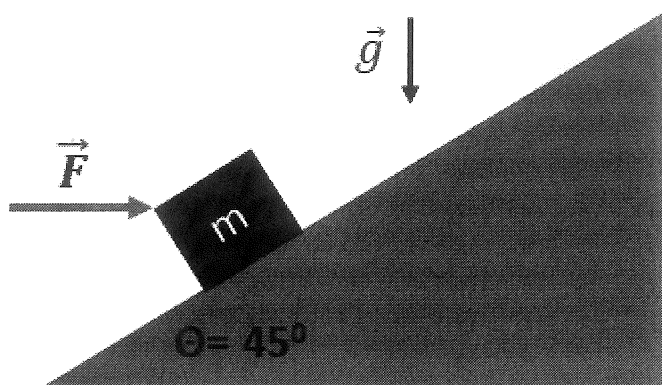


Question 1



$$a) W_F = F \cos \theta \cdot d = F d \frac{\sqrt{2}}{2}$$

$$b) W_G = -mg \sin \theta \cdot d = -mg d \frac{\sqrt{2}}{2}$$

$$F_G = mg$$

$$c) W_N = N \cdot \cos 90 = 0 \text{ J}$$

(a) How much work is done by the constant horizontal force \vec{F} on the block with mass m , when the force pushes the block a distance of d up along the 45° frictionless fixed incline?

(b) How much work is done by the gravitational force on the block during this displacement?

(c) How much work is done by the normal force?

(d) What is the speed of the block (assume that it is zero initially) after this displacement?

$$(\cos 45 = \sin 45 = \sqrt{2}/2)$$

1) work-energy Thm.

$$\Delta KE = W_{\text{total}}$$

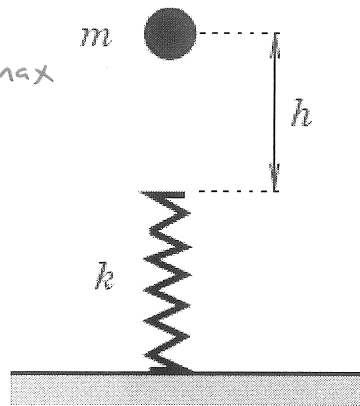
$$W_{\text{tot}} = W_F + W_G + W_N = \frac{d\sqrt{2}}{2} (F - mg)$$

$$\frac{1}{2} \cdot m v^2 - 0 = \frac{d\sqrt{2}}{2} (F - mg)$$

$$v^2 = d\sqrt{2} \left(\frac{F}{m} - g \right)$$

Question 2

b) cont'd: you might also
think that @ $v_{\max} \Rightarrow KE_{\max}$
 KE_{\max} happens @ equilibrium
 $\Sigma F = 0 \Rightarrow mg = kx$
 $x = \frac{mg}{k}$



c) from part b
 $v^2 = \frac{2}{m} [mg(h+x) - \frac{1}{2}kx^2]$
insert $x = \frac{mg}{k}$ for v_{\max}
 $v_{\max} = \sqrt{2gh + \frac{mg^2}{k}}$

An object of mass m is dropped on a spring with a spring constant k from a height h .

- What will be the maximum compression on the spring? (10 pts)
- Find the compression on the spring when the object is at its maximum velocity. (7 pts)
- What is the maximum velocity of the object during its motion? (8 pts)

a) $mg(h+x_{\max}) = \frac{1}{2}kx_{\max}^2 \Rightarrow \frac{1}{2}kx_{\max}^2 - mgx_{\max} - mgh = 0$

use $\Delta (+)$

$$x_{\max} = \frac{mg}{k} + \sqrt{\left(\frac{mg}{k}\right)^2 + \frac{2mgh}{k}}$$

b) For any compression on spring: $mg(h+x) = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
(it might also be zero)
 $v = \sqrt{\frac{2}{m} [mg(h+x) - \frac{1}{2}kx^2]}$
for max $v \Rightarrow \frac{dv}{dx} = 0 \Rightarrow xk = mg \Rightarrow \boxed{x = \frac{mg}{k}}$

Question 3

Handwritten notes and diagram for Question 3:

a) since net force on x is zero
momentum is conserved!
So, x_{cm} does not change
remember $P = M \cdot v_{cm} \Rightarrow P_i = 0$
 $v_{cm} = 0$
So $x_{cm} = \text{const}$

Position on bottom
 $\left[x_0 + \frac{ml}{M+m}, l \right]$

Diagram: A cylinder of mass M is on a horizontal shaft. A ball of mass m is attached to the cylinder by a string of length l . The cylinder is at x_0 from the y-axis. The ball is at an angle θ from the vertical. Gravity \vec{g} points down.

Equations for center of mass:

$$x_{cm} = \frac{Mx_0 + m(x_0 + l)}{M+m}$$

$$x_{cm} = \frac{Mx' + mx'}{M+m}$$

$$x' = x_0 + \frac{ml}{M+m}$$

A ball of mass m is attached to the cylinder of mass M by a massless string of length l . The cylinder is free to slide on a frictionless horizontal shaft on x-axis.

Initially both the cylinder and the ball are at rest, the center of the cylinder is at x_0 distance from y coordinate and the ball is displaced by an angle $\theta_0 = \pi/2$ to the right relative to the vertical. Use the coordinate system indicated in the figure and assume that the motion takes place on the xy plane.

- If the ball is released from its initial position $(x_0 + l, 0)$ with zero initial velocity, what will be its position when it is at the bottom of the swing (when $\theta = 0$)? (10 pts)
- Find the velocities of the ball and cylinder when $\theta = 0$. (15 pts)

Handwritten solution for part b):

b) $P_i = P_f$
 $E_i = E_f$

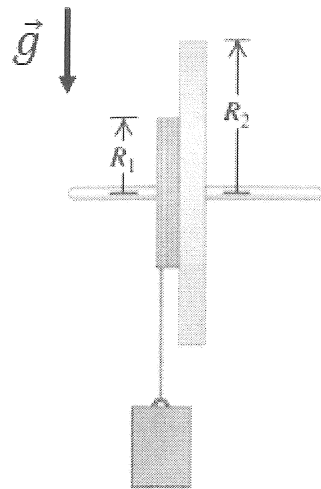
$0 = MV - mv$
 $\frac{1}{2}MV^2 + \frac{1}{2}mv^2 = mgl$

Solving both

$$v = \sqrt{\frac{2Mgl}{M+m}}$$

$$V = \frac{m}{M}v = \sqrt{\frac{2m^2gl}{M(M+m)}}$$

Question 4



$$a) I_{total} = \frac{1}{2} M R_1^2 + \frac{1}{2} M R_2^2 \\ = \frac{1}{2} M (R_1^2 + R_2^2)$$

$$b) E_i = E_f$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I_{tot} \cdot \omega^2 \quad \left(\omega = \frac{v}{R_1} \right)$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{1}{2} M (R_1^2 + R_2^2) \right) \cdot \frac{v^2}{R_1^2}$$

$$\frac{2mgh}{m + \frac{1}{2} M \left(1 + \frac{R_2^2}{R_1^2} \right)} = v^2$$

Two metal disks, one with radius R_1 and mass M and the other with radius R_2 and mass M , are welded together and mounted on a frictionless axis through their common center.

$$(I_{cm} = \frac{1}{2} M R^2)$$

- What is the total moment of inertia of the two disks? (5 pts)
- A string with negligible mass is wrapped around the edge of the smaller disk, and a block of mass m is suspended from the free end of the string. If the block is released from rest at a height of h above the floor, what is its speed just before it strikes the floor? (12 pts)
- Repeat the calculation of part b), this time with the string wrapped around the edge of the larger disk. (8 pts)

$$c) \text{ now } \omega = \frac{v}{R_2} \Rightarrow \frac{2mgh}{m + \frac{1}{2} M \left(\frac{R_1^2}{R_2^2} + 1 \right)} = v^2$$