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Q1-(25 pts) The positions of objects 1 and 2 are given in meters as $\vec{r_1} = (t^2 - 2)\,\hat{\imath}$, and $\vec{r_2} = (2t - 8)\,\hat{\jmath}$ respectively.

a) Find the displacement of object 1 during the time interval from t=1 to t=3 seconds. (6 pts)

$$\Delta\vec{r}_1 = \vec{r}_1(3) - r_1(1)$$
$$= (3^2-2)\hat{\imath} - (1^2-2)\hat{\imath}$$
$$= 7\hat{\imath} + 1\hat{\imath} = 8\hat{\imath}\ (m)$$

$\boxed{8\hat{\imath}\ (m)}$

b) Find the acceleration of object 1? (6 pts)

$$\frac{d^2 r_1}{dt^2} = 2\hat{\imath}\ (m/s^2)$$

$\boxed{2\hat{\imath}\ (m/s^2)}$

c) What is the velocity of object 1 relative to object 2? (6 pts)

$$v_{1/2} = v_1 - v_2 = \frac{d}{dt}(r_1 - r_2)$$
$$= \frac{d}{dt}(t^2-2)\hat{\imath} - (2t-8)\hat{\jmath}$$
$$= 2t\,\hat{\imath} - 2\hat{\jmath}$$

$\boxed{2t\,\hat{\imath} - 2\hat{\jmath}}$

d) What is the closest distance between these two objects during their motion? (7 pts)

$$d_{12} = \sqrt{(t^2-2)^2 + (2t-8)^2}$$

$$\min d_{12} \Rightarrow \frac{d}{dt} d_{12} = 0$$

$$\frac{d}{dt}\left[(t^2-2)^2 + (2t-8)^2\right] = 0$$

$$2(2t-8).2 + 2(t^2-2).2t = 0$$
$$8t - 32 + 4t^3 - 8t = 0 \Rightarrow 4t^3 = 32$$
$$t = 2\ sec.$$

$$d_{1/2} = \sqrt{(2^2-2)^2 + (2.2-8)^2}$$
$$= \sqrt{20}\ m$$

$\boxed{\sqrt{20}\ m}$

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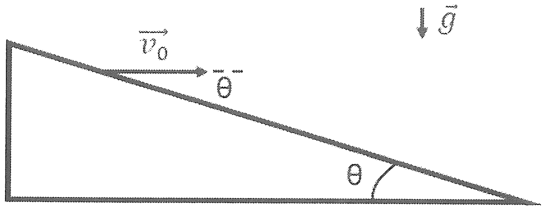
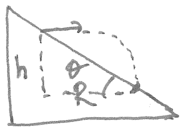
Q2-(25 pts) A projectile is launched with initial speed of v_o on an inclined plane at t=0 as shown in the figure. The inclination angle of the surface θ= 45⁰. The gravitational acceleration is g.



a) Find the time projectile lands on the incline. (12 pts)

$$\boxed{\frac{2v_0}{g}}$$



$R = v_0 \cdot t$ (no acceleration)

$h = \frac{1}{2} g t^2$ (constant acceleration)

$$\tan\theta = \frac{h}{R} = 1 = \frac{1/2\, g t^2}{v_0 t}$$

$$2 v_0 t = g t^2$$

$$0 = t(gt - 2v_0)$$

$$t = \frac{2v_0}{g}$$

b) Find the distance of landing point from the launch point. (13 pts)

$(\sin 45 = \cos 45 = \frac{\sqrt{2}}{2})$

$$\boxed{\sqrt{8}\,\frac{v_0^2}{g}}$$

$$d = \sqrt{h^2 + R^2}$$

$$R = v_0 \cdot \frac{2v_0}{g}$$

$$h = \frac{1}{2} g \left(\frac{2v_0}{g}\right)^2$$

$$d = \sqrt{\frac{1}{4} g^2 \frac{16 v_0^4}{g^2} + \frac{4 v_0^4}{g^2}} = \sqrt{\frac{8 v_0^4}{g^2}}$$

$$d = \sqrt{8}\,\frac{v_0^2}{g}$$

Solution continues on the back side of the page Yes ☐ No ☐

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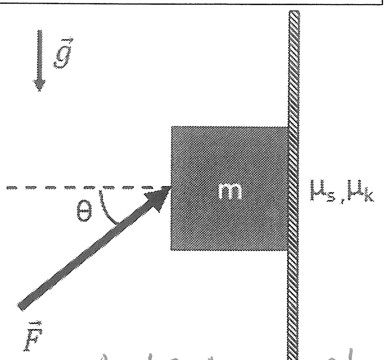
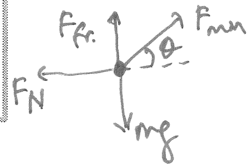
Q3-(25 pts) A block of mass m is pressed against a vertical wall with a constant force of magnitude F at an angle θ with the horizontal, as shown in the figure. The magnitude of the gravitational acceleration is g. The coefficients of static and kinetic frictions between the block and the wall are μ_s and μ_k respectively.



a) Find the minimum magnitude of F for which the block does not slide down. (6 pts)

To prevent slide down, friction must be in upward direction.



$F_{min}\cos\theta - F_N = 0 \quad (\Sigma F_x = 0)$

$F_{min}\sin\theta + F_{fr} - mg = 0 \quad (\Sigma F_y = 0)$

$F_N = F_{min}\cos\theta \qquad F_{fr} = mg - F_{min}\sin\theta$

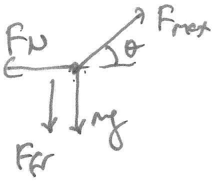
$F_{fr} \le \mu_s F_N$

$mg - F_{min}\sin\theta \le F_{min}\mu_s\cos\theta$

$$\boxed{F_{min} = \frac{mg}{\sin\theta + \mu_s\cos\theta}}$$

b) Find the maximum magnitude of F for which the block does not slide. (6 pts)

To prevent slide up, friction must be downward direction



$F_{max}\cos\theta - F_N = 0 \quad (\Sigma F_x = 0)$

$F_{max}\sin\theta - F_{fr} - mg = 0 \quad (\Sigma F_y = 0)$

$F_N = F_{max}\cos\theta \qquad F_{fr} = F_{max}\sin\theta - mg$

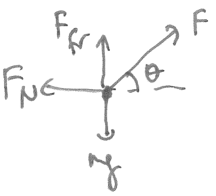
$F_{fr} \le \mu_s N$

$F_{max}\sin\theta - mg \le F_{max}\cos\theta . \mu_s$

$$\boxed{F_{max} = \frac{mg}{\sin\theta - \mu_s\cos\theta}}$$

c) When the magnitude of F is less than the minimum the block starts to slide down. Find its acceleration. (6 pts)

when the block starts to slide down:



$F_{fr} = \text{kinetic} = \mu_k . F_N$

$F\cos\theta - F_N = 0 \qquad F_N = F\cos\theta$

$F\sin\theta + F_{fr} - mg = -ma \Rightarrow F\sin\theta + F\cos\theta\mu_k - mg = -ma$

$a = g - \frac{F}{m}(\sin\theta + \mu_k\cos\theta)$

$$\boxed{a = g - \frac{F}{m}(\sin\theta + \mu_k\cos\theta)}$$

d) Below which angle θ will the block be impossible to move up? (7 pts)

if the block is to move up, we must have.

→ from part b)

$F \ge \frac{mg}{\sin\theta - \mu_s\cos\theta}$

when denominator is zero, it is impossible to move block up!

$\sin\theta - \mu_s\cos\theta = 0 \Rightarrow \theta \le \arctan\mu_s$

$$\boxed{\theta \le \arctan\mu_s}$$

Solution continues on the back side of the page **Yes** □ **No** □

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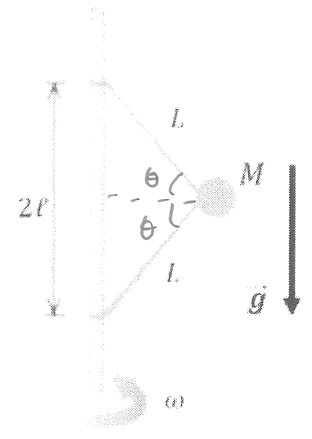
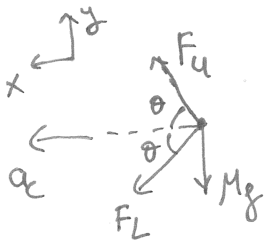
Q4-(25 pts) A block of mass M is attached to a vertical rod by means of two strings. When the system rotates about the axis of the rod with angular velocity ω, the strings are extended as shown in the figure.



a) Draw a free body diagram for the mass M.



$\sin\theta = \dfrac{l}{L}$

b) If the tension in the upper string is twice the tension in the lower string, find the angular velocity ω of the system in terms of the other parameters.

$F_u \cos\theta + F_L \cos\theta = M L \cos\theta\, \omega^2$

$F_u \sin\theta - F_L \sin\theta = Mg$

$F_u + F_L = M L \omega^2$

$F_u - F_L = \dfrac{Mg}{\sin\theta}$

$F_u = 2F_L$

$3F_L = ML\omega^2$

$F_L = \dfrac{Mg}{\sin\theta}$

$\dfrac{1}{3} ML\omega^2 = \dfrac{Mg}{\sin\theta} = \dfrac{Mg}{l/L}$

$\omega^2 = \dfrac{3g}{l}$

$\boxed{\omega = \sqrt{\dfrac{3g}{l}}}$

c) Find the angular velocity at which the tension in the lower string becomes zero.

$F_L = 0$

$F_u = ML\omega^2$

$F_u = \dfrac{Mg}{\sin\theta} = \dfrac{MgL}{l}$

$ML\omega^2 = \dfrac{MgL}{l}$

$\omega^2 = \dfrac{g}{l}$

$\boxed{\omega = \sqrt{\dfrac{g}{l}}}$

Solution continues on the back side of the page **Yes** □ **No** □