 20-N block

i) $W_{G,20\text{-}N} = 20N \times 75 \times 10^{-2} \times \cos\frac{\pi}{2} = 0$

ii) $W_{T,20\text{-}N} = 9N \times 75 \times 10^{-2} \times \cos 0 = 6.75J$

iii) $W_{F,20\text{-}N} = 4N \times 75 \times 10^{-2} \times \cos\pi = -3J$

iv) $W_{N,20\text{-}N} = 20N \times 70 \times 10^{-2} \times \cos\frac{\pi}{2} = 0J$

c) on 12-N block

$W = 2.25J$

on 20-N block

$W = 3.75J$

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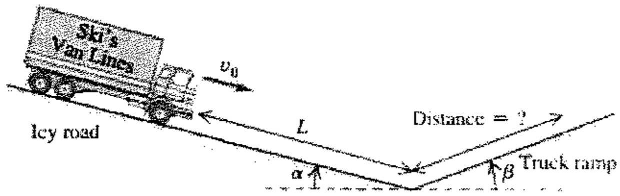
3) (20 Points) A truck with mass *m* has a brake failure while going down an icy mountain road of constant downward slope angle α. Initially the truck is moving downhill at speed v_0. After careening downhill a distance *L* with negligible friction, the truck driver steers the runaway vehicle onto a runaway truck ramp of constant upward slope angle β. The truck ramp has a soft sand surface for which the coefficient of rolling friction is μ_r. What is the distance that the truck moves up the ramp before coming to a halt? Solve by energy methods.



$$\frac{1}{2} m v_0^2 + m g \sin\alpha \cdot L = m g \sin\beta \cdot x + m g \cos\beta \, \mu_r \cdot x$$

$$\frac{v_0^2}{2g} + \sin\alpha L = x \left(\sin\beta + \cos\beta \, \mu_r \right)$$

$$x = \frac{\frac{v_0^2}{2g} + \sin\alpha L}{\left(\sin\beta + \cos\beta \, \mu_r \right)}$$

Solution continues on the back side of the page **Yes** □ **No** □

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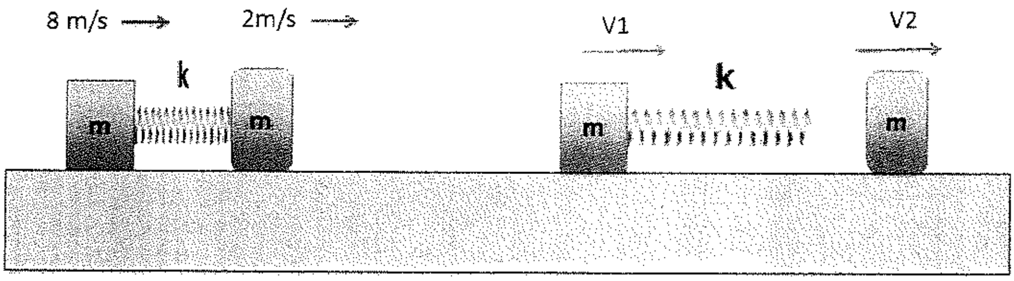
4-(20 Points) As shown in the figure the first block of mass m=1kg is moving to the right with the speed of 8m/s. Similarly, the second identical block (m=1kg) is moving to the right with a speed of 2 m/s. At the same time two blocks are compressing the spring a distance of 0,4 m. The spring constant k is 200 N/m and you can ignore the mass of the spring. (Ignore the mass of the spring.)



a) Find the center of mass velocity of the system.

$$V_{cm} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{8m + 2m}{2m} = 5\ m/s$$

Vcm= 5 m/s

b) Calculate the maximum compression of the spring (Xm).

$$E_{initial} = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \frac{1}{2} k x^2$$

$E_{initial} = E_{max\,comp}$

$$= \frac{1}{2} 1 (8^2 + 2^2) + \frac{1}{2} \cdot 200 (0.4)^2 = 50 J$$

Xm= 0.5 m

$$E_{max,com} = \frac{1}{2} 1 (5^2 + 5^2) + \frac{1}{2} \cdot 200 \cdot x^2 \longrightarrow$$

c) Find the velocities of the two blocks when the spring reaches maximum compression (Xm).

V1'= 5 m/s

V2'= 5 m/s

d) Find the velocities of the blocks V1 and V2 after the blocks are separated as shown in the second figure.

$$\frac{1}{2} m (v_1^2 + v_2^2) = 50 J$$

$$v_1^2 + v_2^2 = 100\ m^2/s^2$$

$$\frac{(v_1 + v_2) m}{2m} = 5 m/s$$

$$v_1 = 0$$

$$v_2 = 10 m/s$$

V1 = 0

V2= 10 m/s

Solution continues on the back side of the page **Yes** □ **No** □

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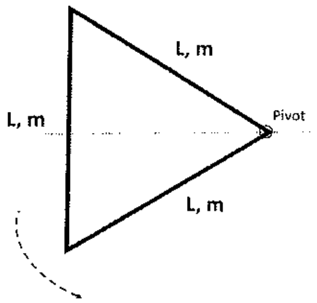
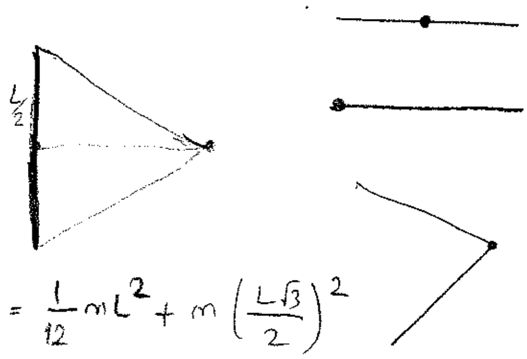
5-(20 Points) - As shown in the figure, a triangular frame consists of three identical rods of length L and mass m is pivoted about a frictionless pin through one corner. The frame is released from the rest in a horizontal position and the frame is rotating downward under the action of the gravitational force. (For a slender rod, the moment of inertia for an axis that goes through the center is $I=(1/12)mL^2$.)





$I = \frac{1}{12} mL^2$

$I = \frac{1}{3} mL^2$

$I = 2\times\left(\frac{1}{3} mL^2\right) = \frac{2}{3} mL^2$

$I = \frac{1}{12} mL^2 + m\left(\frac{L\sqrt{3}}{2}\right)^2$

a) Calculate the total moment of inertia of the frame about the rotation axis through the pivot.

$$I_{total} = \frac{2}{3} mL^2 + \frac{5}{6} mL^2$$
$$= \frac{3}{2} mL^2$$

I = $\boxed{\frac{3}{2} mL^2}$

b) Calculate the maximum angular velocity of the frame

maximum $\omega \longrightarrow$ cm is at lowest

$$3mg \cdot \frac{2}{3}\frac{\sqrt{3}}{2} L = \frac{1}{2} I \omega^2$$

w= $\boxed{\sqrt{\frac{4}{\sqrt{3}}\frac{g}{L}}}$

c) Calculate the maximum the center of mass velocity of the frame.

$$V_{max} = \omega_{max} \cdot r$$

$r = \frac{L\sqrt{3}}{3}$ $\qquad V_{max} = \sqrt{\frac{4 \cdot \cancel{3}}{\sqrt{3} \cdot \cancel{9}_3} \cdot \frac{L^2 g}{L}}$

Vcm= $\boxed{\sqrt{\frac{4}{3\sqrt{3}} g.L}}$

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