## PHYS 101L

# General Physics I Lab: Mechanics Labbook generalphysics.ku.edu.tr 

Spring 2020

| Name of the Student |  |
| :---: | :---: |
| ID Number | : ....................................................................... |
| Signature |  |
| Department | : .............................................................................. |
| Name of the Laboratory Instructor | ... |
| Name of the Laboratory Session | .... |
| Date/Time of the Laboratory Sessio | ........... |

## KOÇ UNIVERSITY

PHYSICS DEPARTMENT

ISTANBUL - TURKEY
www.ku.edu.tr

## Table of Contents

0 Lab Rules, Regulations and Procedures ..... 3
1 Measurement and Error Analysis ..... 11
2 Horizontal Motion and Motion on an Inclined Plane ..... 18
3 Projectile Motion ..... 28
4 Dynamics of Motion ..... 41
5 Conservation of Linear Momentum ..... 52
6 Rotational Motion ..... 63
7 Physical Pendulum ..... 74
8 Appendices ..... 82

## PHYS 101 / 0 Lab Rules, Regulations and Introduction Procedures

### 0.1 Attendance

Attendance to all laboratory sessions is mandatory. Students are eligible to attend makeup laboratory sessions at the end of the semester only by having a medical report approved by Koç University Health Center or an excuse form approved by the Dean of Students covering the missed laboratory date.

### 0.2 Before The Laboratory-Session

$>$ IMPORTANT! Make sure that your own copy of the labbook and a scientific calculator is present when you attend the laboratory session. Students without their own labbook are not allowed to participate in the lab-session.
$>$ The experiment has to be studied from the labbook thoroughly before the lab-session. The introduction and theory sections are essential to understand the objectives of the experiment and its theoretical background. Consulting to external references is strongly recommended.
$>$ Reading the experimental procedure is extremely important to familiarize yourself beforehand with the setup, equipment and measurement techniques to conduct an efficient laboratory session.

### 0.3 During The Laboratory Session

$>$ Follow the announcements of the laboratory Instructor at all times.
$>$ The laboratory session starts as scheduled. There is a grace period of 15 minutes after which admitting the session is not allowed.
$>$ The total duration of the laboratory session is 165 min.s which includes the following subsessions:

- Setting up the experiment
- Performing the experiment and recording data to the databook
- Processing and analyzing the data, plotting graphics

1. Answering the questions in the labbook for the current experiment.

- Preparing an experiment report as follows:
- Experiment report must be hand written in the indicated space of the labbook and handed to the lab TAs at the end of the laboratory session.
- Make sure to fill in the following data in the data-book: Semester, Course, Title of the experiment, Date, ID number, name of the labbook owner, name of the lab-session partner, laboratory section.
$>$ Time extensions are not given unless an exceptional case acknowledged by the lab-instructor (e.g. power outage, malfunctioning equipment) is present.


### 0.4 Finishing The Laboratory Session

$>$ All labbooks must be checked and approved by the laboratory instructor at the end of the lab session. Leaving the laboratory without the instructor's check will render your lab session void.
$>$ The experiment setup must be disassembled, cleaned up and all the electrical and electronic equipment including the computers must be turned off properly. Each setup will be inspected and approved by the instructor. Ignoring these steps will result in a significant penalty to the lab grade.
$>$ Taking equipment out of the laboratory is prohibited and may result in disciplinary action and prosecution.

### 0.5 Physics Laboratory Safety Rules

## Koç University emergency dial 1122.

1. Physics Laboratory equipment include metal, plastic, wood, glass hardware, rotating/moving machinery, electrical, electronic devices. If not properly used, the equipment may cause an accident, fire, explosion, resulting in injuries ranging from minor to lethal.
2. Always wear appropriate clothing during the laboratory session. Hair, clothing parts or jewelry may get caught in the equipment, causing injury.
3. Eating, drinking or smoking in the laboratory is prohibited.
4. Follow the instructor's announcements at all times.
5. If instructed, wear protective equipment (gloves, googles) during the experiment. Protective equipment will be supplied in the laboratory.
6. Inspect the equipment carefully before setup against broken, malfunctioning parts. If there is any problem, inform the lab instructor immediately.
7. Any electrical device must be used in the following sequence:
a. BEFORE PLUG IN:
i. Inspect the outlet cable and its ends against wear and tear.
ii. Make sure the device is not connected to anything else.
iii. Make sure the main switch of the device is at "OFF" position.
b. BEFORE TURN ON: Make sure that output/adjustment knobs of the device are all at off or minimum position.
c. Turn on the device and check that it operates properly.
d. Turn off and make the necessary connections to the device in the setup.
e. Turn on and use the device.
f. Set all the output/adjustment knobs of the device to off or minimum position.
g. Turn off the device.
h. Disconnect from the setup.
i. Unplug the device.

### 0.6 Starting DataStudio for the first time - ScienceWorkshop

Double-clicking the DataStudio icon on your desktop will launch the DataStudio software.

When DataStudio opens, a Welcome to DataStudio navigator screen will appear with four options:


### 0.6.1 ScienceWorkshop Experiment Setup

Click the Setup button to activate the Experiment Setup window. You will use this window to select sensors and set experimental conditions. If the software does not immediately recognize the interface, click the Change button and choose your interface from the list in the Please Choose Data Source window. The Experiment Setup window will then show the selected interface. Create Experiment Open Activity Graph Equation Enter Data.


### 0.6.2 Sensors Panel

The Sensors panel lists all possible sensors. Scroll through the list to find the senor(s) for the experiment. To select a sensor, double-click the icon in the Sensors panel. The software will automatically choose the correct available port.


Now connect the physical sensor into the corresponding channel. When a sensor is selected, an icon will appear in the experiment setup window, with an arrow indicating the appropriate channel for each sensor. Double clicking the sensor icon in this window will open up the sensor properties window where you can set measurement(s), calibration and sampling rate. The available measurements are shown in the summary panel.

### 0.6.3 Using DataStudio to configure experiments and interfaces

DataStudio has a variety of tools to assist with configuring experiments. Using the Summary panel and associated functions helps further define the parameters of the experiment. The displays available provide a powerful method of data visualization. This section deals with creating data displays and describes the function of each.

### 0.6.4 Displaying data



The Summary panel lists measurements currently available, any collected data in the experiment, along with displays.

To display data, the sensor or data must be associated with a display. Dragging a display type from the bottom area of the summary panel up to a sensor in the top area of the summary panel will create a display for the sensor or targeted data set.

Displays can show multiple data types by dragging the sensor or data run from the data summary column into an open display. Some displays will be more useful than others depending on the sensors or experimental conditions.

### 0.6.5 Creating a display for data

You can create or remove a display from the experiment at any time, even during data collection. The following are the types of displays and a description of each:


The graph display
The graph display plots a sensor's data vs. time. To plot one data type against another, drag the data from the data summary (in the Summary panel) to the time axis (x-axis) of the graph. The new data type will replace time, producing an XY plot (i.e. Force vs. Position). Clicking and dragging a number on the axis will directly change the graph scale. Clicking and dragging the axis line, itself, will move the axis in the display window.

The table display

The table display shows the numerical coordinates in paired columns.

### 3.14

The digits display
The digits display shows the instantaneous value of the data as the experiment is running.


The meter display
The meter display shows a pictorial representation of the data using a graphical meter.

## The histogram display

The histogram display plots data points that are lumped together in 'bins' as counts. The area of a bin is proportional to the frequency of the specified data range, or the number of times a specified measurement value has been observed.

## The FFT display

The FFT (Fast Fourier Transform) displays the spectral decomposition of the data. Higher sampling rates will yield finer definition of the data's frequency spectrum. This display does not store data like other displays. It shows a 'time-slice' snapshot of the data.

The oscilloscope display

The oscilloscope display plots a time-based graph, but like the FFT shows a 'time slice' snapshot. The data is not stored. This display is ideal for experiments using fast sampling rates.


The workbook display

The workbook display is a powerful, self-contained authoring environment. Use this feature to create guided scientific inquiry or as a laboratory write-up tool. Workbooks can contain DataStudio displays, graphics, and text.

### 0.6.6 Setting the Experiment Options

Use the Options button to set sampling options. Clicking the Options button in the Experiment Setup window will open the Sampling Options.

### 0.6.7 Manual Sampling Option

This option is used with experiments that require selecting specific data points (instead of collecting continuous data). These data points can then be associated with a parameter that is not measured by a sensor. The associated parameter can be typed in manually. Clicking the "Keep data values only when commanded" check box will activate manual sampling mode. If the data that is kept will have associated manually input data, also check the "Enter a keyboard value when data is kept" checkbox. If the "Prompt for a value" checkbox is checked, when data is 'kept', DataStudio will prompt the user to manually input the associated data. The remaining options are for describing the manually input data. You can describe and name the data, prescribe units, as well as the numerical accuracy.

### 0.6.8 Taking Measurements and Collecting Data

Once the experiment is set up, click the Start button to begin collecting data.


Start button and timer

When the Start button is clicked, it will change into a Stop button. Clicking the Stop button will stop data collection. The experiment timer displays the current timing condition; either how long data has been collected, or a countdown set by an initial timing condition.


If the experiment has been configured for Manual Sampling (see Setting Experiment Options) the start button will change into a Keep/Stop button. Pressing the Keep button during data collection will store a data point. Clicking the red square to the right of the Keep button will stop data collection.

### 0.6.9 Display and Analysis Tools

DataStudio provides a complement of features designed to aid with displaying and analyzing data. Displays may be created or closed at any time before, during or after data collection.


Scale to fit

A graph, FFT, histogram, and meter display can auto-scale using the scale to fit tool. The entire display will automatically adjust the range so the data fills the display window.


Zoom in, Zoom out, Zoom select

The graph and histogram zoom tools change the view of the display window in order to shrink, expand, or focus in on a select portion of the data. To use the zoom select tool, click the tool then draw a box by clicking and dragging around the data area of interest. The graph will zoom in to the area you selected. The Scale to Fit button will return the data back to the optimal view for all data points.


Smart Tool

The Smart Tool activates a set of cross hairs that displays the coordinate data pair of a specific data point. As you get closer to a data point, the Smart Tool will "gravitate" towards the data point. The displayed coordinates appear in parenthesis at the upper right edge of the small box around the cross hairs. The smart tool may also be used to display the difference between two data points.


To change the position of the cross hairs of the Smart Tool, hover your mouse cursor over the center of the smart tool until the pointer turns into two crossed double arrowheads and a hand. Drag the cross hairs of the Smart Tool to the desired location. To constrain the movement of the cross hairs to one axis, hover the pointer over the dashed line that is perpendicular to the axis you want to move along until the pointer turns into a hand. Drag the cross hairs to the new location.


Measuring change - Delta Tool

The Delta Tool is a feature of the Smart Tool for measuring the change in the X and Y coordinates between two data points on a graph display.


To use the Delta Tool: Drag the cross hairs of the Smart Tool to a data point. Hover the mouse pointer over one of the edges of the small box around the cross hairs until the pointer turns into a triangle and hand. Click and drag the triangle to the second data point A dashed box will appear with the selected data points at two of its corners. Along the sides of the dashed box you will see the numbers that are the difference between the coordinate values for the two points. To resize this box, click the corner of the box containing the arrowheads and drag it to a new data point. Clicking and dragging any dashed line will also resize the box (constraining movement to only one dimension).


Statistics can be toggled on and off with this button. Pressing the drop down arrow next to the sigma symbol will display a list of available statistics.


Data can be selectively viewed by using the Data button. Data sets can be toggled on and off by clicking on the Data button, and choosing which sets are to be displayed or hidden. Data can be removed from a display by using the Remove button. Clicking the Remove button will remove the highlighted data set from the display.

Clicking the display settings button will open a menu where display options can be changed. Clicking the down arrow next to the icon will open a menu of common display options. Double clicking in the middle of the display window will also open the display options menu.

## PHYS 101 / 1 Measurement and Error Analysis Experiment 1

### 1.1 Instruments and Materials

- Tape measure
- Stop watch
- Pendulum
- Clamp stand


### 1.2 Aim

In this experiment you will measure $g$, the acceleration due to gravity at the Earth's surface, by measuring the length and the period of a simple pendulum, and evaluate the experimental results.

### 1.3 Introduction

### 1.3.1 Simple Pendulum

A simple pendulum consists of a point-mass, $m$ suspended from a fixed point by a string of length, $l$. When the mass is displaced from vertical, it oscillates about the vertical plane. The period of oscillation, $T$ is the time required for the pendulum to complete one cycle in its motion. This can be determined by measuring the time required for the pendulum to reoccupy a given position. The reason that the pendulum oscillates about the vertical is that if the pendulum is displaced, the force of gravity pulls down on the pendulum. The pendulum begins to move downward. When the pendulum reaches vertical it can't stop instantaneously and continues past the vertical and upward in the opposite direction. The force of gravity slows it down until it eventually stops and begins to fall again. If there is no friction where the pendulum is fixed and there is no air resistance to the motion of the pendulum, this would continue forever. Because it is the force of gravity that produces the oscillation, one might expect the period of oscillation to differ for differing values of gravity. In particular, if the force of gravity is small, there is less force pulling the pendulum downward, the pendulum moves more slowly toward vertical, and the observed period of oscillation becomes longer. Thus, by measuring the period of oscillation of a pendulum, we can estimate the gravitational force or acceleration.


Figure 1.1 Simple Pendulum

As shown in the Figure 1.1, $m$ is the mass of the pendulum bob, $g$ is the magnitude of the gravitational acceleration and $l$ is the length of the string. $m, g$ and $l$ are positive constants. A real pendulum oscillating with a small amplitude may be described as a simple pendulum when the mass of the string is much less than the mass of the bob, and when the length of the string $l$ is much greater than the size of the bob. In this case the bob can be treated as a point-mass located at the center of gravity of the bob. The period of the pendulum will be given by the following equation:

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{l}{g}} \tag{Eq. 1.1}
\end{equation*}
$$

Solving Eq. 1.1 for $g$ yields;

$$
\begin{equation*}
g=4 \pi^{2} \frac{l}{T^{2}} \tag{Eq. 1.2}
\end{equation*}
$$

### 1.3.2 Experimental Errors

Error means the difference between an experimental value and the true value. There are two types of experimental errors: random errors and systematic errors. The difference between the systematic and random errors can be seen by repeating a measurement of a physical quantity several times under the same conditions. Random errors are statistical fluctuations or variations in the measured data produced by the experimenter's inability to take the same measurement in exactly the same way to get exactly the same reading. Therefore, the readings will be spread about the true value as shown in Figure 1.2 (a). You will analyze the ramdom error in your measurements in this experiment. On the other hand, systematic errors are repeating inaccuracies that cause the measurements to constantly be either too high or too low. They are mostly due to defects in the measuring devices which make them continually present throughout the entire experiment. Therefore, the readings will always be displaced far from the true value as shown in Figure 1.2: (a) Measurements with only random errors. (b) Measurements with both random and systematic errors.Figure 1.2 (b). For that reason, systematic errors are difficult to detect and cannot be analyzed statistically.


Figure 1.2: (a) Measurements with only random errors. (b) Measurements with both random and systematic errors.

The Accuracy of an experiment is a measure of how close the result obtained for a given experiment is compared to the true value. The Precision of an experiment is a measure of how exactly the result is determined. For example: The time is 09:20:331. One clock shows the time as $09: 22: 174$, and the other clock shows the time as $09: 20$. So the first clock is more precise as it shows the time to the nearest 1 ms (miliseconds), and the second clock is more accurate as it show the time to the nearest 1s (seconds) but closer to the true value.

### 1.3.3 Experimental Error Calculation

The measured values in experiments are given with their experimental errors like $\alpha=\alpha_{m}+\Delta \alpha$ where $\alpha_{m}$ is the measured value found in the experiment and $\Delta \alpha$ is its experimental error, which can be calculated as the following:

$$
\begin{equation*}
\Delta \alpha=\sqrt{\frac{\sum\left(\left|\left(\alpha_{\text {ave }}-\alpha_{i}\right)^{2}\right|\right)}{N-1}} \tag{Eq. 1.3}
\end{equation*}
$$

where $\alpha_{\text {ave }}$ is the average of measured values, $\alpha_{i}$ is the value of each measurement, and $N$ is the number of measurements. ( $\alpha$ is any given value)

Percent error is the percent difference of the experimental result with the true value. The equation for the percent error is:

$$
\begin{equation*}
\% \text { Error }=\left|\frac{\text { Experimental Result-True Value }}{\text { True Value }}\right| \times 100 \tag{Eq. 1.4}
\end{equation*}
$$

### 1.4 Experimental Procedure

The set up consists of a simple pendulum attached to a clamp stand.

1. Using tape measurer, measure the length, $l$ of the pendulum. Record your result in the TABLE 1.1.
2. Oscillate the pendulum through a small arc (about $5^{\circ}$ on either side of vertical).
3. Use a stop watch to measure the time it takes for the pendulum to complete 10 oscillations.
4. For the same length, repeat the experiment for 4 more times.
5. Record the measured values in TABLE 1.1.
6. Change the length of the pendulum for four times and repeat the procedure 1 to 6 and record your readings in TABLE 1.2-1.5.

TABLE 1.1. VALUES OF LENGTH AND PERIOD OF A SIMPLE PENDULUM (5 PTS)

| $l(m)$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Run \# | Time for 10 oscillations, <br> $t(s)$ | Period of the <br> Pendulum, $T(s)$ | $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

TABLE 1.2. VALUES OF LENGTH AND PERIOD OF A SIMPLE PENDULUM (5 PTS)

| $l(m)$ |  |  |  |
| :--- | :--- | :--- | :--- |
| Run \# | Time for 10 oscillations, <br> $t(s)$ | Period of the <br> Pendulum, $T(s)$ | $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

TABLE 1.3. VALUES OF LENGTH AND PERIOD OF A SIMPLE PENDULUM (5 PTS)

| $l(m)$ |  |  | Time for 10 oscillations, <br> $t(s)$ |  | Period of the <br> Pendulum, $T(s)$ | $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Run \# |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |

TABLE 1.4. VALUES OF LENGTH AND PERIOD OF A SIMPLE PENDULUM (5 PTS)

| $l(m)$ |  |  | Time for 10 oscillations, <br> $t(s)$ |  | Period of the <br> Pendulum, $T(s)$ | $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Run \# |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |

TABLE 1.5. VALUES OF LENGTH AND PERIOD OF A SIMPLE PENDULUM (5 PTS)

| $l(m)$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| Run \# | Time for 10 oscillations, <br> $t(s)$ | Period of the <br> Pendulum, $T(s)$ | $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

### 1.5 Report

1- (15 pts) Calculate $g$, the acceleration due to gravity at Earth's surface with its experimental error for each length, $l$.

2- (25 pts) Plot a graph of length, $l$ versus square of the period of oscillation, $T^{2}$, and calculate the acceleration due to gravity, $g$ from the slope. Find the percent error with the true $g$ in İstanbul $\left(g=9.808 \mathrm{~m} / \mathrm{s}^{2}\right)$.

3- ( 5 pts ) What are the types of errors you encountered in this experiment? Random or systematic? Explain.

4- ( 5 pts ) What is the accuracy of your gravitational acceleration result? Explain.

5- (5 pts) Make two suggestions to increase the precision in your experiment.

6- ( 5 pts ) If you were to double the mass on the end of the pendulum, what would happen to the period?

7- ( 5 pts ) If you were to do this experiment on the moon, what would you expect of the motion of the pendulum?

### 1.5.1 Conclusion (10 pts)

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.)


## Completed by:

Name, Surname and ID of Lab partner(s):

## Department of Lab partner(s): <br> Verified by:

## PHYS 101 / 2 Horizontal Motion and Motion on an Experiment 2

### 2.1 Instruments and Materials

- Signal interface
- Photogate
- Picket fence
- Small cart
- Ideally frictionless metal track
- Leveling device
- Support rod


### 2.2 Aim

The purpose of this experiment is to study the motions with constant velocity and with constant acceleration. Newton's second law will be verified.

### 2.3 Introduction

### 2.3.1 Theory

In this experiment, the relationships between an object's position, velocity, and acceleration will be investigated when the object is moving along a straight line on an almost ideal frictionless surface.

The acceleration of an object is the rate of change of its velocity with time. For an object having a velocity $\vec{v}_{1}$ at time $t_{1}$ and a velocity $\vec{v}_{2}$ at a later time $t_{2}$, the average acceleration in the time interval $\Delta t=t_{2}-t_{1}$ is given by:

$$
\begin{equation*}
\vec{a}=\frac{\vec{v}_{2}-\vec{v}_{1}}{\Delta t} . \tag{Eq. 2.1}
\end{equation*}
$$

If the instantaneous acceleration is constant throughout the time interval $\Delta t$, the velocity changes uniformly with time from $\vec{v}_{1}$ to $\vec{v}_{2}$. For this case, the average velocity $\left(\vec{v}_{\text {avg }}\right)$ of the particle between $t_{1}$ and $t_{2}$ is given as:

$$
\begin{equation*}
\vec{v}_{\text {avg }}=\frac{\vec{v}_{1}+\vec{v}_{2}}{2} . \tag{Eq. 2.2}
\end{equation*}
$$

Considering motion in one dimension, if the object is at positions $x_{1}$ and $x_{2}$ at $t_{1}$ and $t_{2}$, its average velocity in the time interval $\Delta t$ can be written as:

$$
\begin{equation*}
v_{a v g}=\frac{x_{2-} x_{1}}{\Delta t} \tag{Eq. 2.3}
\end{equation*}
$$

In one dimension, Eq. 2.2 and Eq. 2.3 reveal:

$$
\begin{equation*}
\frac{v_{1}+v_{2}}{2}=\frac{x_{2}-x_{1}}{\Delta t} . \tag{Eq. 2.4}
\end{equation*}
$$

When Eq. 2.1 and Eq. 2.4 are combined we obtain

$$
\begin{equation*}
v_{2}^{2}-v_{1}^{2}=2 a\left(x_{2}-x_{1}\right) . \tag{Eq. 2.5}
\end{equation*}
$$

When an object is released down an incline, it will move in one dimension and its speed will increase with a constant acceleration (neglecting friction) given as $a=g \sin \theta$, where $\theta$ is the angle of inclination and $g$ is the gravitational acceleration. Figure 2.1 shows a vector diagram of the object's acceleration. A plot of the acceleration along the inclined plane $\left(a_{t}\right)$ versus $\sin \theta$ should yield a straight line whose slope is equal to $g$.


Figure 2.1: Vector components of $g$

### 2.3.2 Experimental Error Calculation

Experimental errors determine the precision of a measurement. Any measured quantity $\alpha$ will be expressed as $\alpha_{m}$ that satisfies the equation $\alpha=\alpha_{m} \mp \Delta \alpha$ where $\alpha_{m}$ is the result of a measurement and $\Delta \alpha$ reveals the precision of the measurement as a result of the experimental errors. For a series of measurements, $\Delta \alpha$ is obtained by the standard deviation of the measured quantities $\alpha_{i}$ as:

$$
\begin{equation*}
\Delta \alpha=\sqrt{\frac{1}{N-1} \sum_{i=1}^{N}\left|\alpha_{i}-\alpha_{a v g}\right|^{2}} \tag{Eq. 2.6}
\end{equation*}
$$

where $\alpha_{a v g}=\frac{1}{N} \sum_{i=1}^{N} \alpha_{i}$ is the average of the measured values, $\alpha_{i}$, and $N$ is the total number of measurements.

### 2.4 Experiment

In the first part of this experiment, you will investigate the motion of an object moving along a line in a horizontal plane. The second part will involve the motion of the same object down an inclined plane under the influence of gravity.

### 2.4.1 Procedure for Horizontal Motion Experiment

2. Adjust the height of your track with the variable screw provided at one end. The track is level, if the bubble in the small tube on the leveling device is perfectly centered between the two lines on the tube.
3. The metal track on your desk serves as an almost frictionless plane (flat surface) for the motion of the studied object, which is a small cart. Unless the track is level, your results will produce significant error.
4. Place the cart on the track with the wheels in the grooves and test it by pushing it gently and then releasing it. The cart should slide very smoothly. Overhanging the track are the two photogates initially set about 20 cm apart.
5. Photogates are devices that sensitively record the time between successive regions of darkness and illumination on the picket fence. The picket fence is the small plastic card with rows of black and clear regions, which should be placed vertically on the top of your cart. The data from the photogates is transferred to the interface by the cables. The computer records the distance traveled by the cart as a function of time.
6. Fit the picket fence vertically onto the cart with the $1-\mathrm{cm}$ row facing up, and adjust the height of the photogates so that the $1-\mathrm{cm}$ uppermost row (the row with the smallest black and clear regions) will run past the eyes of the photogates as the cart slides down the track (see Figure 2.2).
7. There is a small red light on top of the photogates, which should blink 12 times as you push the cart slowly through the photogates.
8. If it blinks fewer times (such as 6 times), the height of the photogates needs to be raised. If it just lights up but doesn't blink as you roll the cart through the photogate that means you are rolling it too fast. Slow down and try again. If it doesn't light up at all, the photogates need to be lowered.


Figure 2.2: Horizontal Motion Experimental Setup.
9. Connect the photogate to digital channel 1 of the interface (see Figure 2.3). Make sure that, the photogate, the cart will pass through first, is connected to channel 1, and the other photogate to channel 2 . Turn on the interface using the switch on the back.


Figure 2.3: Proper wire connections to the signal interface.
10. Turn on the personal computer (PC). Open "DataStudio" software (If it still does not work, contact your instructor for help). Create a new experiment in the software. Introduce "photogate- picket fence" to the channel 1 and 2.
11. Change the "band spacing" setting to a new value of 0.01 m . To change the "band spacing" click on the first channel, then, you will find the band spacing option under "constants". Repeat the procedure for the second channel.
12. You are ready to start taking data. To record data, click "START" and then give a light push to the cart.
13. Make sure you are not touching the cart, before it enters the first photogate.
14. Click "STOP" after the cart has passed through the second photogate.
15. Make sure you click on "STOP", before the cart bounces back from the end of the track and reenters the second photogate. The data is labeled as "Run \# (a number)", e.g., "Run \#1" for the first trial.
16. Use the "TABLE" function to determine the cart's velocity at the photogates as functions of time.
17. Make sure you have 12 data points! If you do not, check the height of the photogates as described above. Notice that the position and velocity have different times. Also note that you can have your data presented as a graph by using the "GRAPH" option.
18. Record the velocity and corresponding time readings to TABLE 2.1.
19. We identify $v_{1}$ (or $v_{2}$ ) as the velocity measured at gate 1 (or 2 ), and $\mathrm{t}_{1}$ (or $\mathrm{t}_{2}$ ) as the time measured by gates 1 (or 2 ). To ensure consistency choose the FIRST value from each set of data points. We understand, therefore, that by $t_{1}$ we mean the instant of the measurement $v_{1} . \Delta x$ in TABLE 2.1 is the separation between the photogates (which you must measure from the same side on each photogate)
20. Repeat the procedure four more times.

### 2.4.2 Procedure for Motion on an Inclined Plane Experiment



Figure 2.4: Inclined Plane Experimental Setup

1. One end of the track is provided with a variable screw, which is used for leveling the track. Raise this end to a height H of about $\underline{\mathbf{x}} \mathrm{cm}$, and screw it tightly to the supporting rod using the rod clamp. Measure the height H from the tabletop to the bottom surface of the track at the very end of the raised end of the track (see Figure 2.4). Also measure L, the full length of the track.
2. Click on "START" and release the cart above the first gate.
3. Your partner should be waiting in alert, to prevent the cart from hitting the lower end of the track or falling off. The partner should wait, until the cart has cleared through the second gate, and then catch it before it hits the lower end. Don't forget to click on "STOP" after the cart is through the second gate.
4. Use the "TABLE" function to observe the recorded velocity values.
5. Record the velocity readings in both photogates in TABLE 2.2.
6. Using the "GRAPH" option, plot a graph of acceleration vs. time for each run. Repeat the experiment four more times for lower heights, $H$, keeping $\Delta x$, the separation between the photogates constant which you must measure from the same side on each photogate. In what follows, $v_{1}$ and $v_{2}$ are again the first values of each set of data points, and $a$ is the acceleration that you will calculate using. Remember to consider significant figures.
7. Do not delete your data before answering all questions.

$$
\Delta x=
$$

. (m)

TABLE 2.1. VALUES FOR HORIZONTAL MOTION (10 PTS)

| Run \# | $\begin{aligned} & v_{1} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $t_{1}$ (s) | $v_{2}(\mathrm{~m} / \mathrm{s})$ | $v_{2} \quad(\mathrm{~s})$ | $\begin{aligned} & \Delta t=t_{2}- \\ & t_{1}(\mathrm{~s}) \end{aligned}$ | $\begin{aligned} & v_{\text {avg }}{ }^{a}= \\ & \frac{x_{2}-x_{1}}{\Delta t} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $\begin{aligned} & v_{a v g}^{b}= \\ & \frac{v_{1}+v_{2}}{2} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $\begin{aligned} & \Delta v= \\ & v_{2}-v_{1} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $\begin{aligned} & a=\frac{\Delta v}{\Delta t} \\ & \left(\mathrm{~m} / \mathrm{s}^{2}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |

TABLE 2.2. VALUES FOR MOTION ON AN INCLINED PLANE (10 PTS)

| Run | $H$ | $L$ | $\sin \theta$ | $v_{1}$ | $v_{2}$ | $v_{1}{ }^{2}$ | $v_{2}{ }^{2}$ | $\Delta v^{2}$ <br> $=_{2}^{2}$ <br> $(\mathrm{~cm})$ <br> $(\mathrm{cm})$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

### 2.5 Report

### 2.5.1 Answer the following questions related to the horizontal motion experiment

1. (5 pts) After giving a slight push to the cart, what sort of motion would you expect from the cart on an ideal frictionless horizontal track (i.e., constant velocity, constant acceleration, changing acceleration, etc.)?
2. ( 5 pts) Considering that, friction exists in reality between the cart and the track, what deviations would you expect from your answer to question 1 ?
3. (5 pts) Compare $v_{\text {avg }}{ }^{a}$ with $v_{\text {avg }}{ }^{b}$. Do you expect them to be equal to each other? Explain.
4. (5 pts) Do your results confirm your predictions in questions 1-3? Explain.
5. (5 pts) What is your experimental error for acceleration? Show your calculations.

### 2.5.2 Answer the following questions related to the motion on an inclined plane experiment

1. ( 5 pts ) What kind of motion would you expect in this experiment (i.e., constant velocity, constant acceleration, changing acceleration, etc.)?
2. ( 5 pts ) How does the acceleration determined from TABLE 2.2 compare to the acceleration vs. time graphs in the readings from the both photogates?
3. ( 5 pts ) Calculate the measured value of $g$ from TABLE 2.2. What is your experimental error? Show your calculations.
4. ( 25 pts ) Plot a graph of $a$ versus $\sin \theta$, and draw a straight line that "best fits" (as many points above the line as below, all approximately equally spaced from the line) the 6 data points
5. ( 5 pts ) Find the percent error in the gravitational acceleration obtained from TABLE 2.2 and from the plot. (Take $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$ )

### 2.5.3 Conclusion (10 pts)

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.)

| T | L | - |  |  | , |  |  | $\square$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | - |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | - |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Completed by:
Name, Surname and ID of Lab partner(s):

## Department of Lab partner(s)

Verified by

## PHYS 101 / 3 Projectile Motion <br> Experiment 3

### 3.1 Instruments and Materials

- Signal interface
- Photogate
- Picket fence
- Projectile launcher and plastic balls
- Measuring tape
- Time of flight accessory
- Carbon paper, white paper


### 3.2 Aim

This experiment studies the relationship between the initial velocity, angle of projectile, initial height, time of flight and range in projectile motion.

### 3.3 Introduction

### 3.3.1 Theory of Projectile Motion

A projectile is an object with a certain initial velocity that moves under the effect of gravitational force. If air resistance is neglected, the motion of a projectile is influenced only by the constant gravitational acceleration, which is directed towards the planet Earth and does not depend on the velocity.


Figure 3.1. Diagram of a projectile motion.
If the projectile is launched with an initial velocity $\vec{v}_{0}$ at an angle $\theta, x$ and $y$ components of the velocity are given as

$$
\begin{align*}
& v_{0 x}=v_{0} \cos \theta  \tag{Eq. 3.1}\\
& v_{0 y}=v_{0} \sin \theta \tag{Eq. 3.2}
\end{align*}
$$

Since there is no acceleration in the horizontal direction, the horizontal component $v_{0 x}$ of the projectile's initial velocity remains unchanged throughout the motion and vertical component acceleration is $\vec{a}=$ $-\vec{g}$. So the the horizontal and vertical components of the velocity at any later time $t$ are

$$
\begin{gather*}
v_{x}=v_{0 x}  \tag{Eq. 3.3}\\
v_{y}=v_{0 y}-g t \tag{Eq. 3.4}
\end{gather*}
$$

When the projectile is launched from an initial height the horizontal and vertical distances travelled by the ball at any time $t$, are given by

$$
\begin{gather*}
x=v_{0 x} t=\left(v_{0} \cos \theta\right) t  \tag{Eq. 3.5}\\
y=y_{0}+\left(v_{0} \sin \theta\right) t-\frac{1}{2} g t^{2} \tag{Eq. 3.6}
\end{gather*}
$$

The range $R$ of a projectile is the horizontal distance that the projectile travels before hitting the ground. When the projectile is fired with an angle $\theta$ with horizontal, the vertical distance $h$ and the range $R$ are given by

$$
\begin{gather*}
-h=y_{f}-y_{0}=v_{0 y} T-\frac{1}{2} g T^{2}=\left(v_{0} \sin \theta\right) T-\frac{1}{2} g T^{2}  \tag{Eq. 3.7}\\
R=v_{x} T=v_{o x} T=\left(v_{0} \cos \theta\right) T \tag{Eq. 3.8}
\end{gather*}
$$

The time of flight, $T$ is found by solving the second order Eq. 3.7 for the vertical motion.

When the projectile is fired horizontally, i.e., if $\theta=0$, the vertical distance, $h$ is calculated by simplifying Eq. 3.7:

$$
\begin{equation*}
-h=y_{f}-y_{0}=-\frac{1}{2} g T^{2} \tag{Eq. 3.9}
\end{equation*}
$$

When the projectile launched with no initial height, the object reaches its maximum height exactly halfway through its motion. At this point the motion is horizontal, so we can find $t$ of this instant by setting $v_{y}=0$ in Eq. 3.4:

$$
\begin{equation*}
V_{0 y}-g t=0 \Rightarrow t=\frac{V_{0 y}}{g}=\frac{V_{0} \sin \theta}{g} \tag{Eq. 3.10}
\end{equation*}
$$

So, the total flight time $T$ is

$$
\begin{equation*}
T=2 t=2 \frac{V_{0} \sin \theta}{g} \tag{Eq. 3.11}
\end{equation*}
$$

The range $R$ of a projectile launched from the ground is the horizontal distance that the projectile travels before touching the ground. The quantity $R$ is given by

$$
\begin{equation*}
R=v_{x} T=\left(v_{0} \cos \theta\right) 2 t=\left(v_{0} \cos \theta\right) 2 \frac{v_{0}}{g} \sin \theta=\frac{v_{0}^{2}}{g} 2 \sin \theta \cos \theta ; R=\frac{v_{0}^{2}}{g} \sin (2 \theta) \tag{Eq. 3.12}
\end{equation*}
$$

### 3.3.2 RMS (root mean square) Calculation

Root mean square of a measurement $\alpha$ is calculated using the following formula:

$$
\begin{equation*}
\alpha_{R M S}=\sqrt{\frac{\sum_{i=1}^{N} \alpha_{i}^{2}}{N}} \tag{Eq. 3.13}
\end{equation*}
$$

where $N$ is the number of measurements.

### 3.4 Experiment

In the first part of this experiment, you will investigate the projectile motion of an object fired from an initial height horizontally. The initial velocity of the projectile will be measured by the two photogates attached to the end of the projectile launcher and compared to the initial velocity value calculated using the range, $R$ and the time of flight, $T$.

Then you will investigate the projectile motion of an object fired from an initial height with an angle $\theta$. The measured range, $R$ of the projectile will be compared to the range, $R$ value calculated using the time of flight, $T$ and the measured initial velocity, $v_{0}$.

In the second part of the experiment, you will estimate the firing angle for maximum range. You should note in this part that the projectile launcher used in the experiment has an initial height.

In the third part of the experiment, you will investigate the projectile motion of an object fired from an initial height with an angle $\theta$, measure the time of flight of the projectile, $T$ and calculate the acceleration due to gravity, $g$.

### 3.4.1 Procedure for Initial Velocity and Range Measurements



Figure 3.2. Projectile launched horizontally.

1. Turn on the Signal Interface, and then turn on the computer and open the "DATASTUDIO". Two photogates should be attached to the launcher to measure the initial velocity. The first photogate should be plugged into channel 1 and the second photogate should be plugged into channel 2 of the Signal Interface. The distance between the photogates should be about 10 cm .
2. Introduce "PHOTOGATE" to the digital channel 1 and channel 2 of the interface. Size of the object is irrelevant in this case so you can leave the default value.
3. Check that the Projectile Launcher is safely clamped to the table.
4. Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot horizontally. If the ball does not stay in the launcher, increase the angle slightly. Read the barrel angle using the plumb bob on the side of the launcher.


Figure 3.3. Ange adjustment on the projectile launcher.
5. Measure the vertical distance, $h$ from the bottom of the ball as it leaves the barrel to the floor. Record this distance in TABLE 3.1. Note that the position of the ball just before the ball leaves the launcher is marked on the launcher as below.


Figure 3.4. The initial position of the ball leaving the launcher.
6. Put the plastic ball into the Projectile Launcher and cock it to the short-range position. Do not look into the launcher or stay in front of it. Make sure that you are not firing at another person or at anything breakable like glass windows, doors, etc. Keep in mind that the balls are hard and bounce off the floor.


Figure 3.5. Range indications on the projectile launcher.
7. Fire one shot to locate where the ball hits the tabletop. At this position, tape a piece of white paper to the tabletop. Place a piece of carbon paper on top of this paper and tape it down. When the ball hits on the tabletop, it will leave a mark on the white paper.
8. To acquire data click "START" and fire one shot. To stop data collection, click "STOP".
9. Repeat the procedure four more times.
10. Measure the range $R$ along the floor from the release point of the ball to the landing point and record your measurement in TABLE 3.1.
11. Usig the measured vertical and horizontal distances, calculate the time of flight, $T$ and then the initial velocity, $v_{0, \text { calc }}$ as explained in the introduction.
12. Change the firing angle $\theta$ to $20^{\circ}$ and repeat the steps 5 to 11 .


Figure 3.6. Projectile launched with an angle $\theta$ with the horizontal.
Record your results in TABLE 3.2 as described in the introduction, first find the time of flight, T and then compute the expected range R_calc.

### 3.4.2 Procedure for Finding Angle of Maximum Range

1. Adjust the angle of the Projectile Launcher to $30^{\circ}$.
2. Fire one shot to locate where the ball hits the tabletop. At this position, tape a piece of white paper to the tabletop. Place a piece of carbon paper on top of this paper and tape it down.
3. To acquire data click "START" and using the short firing range fire one shot.
4. To stop data collection, click "STOP".
5. Measure and record the range in TABLE 3.3.
6. Make three measurements at each angle and record results in TABLE 3.3.
7. Repeat the procedure six more angles. Before proceeding to the next angle, make sure that you have collected enough data to answer the questions. Collect more data if necessary.

### 3.4.3 Procedure for $g$ Measurement

1. Adjust the angle of the Projectile Launcher to $60^{0}$.
2. Fire one shot to locate where the ball hits the tabletop. At this position, tape a piece of white paper to the tabletop. Place a piece of carbon paper on top of this paper and tape it down.
3. To acquire data click "START" and fire one shot to locate where the ball hits the tabletop.
4. Using the short firing range of the Projectile Launcher fire one shot. To stop data collection, click "STOP".
5. Repeat the procedure four more times.
6. Measure the times the ball passed through the first photogate and the second photogate by using "TABLE" option.
7. Record the time and range measurements in TABLE 3.4.
8. Calculate the initial speed of the ball for each run and record your results in TABLE 3.4.
9. Calculate the initial speed of the ball for each run and record your results in TABLE 3.4.
10. Calculate the acceleration due to gravity for each run and record your results in TABLE 3.4.

TABLE 3.1. VALUES FOR TIMES, VELOCITIES AND DISTANCES (10 PTS)
Vertical distance $h$ :
Distance between the photogates $\Delta x$ :
Launcher angle: $\theta=0^{0}$.

| Trial | Time (PG\#1) | $\begin{aligned} & \text { Time } \\ & \text { (PG\#2) } \end{aligned}$ | $v_{0}^{\text {meas }}(\mathrm{m} / \mathrm{s})$ | $\begin{aligned} & v_{0 x} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $\begin{aligned} & v_{0 y} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $\begin{array}{ll} \text { Range, } & R \\ (\mathrm{~m}) \end{array}$ | Time of flight, $T$ (s) | $v_{0}^{\text {calc }}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |
|  |  |  | $\begin{aligned} & v_{0}^{\text {meas }}(a v g) \\ & = \end{aligned}$ |  |  |  |  | $v_{0}^{\text {calc }}(a v g)=$ |

TABLE 3.2. VALUES FOR TIMES, VELOCITIES AND DISTANCES (10 PTS)
Vertical distance $h$ :
Distance between the photogates $\Delta x$ :
Launcher angle: $\theta=20^{\circ}$.

| Trial | Time (PG\#1) | Time (PG\#2) | $\begin{aligned} & v_{0}^{\text {meas }} \\ & (\mathrm{m} / \mathrm{s}) \end{aligned}$ | $\begin{array}{r} v_{o x} \\ (\mathrm{~m} / \mathrm{s}) \end{array}$ | $\begin{gathered} v_{o y} \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | Range, $R^{\text {meas }}$ (m) | Time of flight, $T$ (s) | $\begin{aligned} & \text { Range, } R^{\text {calc }} \\ & (\mathrm{m}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $R_{\text {avg }}^{\text {meas }}=$ |  | $R_{\text {avg }}^{c a l c}=$ |

TABLE 3.3. VALUES FOR TIMES, VELOCITIES AND DISTANCES (10 PTS)
Vertical distance $h$ :
Distance between the photogates $\Delta x$ :

| Angle | Range \# 1 | Range \# 2 | Range \# 3 | Average Range, <br> $R$ |
| :---: | :--- | :--- | :--- | :--- |
| $30^{0}$ |  |  |  |  |
| $35^{0}$ |  |  |  |  |
| $40^{0}$ |  |  |  |  |
| $45^{0}$ |  |  |  |  |
| $50^{0}$ |  |  |  |  |
| $55^{0}$ |  |  |  |  |
| $60^{0}$ |  |  |  |  |

TABLE 3.4. VALUES FOR TIMES AND VELOCITIES (10 PTS)
Vertical distance $h$ :
Distance between the photogates $\Delta x$ :
Launcher angle: $\theta=60^{\circ}$.

| Trial | Time (PG \#1) | Time (PG \#2) | Range (m) | $\begin{gathered} v_{0}^{\text {meas }} \\ (\mathrm{m} / \mathrm{s}) \end{gathered}$ | Time of Flight, $T$ (s) | $g$ $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
|  |  |  |  |  |  | $g_{a v g}=$ |

### 3.5 Report

### 3.5.1 Answer the following questions related to initial velocity and range measuremets $\left(\theta=0^{0}\right)$.

Q1-(10 pts) Calculate the rms (root mean square) deviations for the averages of the measured and the calculated velocity values.

Q2-(5 pts) What is the percent deviation between the calculated and the measured velocities?

### 3.5.2 Answer the following questions related to initial velocity and range measurements $\left(\theta=20^{0}\right)$.

Q3-(10 pts) Calculate the rms (root mean square) deviations for the averages of the measured and the calculated range values

Q4-(5 pts) How do the measured and the calculated ranges compare? Explain the possible reasons for differences.

### 3.5.3 Answer the following questios related to the angle $\boldsymbol{\theta}$ for maximum range measurements

Q5-(15 pts) Plot a graph on graph paper of horizontal distance versus $\theta$ to determine which angle gives the longest range.

Q6-(5 pts) Calculate the angle for the maximum range theoretically when $h=0$ (initial height above the ground). Compare this number with your answer to Q5? Explain the differences?

### 3.5.4 Answer the following question related to $\boldsymbol{g}$ measurements.

Q7-(10 pts) Calculate the experimental value for acceleration due to gravitiy, $g$ with its experimental error. Compare your result with the actual value of $g\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$ and explain the differences.

### 3.5.5 Conclusion ( $\mathbf{5} \mathbf{~ p t s}$ )

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.

|  |  |  |  |  |  |  | - |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  | - |  |  |  | $\square$ | $\square$ |  |  |  |  | $\square$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| \# |  |  |  |  |  |  | $\pm$ |  |  |  |  | $\square$ |  |  | \% |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\square$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\pm$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\#$ |  |  |
|  |  |  |  |  |  |  | - |  |  |  |  |  |  |  | $\square$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\square$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | - |  |  | \# |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | - |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Completed by:

Name, Surname and ID of Lab partner(s):

Department of Lab partner(s):
Verified by:

## PHYS 101 / 4 Dynamics of Motion Experiment 4

### 4.1 Instruments and Materials

- a signal interface
- one photogate
- a mass set and a mass hanger
- a small cart
- a metal track


### 4.2 Aim

The purpose of this experiment is to verify Newton's 2nd law and find the value for the coefficient of friction $\mu$ by measuring the acceleration of a cart while subject to varying net applied forces in horizontal plane and inclined plane, then calculate the coefficient of friction $\mu$ in static equilibrium.

### 4.3 Introduction

### 4.3.1 Theory

Newton's $2^{\text {nd }}$ law states that, the net force acting on an object is equal to the product of the mass and the acceleration of the object. The direction of the force is same as that of the acceleration. In equation form, we write:

$$
\begin{equation*}
\vec{F}_{n e t}=m \vec{a} \tag{Eq. 4.1}
\end{equation*}
$$

In this equation, $\vec{F}_{n e t}$ is the vector sum of all the forces acting on the object, $m$ is the mass of the object, and $\vec{a}$ is its acceleration. Remember that, acceleration and force are vector quantities.

When we consider an object in a horizontal motion and subject to an external force as the system shown below; the force diagram of the system, where $\vec{N}$ is the normal force exerted by the surface on the object:


Figure 4.1. Force diagram of the system

For mass, $m$ there is no acceleration in the vertical direction. By adding all the forces acting on the object in the vertical direction and applying Newton's second law, one obtains:

$$
\begin{array}{r}
F_{n e t, v e r t i c a l}=m a=0 \\
F_{n e t, v e r t i c a l}=N-m g=0 \tag{Eq. 4.3}
\end{array}
$$

So the normal force is

$$
\begin{equation*}
N=m g \tag{Eq. 4.4}
\end{equation*}
$$

Knowing that the magnitude of the frictional force is given by $F_{\text {friction }}=\mu N$, one can write

$$
\begin{equation*}
F_{\text {friction }}=\mu m g \tag{Eq. 4.5}
\end{equation*}
$$

where $\mu$ is the coefficient of friction between the object and the surface. For the motion along the string, both the object and the hanger have the same magnitude of the acceleration a (they are moving together). Then, taking $M$ as the total mass of the system $\left(M=m+m_{H}\right)$, we write for the forces along the string

$$
\begin{gather*}
F_{n e t, \text { string }}=M a  \tag{Eq. 4.6}\\
m_{H} g-\mu m g=m a+m_{H} a  \tag{Eq. 4.7}\\
m_{H} g-\mu m g=\left(m+m_{H}\right) a \tag{Eq. 4.8}
\end{gather*}
$$

Note that the frictional force opposes the motion. Dividing $m_{H} g-\mu m g=\left(m+m_{H}\right) a$ Eq. 4.8 by $m g$ and adding $\mu$ to both sides we get

$$
\begin{equation*}
\frac{m_{H}}{m}=\left(m+m_{H}\right)\left\{\frac{a}{m g}\right\}+\mu \tag{Eq. 4.9}
\end{equation*}
$$

To find the coefficient of friction $\mu$, you will use Eq. 4.9 in horizontal motion.

When the track is inclined by an angle $\theta$, the Eq. 4.8 can be written as;

$$
\begin{equation*}
m_{H} g-m g \sin \theta-\mu m g \cos \theta=\left(m+m_{H}\right) a \tag{Eq. 4.10}
\end{equation*}
$$

When the system is in static equilibrium;

$$
\begin{equation*}
m_{H} g-m g \sin \theta-\mu m g \cos \theta=0 \tag{Eq. 4.11}
\end{equation*}
$$

### 4.3.2 Linear Regression Method

Let us assume that we have a set of paired variables such as

$$
\begin{equation*}
\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{n}, y_{n}\right) \tag{Eq. 4.12}
\end{equation*}
$$

The equation for a straight line can be expressed as $y=A x+B$ where $y$ is estimated value of the dependent variable, $A$ is the intercept of the straight line and $B$ is the slope of the straight line.
$A$ and $B$ can be found by using following formulas:

$$
\begin{align*}
& A=\frac{n \sum_{i=1}^{n} x_{i} y_{i}-\sum_{i=1}^{n} x_{i} \sum_{i=1}^{n} y_{i}}{n \sum_{i=1}^{n} x_{i}^{2}-\left(\sum_{i=1}^{n} x_{i}\right)^{2}}  \tag{Eq. 4.13}\\
& B=\frac{1}{n}\left(\sum_{i=1}^{n} y_{i}-A \sum_{i=1}^{n} x_{i}\right) \tag{Eq. 4.14}
\end{align*}
$$

### 4.4 Experiment

In the first part of the experiment, you will determine the kinetic friction coefficient between the cart and the track and total mass of the system when the cart is in horizontal motion. You will use linear regression method to plot a graph. In the second part, the track will be inclined by an angle, and the kinetic friction coefficient will be calculated and compared by varying the mass of the cart and the angle of the incline. In the last part, the system will be kept in equilibrium in an inclined plane and static friction coefficient will be calculated and compared by varying the mass of the cart and the angle of the incline.

### 4.4.1 Procedure for finding coefficient of friction $\boldsymbol{\mu}$ in horizontal motion

In the mechanical system is shown in Figure 4.2, the cart is pulled along a horizontal surface by a string that passes over a pulley and is attached to a hanger, upon which various masses can be placed. The force of gravity on the hanger causes it to accelerate, which then accelerates the cart (with the same magnitude) along the track. By varying the mass on the hanger, you can vary the net force applied to the cart.


Figure 4.2. Experimental Setup for horizontal motion.

1. Turn on the Signal Interface, and then turn on the computer and open the "DATASTUDIO". The photogate should be plugged into channel 1 of the Signal Interface. Then introduce "photogate \& picket fence" to the interface.
2. Change the "band spacing" setting to a new value of 0.01 m by double clicking on the channel logo and click OK.
3. On your track is the dynamics cart attached to a mass hanger with a string running smoothly over an ideally frictionless pulley. Weigh the cart and the hanger, and record their masses. Initially the mass hanger is empty. Place 20 grams of mass on the hanger of 10 grams and 500 grams on top of the cart. At this point we define the total mass of the cart as $m$, and the total mass of the hanger as $m_{H}$.
4. Place the hanger along with the masses in a hanging position, running the string gently over the pulley, making sure that it is in the groove of the pulley and that the string runs parallel to the track when stretched over the pulley. Pull the cart away to the other end of the track until the mass hanger is just below the pulley. Make sure that the photogate is in front of the cart and that it is not too far away (move the photogate if necessary).
5. Release the cart, immediately click "START" before the cart enters the photogate, and then click "STOP" after the cart passes through the photogate. Stop the cart with your hand before the cart hits the end of the track or the hanger hits the floor.
6. Record the average acceleration in TABLE 4.1.
7. To change the net force being applied to the cart while keeping the total system mass constant, transfer masses from the top of the cart to the hanger. Make sure the mass that you put on the hanger does not exceed 100 g . Record the new values of $m$ and $m_{H}$, and repeat the experiment four more times, remembering each time to transfer some mass from the cart to the hanger below, and to record the average acceleration. Also calculate $\left\{m_{H} / m\right\}$ and $\{a / m g\}$, and record these values under the appropriate headings in TABLE 4.1.

Consider Eq. 4.9 which resembles the equation of a line $y=A x+B$. Graphing $y$ versus $x$ for an equation of this form gives a straight line with slope $A$ and $y$-intercept $B$. Thus, we have the following correspondences:

$$
y=m_{H} / m \quad A=m+m_{H} \quad x=a / m g \quad B=\mu
$$

Note that $A=m+m_{H}$ is constant in all runs. In the graph of $\left\{m_{H} / m\right\}$ versus $\{a / m g\}, y$-intercept gives $\mu$, and the slope of the line drawn by linear regression method gives $m+m_{H}$.

### 4.4.2 Procedure for finding coefficient of friction, $\mu$ in motion on an inclined plane.

This time you will measure the force when the cart is moving on an inclined plane.


Figure 4.3. Experimental Setup for inclined plane.

1. One end of the track is provided with a variable screw. Raise this end to a height H of about 15 cm , and screw it tightly to the supporting rod using the rod clamp. Measure the height from the table top to the bottom surface of the track at the very end of the raised end of the track. Also measure L , the full length of the track.
2. Place 90 g on the hanger. Making the total hanger mass 100 g .
3. Release the system and record the acceleration of the cart in TABLE 4.2.
4. Repeat the procedure for lower heights for four more times and record your readings in TABLE 4.2.
5. Add 500 g on top of the cart. Repeat the steps 2 to 4 and record your readings in TABLE 4.3.

### 4.4.3 Procedure for coefficient of friction, $\mu$ in static equilibrium on an inclined plane.

This time you will measure the force to keep the cart in equilibrium in an inclined plane.

1. Raise this end to a height H of about 15 cm , and screw it tightly to the supporting rod using the rod clamp. Measure the height from the table to the bottom surface of the track at the very end of the raised end of the track. Also measure $L$, the full length of the track.
2. Place masses on the hanger to achieve static equilibrium in the system. Record the mass on the hanger in TABLE 4.4.
3. Repeat the procedure for lower heights for four more times.
4. Add 500 g on top of the cart. Repeat the steps $2 \& 3$ and record your readings in TABLE 4.5.

TABLE 4.1. VALUES FOR FINDING $\mu$ IN HORIZONTAL PLANE (5 PTS)
$\left(m+m_{H}\right)_{\text {scale }}=1030 \mathrm{~g}$

| Run \# | $\mathrm{m}(\mathrm{kg})$ | $\mathrm{m}_{\mathrm{H}}(\mathrm{kg})$ | $\mathrm{a}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathrm{m}_{\mathrm{H}} / \mathrm{m}$ | $\mathrm{a} / \mathrm{mg}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |

TABLE 4.2. VALUES FOR FINDING $\mu$ IN INCLINED PLANE (5 PTS)
$m=500 \mathrm{~g}$

| Run \# | L | H | $\operatorname{Sin} \theta$ | $\operatorname{Cos} \theta$ | $\mathrm{m}_{\mathrm{H}}$ | $\mathrm{a}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mu$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |

TABLE 4.3. VALUES FOR FINDING $\mu$ IN INCLINED PLANE (5 PTS)
$m=1000 \mathrm{~g}$

| Run \# | L | H | $\operatorname{Sin} \theta$ | $\operatorname{Cos} \theta$ | $\mathrm{m}_{\mathrm{H}}$ | $\mathrm{a}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mu$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |

TABLE 4.4. VALUES FOR FINDING $\mu$ IN STATIC EQUILIBRIUM (5 PTS) $m=500 g$

| Run \# | L | H | $\operatorname{Sin} \theta$ | $\operatorname{Cos} \theta$ | $m_{H}$ | $\mu$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |

TABLE 4.5. VALUES FOR FINDING $\mu$ IN STATIC EQUILIBRIUM (5 PTS)
$m=1000 \mathrm{~g}$

| Run \# | L | H | $\operatorname{Sin} \theta$ | $\operatorname{Cos} \theta$ | $m_{H}$ | $\mu$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |

### 4.5 Report

### 4.5.1 Answer the following questions related to finding $\boldsymbol{\mu}$ in motion in horizontal plane

1. (25 pts) Using linear regression method, plot the graph of $\left\{m_{H} / m\right\}$ versus $\{a / m g\}$, and find the total mass of the system, $\left(m+m_{H}\right)$ and the coefficient of friction between the track and the cart $\mu$.
2. ( 5 pts ) Compare the total mass of the system measured by a scale and claculated from the slope of the graph.
3. (5 pts) We have made several assumptions in the experiment. State one of these assumptions; did the experiment invalidated or confirmed this assumption?

### 4.5.2 Answer the following questions related to finding $\boldsymbol{\mu}$ in motion in inclined plane

4. (10 pts) Calculate the average of the coefficient of kinetic friction using the measurements in TABLE 4.2 and TABLE 4.3.
5. ( 5 pts ) How does the mass of the cart effect the coefficient of kinetic friction between the cart and the track? Explain.
6. ( 5 pts ) How does the angle of the incline effect the coefficient of kinetic friction between the cart and the track? Explain.

### 4.5.3 Answer the following questios related to $\boldsymbol{\mu}$ in static equilibrium

(10 pts) Calculate the static friction between the cart and the track using the measurements in TABLE 4.4 and TABLE 4.5
7. (5 pts) Compare the coefficient of kinetic and static friction between the cart and the track you calculated. Do your results agree with the theory? Explain.

### 4.5.4 Conclusion (10 pts)

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.)


## Completed by:

Name, Surname and ID of Lab partner(s):

## Department of Lab partner(s): <br> Verified by:

## PHYS 101 / 5 Conservation of Linear Momentum Experiment 5

### 5.1 Instruments and Materials

- Two dynamics carts with picket fences
- Dynamics cart track
- Two photogates with mounting bracket
- Computer interface


### 5.2 Aim

The purpose of this experiment is to investigate the linear momentum conservation and the kinetic energy conservation of two objects before and after an elastic or inelastic collision.

### 5.3 Introduction

### 5.3.1 Conservation of Momentum

The linear momentum of an object is the product of its mass and velocity. Newton's second law tells us that the motion of a system of $N$ particles is determined by the external forces acting on each individual particle separately,

$$
\begin{equation*}
\sum_{i=1}^{N} \vec{F}_{i}^{e x t}=\sum_{i=1}^{N} \frac{d \vec{p}_{i}}{d t} \tag{Eq. 5.1}
\end{equation*}
$$

where $\vec{p}_{i}=m_{i} \vec{v}_{i}$ is the linear momentum of the $i$ th particle.

If the net external force is zero, that is $\sum_{i=1}^{N} \vec{F}_{i}^{\text {ext }}=0$, then the total momentum $\vec{p}=\sum_{i=1}^{N} \vec{p}_{i}$ of the system is conserved

$$
\begin{equation*}
\vec{p}_{\text {final }}=\vec{p}_{\text {initial }} \tag{Eq. 5.2}
\end{equation*}
$$

During a collision in a straight line, $\vec{p}_{\text {final }}=\vec{p}_{\text {initial }}$
is expressed in terms of masses and velocities of the objects as

$$
\begin{equation*}
m_{1} v_{1 x, \text { final }}+m_{2} v_{2 x, \text { final }}=m_{1} v_{1 x, \text { initial }}+m_{2} v_{2 x, \text { initial }} \tag{Eq. 5.3}
\end{equation*}
$$

### 5.3.2 Conservation of Mechanical Energy

In a system of $N$ particles, total kinetic energy of the system is defined by $K=\sum_{i=1}^{N} \frac{1}{2} m_{i} v_{i}^{2}$ where $m_{i}$ is the mass of the $i$ th particle and $v_{i}$ is its velocity. According to the work-energy theorem, the change in the kinetic energy is equal to the work done $W$ on the system.

$$
\begin{equation*}
W=K_{\text {final }}-K_{\text {initial }} . \tag{Eq. 5.4}
\end{equation*}
$$

Work done by a conservative force is independent of path and this enables us to make use of a welldefined potential energy function that is defined as the negative integral of the net force along any given path. Considering the motion of the system from its initial to its final state, this is nothing but the work done in the process:

$$
\begin{gather*}
K_{\text {final }}-K_{\text {initial }}=W=-\left(U_{\text {final }}-U_{\text {initial }}\right)  \tag{Eq. 5.5}\\
K_{\text {final }}+U_{\text {final }}=K_{\text {initial }}+U_{\text {initial }} \tag{Eq. 5.6}
\end{gather*}
$$

In this case, the term on the left is the total energy after the collision while the right hand side is the total energy before the collision. Then $K_{\text {final }}+U_{\text {final }}=K_{\text {initial }}+U_{\text {initial }}$ Eq. 5.6 can be rewritten as

$$
\begin{equation*}
E_{\text {final }}=E_{\text {initial }} \tag{Eq. 5.7}
\end{equation*}
$$

where $E=K+U$ is the total mechanical energy. Considering the collision of two objects in an isolated system, the net external force acting on the system is zero; because the normal force cancels the gravitational force and we ignore the small frictional force. Consequently, there is no change in the potential energy and Eq. 5.7 can be simplified as

$$
\begin{equation*}
K_{\text {final }}=K_{\text {initial }} \tag{Eq. 5.8}
\end{equation*}
$$

During a collision in a straight line, in terms of masses and velocities of the objects Eq. 5.8 is written as

$$
\begin{equation*}
\frac{1}{2} m_{1} v_{1 x, \text { final }}^{2}+\frac{1}{2} m_{2} v_{2 x, \text { final }}^{2}=\frac{1}{2} m_{1} v_{1 x, \text { initial }}^{2}+\frac{1}{2} m_{2} v_{2 x, \text { initial }}^{2} \tag{Eq. 5.9}
\end{equation*}
$$

### 5.4 Experiment

In this experiment, you will perform the elastic and inelastic collisions of two carts on a straight track. You will measure their masses and the velocities and calculate momentum and kinetic energy of two carts before and after they collide with each other. Then analyze the momentum and kinetic energy conservation in both type of collisions.

### 5.4.1 Procedure for Elastic Collision

1. Check your equipment set-up. The first and the second photogates should be located at 30 cm . and 70 cm . marks on the track respectively. The first photogate must be plugged into channel 1 and the second photogate must be plugged into channel 2.
2. Change the "Band Spacing" setting to a new value of $\mathbf{0 . 0 1 m}$ by clicking on a channel logo. Click OK, and do the same for the second channel.
3. Make sure the cables of the photogates do not block the motion of the carts.
4. At the beginning of the each data run, position the cart \#1 in front of the first photogate and cart \#2 in front of the second photogate. Orient the carts as shown in Figure 5.1. Springs on the carts have important role in the experiment, pay attention to the direction of the carts.


Figure 5.1. Experimental setup for elastic collision.
5. To acquire data click "START" then give a push to cart \#1. Click "STOP" after two carts have (or the moving cart has) left the area between two photogates.
6. Add or take away black bars as appropriate to have the total cart mass indicated in TABLE 5.1 for each run.
7. Record the velocities of the two carts in TABLE 5.1.
8. Always take the velocity reading on the last picket on a cart just before the collision and the velocity reading on the first picket on a cart just after the collision.
9. If both carts go through a single photogate, your second measurement will be the $14^{\text {th }}$. data on the table. As the first 12 data will belong to the picket fence on the first cart. The following, very small velocity data will indicate the large time gap between the carts. The $14^{\text {th }}$ data will be the velocity in the first picket on the second cart.
10. If a cart goes through one photogate twice, your second measurement will again be the $14^{\text {th }}$. data on the table for the same reason.
11. If we take the positive direction as the direction in which you initially pushed cart \#1, then, each time a cart passes through a photogate in this direction, the velocity should be recorded as a positive number.
12. Conversely, each time a cart passes through a photogate in the opposite direction, the velocity should be negative.
13. Compute the total momentum before and after the collision and the percentage change in the momentum of the carts for each combination of masses and record the results in the TABLE 5.2.
14. Compute the total kinetic energy before and after the collision and the percentage change in the kinetic energy of the carts for each combination of masses and record the results in the TABLE 5.3.

### 5.4.2 Procedure for Inelastic Collision

1. Again the first and the second photogates should be located at 30 cm . and 70 cm . marks on the track respectively. The first photogate must be plugged into channel 1 and the second photogate must be plugged into channel 2. The "Band Spacing" is kept in the value of $\mathbf{0 . 0 1 m}$.
2. Make sure the cables of the photogates do not block the motion of the carts.
3. At the beginning of the each data run, position the cart \#1 in front of the first photogate and cart \#2 in front of the second photogate. Orient the carts as shown in Figure 5.2. Velcro sticky pads have important role in the experiment, pay attention to the direction of the carts so that the two carts stick after the collision as shown in Figure 5.3.

Before the Collision


Figure 5.2. Experimental setup for inelastic collision. Before the collision.


Figure 5.3. Experimental setup for inelastic collision. After the collision.
4. To acquire data click "START" then give a push to cart \#1. Click "STOP" after two carts have (or the moving cart has) left the area between two photogates.
5. Add or take away black bars as appropriate to have the total cart mass indicated in the TABLE 5.4 for each run.
6. Record the velocities of the carts in TABLE 5.4.
7. Always take the velocity reading on the last picket on a cart just before the collision and the velocity reading on the first picket on a cart just after the collision.
8. After the inelastic collision, the carts will stick together and go through the second photogate together. The final velocity measurement will be the reading on the first picket of the two carts combined just after the collision.
9. If we take the positive direction as the direction in which you initially pushed cart \#1, then, each time a cart passes through a photogate in this direction, the velocity should be recorded as a positive number.
10. Conversely, each time a cart passes through a photogate in the opposite direction, the velocity should be negative.
11. Compute the total momentum before and after the collision and the percentage change in the momentum of the carts for each combination of masses and record the results in the TABLE 5.5.
12. Compute the total kinetic energy before and after the collision and the percentage change in the kinetic energy of the carts for each combination of masses and record the results in the TABLE 5.6.

TABLE 5.1. MASS AND VELOCITY VALUES FOR ELASTIC COLLISION (10 PTS)

|  |  | CART \# 1 | CART \# 2 |
| :---: | :---: | :---: | :---: |
| 1 | $M(\mathrm{~kg})$ | 0.5 | 0.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 2 | $M(\mathrm{~kg})$ | 1 | 0.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 3 | $M(k g)$ | 0.5 | 1 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 4 | $M(k g)$ | 1.5 | 0.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 5 | $M(k g)$ | 0.5 | 1.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |

TABLE 5.2. MOMENTUM VALUES FOR ELASTIC COLLISION (10 PTS)

| RUN \# | Total Initial Momentum <br> $(\mathrm{kgm} / \mathrm{s})$ | Total final Momentum <br> $(\mathrm{kgm} / \mathrm{s})$ | Percent Change <br> in Momentum |
| :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

TABLE 5.3. KINETIC ENERGY VALUES FOR ELASTIC COLLISION (10 PTS)

| RUN \# | Total Initial Kinetic <br> Energy <br> (Joule) | Total Final Kinetic Energy <br> (Joule) | Percent Change in <br> Kinetic Energy |
| :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

TABLE 5.4. MASS AND VELOCITY VALUES FOR INELASTIC COLLISION (10 PTS)

|  |  | CART \# 1 | CART \# 2 |
| :---: | :---: | :---: | :---: |
| 1 | $M(\mathrm{~kg})$ | 0.5 | 0.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 2 | $M(\mathrm{~kg})$ | 1 | 0.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 3 | $M(\mathrm{~kg})$ | 0.5 | 1 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 4 | $M(\mathrm{~kg})$ | 1.5 | 0.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(\mathrm{m} / \mathrm{s})$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
| 5 | $M(\mathrm{~kg})$ | 0.5 | 1.5 |
|  | $v_{\text {initial }}(\mathrm{m} / \mathrm{s})$ |  | 0 |
|  | $v_{\text {final }}(m / s)$ |  |  |
|  | $M v_{\text {initial }}(\mathrm{kgm} / \mathrm{s})$ |  |  |
|  | $M v_{\text {final }}(\mathrm{kgm} / \mathrm{s})$ |  |  |

TABLE 5.5. MOMENTUM VALUES FOR INELASTIC COLLISION (10 PTS)

| RUN \# | Total Initial Momentum <br> $(\mathrm{kgm} / \mathrm{s})$ | Total final Momentum <br> $(\mathrm{kgm} / \mathrm{s})$ | Percent Change <br> in Momentum |
| :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

TABLE 5.6. KINETIC ENERGY VALUES FOR INELASTIC COLLISION (10 PTS)

| RUN \# | Total Initial Kinetic <br> Energy <br> (Joule) | Total Final Kinetic Energy <br> (Joule) | Percent Change in <br> Kinetic Energy |
| :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

### 5.5 Report

### 5.5.1 Answer the following questions related to elastic collision

1. ( 5 pts ) How does the total momentum before the elastic collision compare with the total momentum after the collision? Use your results in TABLE 5.2. If the momentum was not conserved, what might have caused the change?
2. (10 pts) Compare the total kinetic energy before and after the elastic collision by using the results in TABLE 5.3. If there is a change in the total kinetic energy, what might have caused to this change (where did the energy go)?

### 5.5.2 Answer the following questions related to inelastic collision

1. ( 5 pts ) How does the total momentum before the inelastic collision compare with the total momentum after the collision? Use the results in TABLE 5.5. If the momentum was not conserved, what might have caused the change?
2. (10 pts) Compare the total kinetic energy before and after the inelastic collision by using the results in TABLE 5.6. If there is a change in the total kinetic energy, what might have caused to this change (where did the energy go)?

### 5.5.3 Conclusion ( 10 pts )

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.)

Completed by:
Name, Surname and ID of Lab partner(s):
Department of Lab partner(s):
Verified by:

## PHYS 101 / 6 Rotational Motion Experiment 6

### 6.1 Instruments and Materials

- Signal interface
- One photogate
- Mass set and a mass hanger
- Rotation apparatus
- Two disks
- Two screws and a screwdriver


### 6.2 Aim

The purpose of this experiment is investigate the relationship between the torque, rotational acceleration, linear acceleration and moment of inertia of a solid disk rotating about its central axis.

### 6.3 Introduction

### 6.3.1 Theory

We can describe the motion of the rotating body by specifying the angular position with respect to a reference line.


Figure 6.1. Rotation axis (out of page).

The average angular velocity of an object rotated by an angular displacement $\Delta \theta=\theta_{2}-\theta_{1}$ in time interval $\Delta t=t_{2}-t_{1}$ is:

$$
\begin{equation*}
\omega_{a v r}=\frac{\theta_{2}-\theta_{1}}{t_{2}-t_{1}}=\frac{\Delta \theta}{\Delta t} \tag{Eq. 6.1}
\end{equation*}
$$

The instantaneous angular velocity $\omega$ can be found by taking the limit of an infinitesimal time interval. Thus,

$$
\begin{equation*}
\omega=\lim _{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}=\frac{d \theta}{d t} \tag{Eq. 6.2}
\end{equation*}
$$

Therefore, if $\theta(t)$ is known, the angular velocity $\omega$ can be found by differentiation. In general the angular velocity of a rotating body may not be constant, i.e., the body may have an angular acceleration. Let $\omega_{2}$ and $\omega_{1}$ be the angular velocities at times $t_{2}$ and $t_{1}$, respectively. The average angular acceleration of the rotating body in the interval from $t_{1}$ to $t_{2}$ is defined as

$$
\begin{equation*}
\alpha_{a v r}=\frac{\omega_{2}-\omega_{1}}{t_{2}-t_{1}}=\frac{\Delta \omega}{\Delta t} \tag{Eq. 6.3}
\end{equation*}
$$

where $\Delta \omega$ is the change in the angular velocity that occurs during the time interval $\Delta t$. The instantaneous angular acceleration $\alpha$ is the limiting value of $\alpha_{a v r}$ when $\Delta t$ approaches zero

$$
\begin{equation*}
\alpha=\lim _{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}=\frac{d \omega}{d t} \tag{Eq. 6.4}
\end{equation*}
$$

Figure 6.2 shows the apparatus, you will use in this experiment.


Figure 6.2. Experimental Rotation Apparatus.

Let us call the mass of the rotating disk as $M$ and its radius as $R$, and the mass of the hanging object as $m$. The motion of the hanging object and the rotating disk can be analyzed using the free body diagrams. If we take the positive direction of the y-coordinate for $m$ to be downward, Newton's second law applied to the hanging object gives

$$
\begin{equation*}
m g-T=m a \tag{Eq. 6.5}
\end{equation*}
$$

where $T$ is the tension in the rope and $a$ is the acceleration of the hanging object. This tension will create a torque on the disk. Since the tension force and the radial vector are perpendicular, the vector torque equation

$$
\begin{equation*}
\overrightarrow{\boldsymbol{\tau}}=\vec{R} \times \vec{T} \tag{Eq. 6.6}
\end{equation*}
$$

can be written as $\tau=R T$. Then, we ge

$$
\begin{equation*}
\tau=R m(g-a) \tag{Eq. 6.7}
\end{equation*}
$$

Since the rope does not slip, the acceleration of the rope is the same as the tangential acceleration of a point on the cylinder rim. Therefore, the acceleration of the hanging mass $a$ and the rotational acceleration of the disk $\alpha$ are related as $a=\alpha R$. Note that the moment of inertia of a solid disk around its symmetry axis is

$$
\begin{equation*}
I_{\text {disk }}=M R^{2} / 2 \tag{Eq. 6.8}
\end{equation*}
$$

When another solid disk with the same axis of rotation is attached to the initial solid disk, then the moment of inertia of the combined system will be the sum of the moment of inertia of the two disks

$$
\begin{equation*}
I_{\text {Total }}=I_{\text {disk } 1}+I_{\text {disk } 2} \tag{Eq. 6.9}
\end{equation*}
$$

Using the relation between the torque and the rotational acceleration, $\tau=I_{\text {total }} \alpha$, one can determine the moment of inertia using the measured acceleration of the hanging mass as

$$
\begin{equation*}
I_{\text {total }}=\frac{g-a}{a} m R^{2} \tag{Eq. 6.10}
\end{equation*}
$$

From Eq. 6.8 and Eq. 6.10we can write

$$
\begin{equation*}
a=\frac{m g R}{m R+\frac{I_{\text {total }}}{R}} \tag{Eq. 6.11}
\end{equation*}
$$

which enables us to determine whether the experimental acceleration results are in the expected range.

### 6.4 Experiment

In the first part of the experiment, you will measure the acceleration of a hanging mass attached to the rim of a solid disk. You will calculate the moment of inertia of the disk using Eq. 6.8 and measure the moment of inertia using Eq. 6.10 and compare your results. Then, you will attach an additional disk with the same axis of rotation, measure the acceleration, calculate and compare the theoretical (using Eq. 6.8 and Eq. 6.9) and experimental moment of inertia (using Eq. 6.10) of the two disk system. In the last step, you will attach another disk with the mass and radius, and repeat the same procedure for the three disk system.

### 6.4.1 Procedure

1. Turn on the Signal Interface, and then turn on the computer and open the "DATASTUDIO". The photogate should be plugged into channel 1 of the Signal Interface. Then introduce "photogate \& picket fence" to the interface.
2. Change the "band spacing" setting to a new value of 0.11 m . ( 0.11 m is the distance between the holes on the disk).
3. Measure and record the radius and mass of the pulley, and moment of inertia of the pulley in TABLE 6.1.
4. Next, place the hanger along with the masses in a hanging position. Run the string gently over the pulley; make sure it is in the groove of the pulley.
5. Put 30 g mass on the hanger, wind the rope on the rim of the pulley and hold it.
6. Release the pulley, and immediately click "START", remembering to click "STOP" before the hanging mass hits the tabletop.
7. Click on table option, drag it over to channel 1 and select acceleration and click OK. Use the $\Sigma$ function to obtain the average acceleration and record the result in TABLE 6.2.
8. Repeat the procedure four more times, follow the same procedure as in step 7 and record the acceleration results in TABLE 6.2.
9. Place 50 g mass on the hanger, repeat the steps 5-8 and record your results in TABLE 6.2.
10. Measure and record the radius and mass and total moment of inertia of pulley and Disk 1 in TABLE 6.3.
11. Attach Disk 1 to the pulley using the two screws and a screwdriver. Make sure that it is fixed properly (not very tight or loose). Note then the moment of inertia of combined system of the pulley (a solid disk) and the attached disk is the sum of their moment of inertia ( $I_{\text {Total }}=$ $\left.I_{\text {pulley }}+I_{\text {disk }}\right)$.
12. Repeat the steps $4-9$ and record your results in TABLE 6.4.
13. Measure and record the radius, mass and total moment of inertia of pulley, Disk 1 and Disk 2 in TABLE 6.5.
14. Attach Disk 2 to the pulley and Disk 1 system.
15. Repeat the steps $4-9$ and record your results in TABLE 6.6.
16. Remove the Disks and place the screws and screw driver in the given container.

TABLE 6.1. VALUES FOR MASS, RADIUS AND THEORETICAL MOMENT OF INERTIA OF THE PULLEY (2 PTS)

| $\mathrm{R}_{\text {pulley }}$ |  |
| :---: | :--- |
| $\mathrm{M}_{\text {pulley }}$ |  |
| $I_{\text {pulley }}$ |  |

TABLE 6.2. VALUES FOR MASS AND ACCELERATION OF THE HANGING OBJECT AND THE EXPERIMENTAL MOMENT OF INERTIA OF THE PULLEY (8 PTS)


TABLE 6.3. VALUES FOR MASS, RADIUS AND THEORETICAL MOMENT OF INERTIA OF THE PULLEY AND DISK 1 (3 PTS)

| $\mathrm{R}_{\text {pulley }}$ |  |
| :---: | :--- |
| $\mathrm{M}_{\text {pulley }}$ |  |
| $I_{\text {pulley }}$ |  |
| $\mathrm{R}_{\text {Disk1 }}$ |  |
| $\mathrm{M}_{\text {Disk1 }}$ |  |
| $I_{\text {Disk1 }}$ |  |
| $I_{\text {Total }}$ |  |

TABLE 6.4. VALUES FOR MASS AND ACCELERATION OF THE HANGING OBJECT AND THE EXPERIMENTAL MOMENT OF INERTIA OF THE PULLEY AND DISK 1 SYSTEM (8PTS)

| Run \# | M | a |  |
| :--- | :--- | :--- | :--- |
| 1 | $\mathrm{~m}_{1=}$ |  | $I_{\text {exp }}$ |
| 2 | $\mathrm{~m}_{1=}$ |  |  |
| 3 | $\mathrm{~m}_{1=}$ |  |  |
| 4 | $\mathrm{~m}_{1=}$ |  | $I_{1}$ |
| 5 | $\mathrm{~m}_{2=}$ |  |  |
| 1 | $\mathrm{~m}_{2=}$ |  |  |
| 2 | $\mathrm{~m}_{2=}$ |  | $I_{\text {total }}$ |
| 3 | $\mathrm{~m}_{2=}$ |  |  |
| 5 | $\mathrm{~m}_{2=}$ |  |  |
|  |  |  |  |

TABLE 6.5. VALUES FOR MASS, RADIUS AND THEORETICAL MOMENT OF INERTIA OF THE PULLEY, DISK 1 AND DISK 2 (5 PTS)

| $\mathrm{R}_{\text {pulley }}$ |  |
| :---: | :--- |
| $\mathrm{M}_{\text {pulley }}$ |  |
| $I_{\text {pulley }}$ |  |
| $\mathrm{R}_{\text {Disk1 }}$ |  |
| $\mathrm{M}_{\text {Disk1 }}$ |  |
| $I_{\text {Disk1 }}$ |  |
| $R_{\text {Disk2 }}$ |  |
| $\mathrm{M}_{\text {Disk2 }}$ |  |
| $I_{\text {Disk2 }}$ |  |
| $I_{\text {Total }}$ |  |

TABLE 6.6. VALUES FOR MASS AND ACCELERATION AND THE EXPERIMENTAL MOMENT OF INERTIA OF THE PULLEY,DISK1 AND DISK2 SYSTEM (8 PTS)

| Run \# | M | a |  |
| :--- | :--- | :--- | :--- |
| 1 | $\mathrm{~m}_{1=}$ |  | $I_{\text {exp }}$ |
| 2 | $\mathrm{~m}_{1=}$ |  |  |
| 3 | $\mathrm{~m}_{1=}$ |  |  |
| 4 | $\mathrm{~m}_{1=}$ |  | $I_{1}$ |
| 5 | $\mathrm{~m}_{1=}$ |  |  |
| 1 | $\mathrm{~m}_{2=}$ |  |  |
| 2 | $\mathrm{~m}_{2=}$ |  | $I_{2}$ |
| 3 | $\mathrm{~m}_{2=}$ |  | $I_{\text {total }}$ |
| 4 |  |  |  |

### 6.5 Report

6.5.1 Questions (give detailed answers to each question)

1. ( 15 pts ) Calculate the true and experimental values of moment of inertia $I$ of pulley, two disk system of pulley and disk 1 , three disk system of pulley, disk 1 and disk 2.
2. ( 5 pts ) Find the percent error of the experimental values of $I$. If your error is large (more than $5 \%$ ), what might have been the cause of the encountered errors?
3. (15) Calculate the acceleration of the hanging mass $a$ theoretically for each run for pulley, two disk system of pulley and disk 1, three disk system of pulley, disk 1 and disk 2.
4. ( 5 pts ) Find the percent error for the experimental results of $a$. If your error is large (more than $5 \%$ ), what might have been the cause of the encountered errors?
5. (15 pts) Plot a graph of moment of inertia, I versus mass of rotating solid disk $M$, considering all three cases.
6. Determine the radius of the two disks, $r$. Compare your result with the measured radii of the Disk1 and Disk2.

### 6.5.2 Conclusion (10 pts)

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.)

| - | - |  |  | - |  |  |  | - | T |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | , |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Completed by:
Name, Surname and ID of Lab partner(s):

| Department of Lab partner(s): |
| :--- |
| Verified by: |

## PHYS 101 / 7 Physical Pendulum <br> Experiment 7

### 7.1 Instruments and Materials

- Signal interface
- Photogate
- Pendulum bar
- Support rod and clamp
- Measuring tape
- Rectangular shape


### 7.2 Aim

The purpose of this experiment is to measure the acceleration due to gravity using a physical pendulum, and determine the moment of inertia of a physical pendulum from the period of oscillation.

### 7.3 Theory

A physical pendulum is a rigid object which swings freely about some pivot point. In this experiment, the physical pendulum is a thin bar with length L , and it pivots about a fixed point a distance d from the center of mass shown in Figure 7.1.


Figure 7.1. Physical pendulum.
The torque of the gravitational force with respect to the point $O$ is given by

$$
\begin{equation*}
\vec{\tau}=\vec{r} \times m \vec{g} \tag{Eq. 7.1}
\end{equation*}
$$

where $\vec{r}$ is the vector pointing from the point $O$ to the center of mass. For the configuration given in Figure 7.1, the torque can be written as,

$$
\begin{equation*}
\tau=-m g d \sin \theta \tag{Eq. 7.2}
\end{equation*}
$$

Where, $\theta$ is the angle between the position vector $\vec{r}$ and the gravitational force $m \vec{g}, d$ is the distance between center of mass and the point $O$. The minus sign is due to the fact that the $\tau$ acts in the direction opposite to the direction of increase in $\theta$.

The fundamental equation of rotational dynamics is:

$$
\begin{equation*}
\tau=I \frac{d^{2} \theta}{d t^{2}} \tag{Eq. 7.3}
\end{equation*}
$$

and in our case, $I$ is the moment of inertia of the physical pendulum relative to the point $O$. Combining Eq. 7.2 and Eq. 7.3 yields the equation of motion of this oscillation:

$$
\begin{equation*}
I \frac{d^{2} \theta}{d t^{2}}+m g d \sin \theta=0 \tag{Eq. 7.4}
\end{equation*}
$$

Using the approximation $\sin \theta \approx \theta$ for sufficiently small oscillations, Eq. 7.4 reduces to

$$
\begin{equation*}
\frac{d^{2} \theta}{d t^{2}}+\frac{m g d}{I} \theta=0 \tag{Eq. 7.5}
\end{equation*}
$$

This is in the same form as the general equation of motion for a simple harmonic oscillator given by

$$
\begin{equation*}
\frac{d^{2} \theta}{d t^{2}}+\omega^{2} \theta=0 \tag{Eq. 7.6}
\end{equation*}
$$

where $\omega$ is the angular frequency of the oscillation. Comparing Eq. 7.5 and Eq. 7.6 yields

$$
\begin{gather*}
\omega=\sqrt{\frac{m g d}{I}}  \tag{Eq. 7.7}\\
T=\frac{2 \pi}{\omega}=2 \pi \sqrt{\frac{I}{m g d}} \tag{Eq. 7.8}
\end{gather*}
$$

and
where $I$ is the moment of inertia about the pivot point, $M$ is the total mass, and $g$ the acceleration due to gravity. In the first part of the experiment, we will use a physical pendulum to measure $g$. The moment of inertia of the physical pendulum relative to the point $O$ is related to the moment of inertia about the center of mass $I_{C M}$ by the parallel axis theorem;

$$
\begin{equation*}
I=I_{C M}+m d^{2} \tag{Eq. 7.9}
\end{equation*}
$$

Squaring Eq. 7.8 and applying the parallel axis theorem;

$$
\begin{equation*}
T^{2}=4 \pi^{2} \frac{I}{m g d}=\frac{4 \pi^{2}}{m g d}\left(I_{C M}+m d^{2}\right)=\frac{4 \pi^{2}}{g d}\left(\frac{I_{C M}}{m}+d^{2}\right) \tag{Eq. 7.10}
\end{equation*}
$$

For a uniform bar
For a uniform bar with length $L$, the moment of inertia about the center of mass is;

$$
\begin{equation*}
I_{C M}=\frac{1}{12} m L^{2} \tag{Eq. 7.11}
\end{equation*}
$$

Then the Eq. 7.10 yields;

$$
\begin{equation*}
\frac{g}{4 \pi^{2}} T^{2} d=d^{2}+\frac{1}{12} L^{2} \tag{Eq. 7.12}
\end{equation*}
$$

Eq. 7.12 can be rearranged as;

$$
\begin{equation*}
T^{2} d=\frac{4 \pi^{2}}{g} d^{2}+\frac{4 \pi^{2}}{12 g} L^{2} \tag{Eq. 7.13}
\end{equation*}
$$

For a rectangular plate
When we rearrange Eq. 7.10 as below, the moment of inertia of a physical pendulum can be related to its period of oscillation, $T$, mass, $M$, and the distance from the pivot point to the center of gravity, $d$.

$$
\begin{equation*}
I_{C M}=\frac{T^{2} m g d}{4 \pi^{2}}-m d^{2} \tag{Eq. 7.14}
\end{equation*}
$$

The moment of inertia about the center of mass for a thin uniform rectangular plate with length a and width bis;

$$
\begin{equation*}
I_{C M}=\frac{1}{12} m\left(a^{2}+b^{2}\right) \tag{Eq. 7.15}
\end{equation*}
$$

### 7.4 Experiment

In the first part of the experiment, we change the pivot distance from the center of mass of the bar and measure the corresponding period of oscillations. Eq. 7.13 is a linear equation resembling $y=a x+b$ with the variables $T^{2} d$ and $d^{2}$. When we plot $T^{2} d$ vs. $d^{2}$, we get a straight line with slope of $\frac{4 \pi^{2}}{g}$. We also determine d for the mimimum period of oscillation by plotting $T$ vs $d$ graph. In the second part of the experiment we measure the period of oscillation of a rectangular plate to determine its moment of inertia.

### 7.4.1 Procedure for measuring the acceleration due to Gravity using a physical pendulum.

1. Attach the pendulum bar with several small holes to the support rod and clamp.
2. Using the bubble level provided, check that the clamp (pivot rod) is level.
3. Position the photogate near the bottom of the pendulum bar so that the end of the bar blocks the photogate beam when it hangs straight down.
4. Set the photogate-interface-PC connections and select "pendulum" on datastudio.
5. Keep the amplitude of the oscillation quite small, just large enough to clear the photogate.
6. Let the physical pendulum oscillate for several periods before starting the datastudio, to eliminate initial sideways wobbling motion.
7. Click "START" to take measurement for about 10 seconds and record the average period $T$ on your data
8. Repeat the procedure for two more times and record your readings on Error! Reference source n ot found..
9. Repeat the procedure for five different d values and record your readings on Error! Reference $s$ ource not found..

### 7.4.2 Procedure for measuring moment of inertia based on the period of oscillation.

1. Measure and record the mass M , and the side lengths $\mathrm{a}, \mathrm{b}$, of the rectangular plate.
2. Measure the center of mass point of the plate.
3. Attach the plate to the pivot rod at one edge with the rotation axis being perpendicular to the plate.
4. Position the photogate near the bottom of the plate so that the end of the plate blocks the photogate beam when it hangs straight down.
5. Keep the amplitude of the oscillation quite small, just large enough to clear the photogate.
6. Let the plate to oscillate for several periods before starting the data studio, to eliminate initial sideways wobbling motion.
7. Click "start" to take measurement for about 10 seconds and record the average period, T.
8. Repeat the procedure for 3 more times and calculate the average period, $T_{\text {ave }}$.
9. Record the values on TABLE 7.2.

TABLE 7.1. PIVOT LENGTH AND PERIOD OF OSCILLATION VALUES (10 PTS)


TABLE 7.2. PERIOD OF OSCILLATION AND MOMENT OF INERTIA VALUES FOR RECTANGULAR PLATE (10 PTS)

| Mass of the plate, $M$ |  |
| :--- | :--- |
| Length of the plate, $a$ |  |
| Width of the plate, $b$ |  |
| Distance from pivot to center of mass, $d$ |  |
| Run\#1 for period of oscillation, $T$ |  |
| Run\#2 for period of oscillation, $T$ |  |
| Run\#3 for period of oscillation, $T$ |  |
| Average measured period of oscillation, $T_{\text {ave }}$ |  |
| Calculated value for moment of inertia, $I_{\text {calculated }}$ |  |

### 7.5 Report

### 7.5.1 Answer the following questions related to the first part of the experiment

1. ( 25 pts ) Plot the graph of $\mathrm{T}^{2} \mathrm{~d}$ vs. $\mathrm{d}^{2}$. Indicate the value of the slope, calculate the acceleration due to gravity, $g$ and find the percent error with the true acceleration due to gravity in Istanbul ( $g=9.808 \mathrm{~m} / \mathrm{s}^{2}$ ).
2. ( 10 pts ) Show that the distance from the pivot point to the center of mass of a long bar that produces the minimum period of oscillation for the bar at small amplitudes is;

$$
d=\sqrt{\frac{I_{C M}}{M}}
$$

3. (20 pts) Plot a $T$ vs. $d$ graph and in order to determine the value of length, $d$ for the minimum period of oscillation. Find the recent error with the experimental and calculated $d$ values.

### 7.5.2 Answer the following questions related to the second part of the experiment

1. ( 15 pts ) Calculate the moment of inertia of the thin rectangular plate using the measured average period of oscillation, $T_{\text {ave }}$. How does your value for the calculated moment of inertia compare to the true moment of inertia of a thin rectangular plate found by Eq. 7.15?

### 7.5.3 Conclusion ( 10 pts )

(Summarize what you have learned from the experiment in a few sentences and discuss the main errors you encountered and possible measurement improvements.)


Completed by:
Name, Surname and ID of Lab partner(s):

Department of Lab partner(s):
Verified by:

## 8 Appendices

### 8.1 APPENDIX A. SOME IMPORTANT POINTS ON DRAWING A GRAPH

 | Names and units of axes: |
| :--- |
| The names and units of the axes must be written on |
| the axes clearly. |



### 8.2 APPENDIX B. THE INTERNATIONAL SYSTEM OF UNITS*

The Système International d'Unites, abbreviated SI, is the system developed by the General Conference on Weights and Measures and adopted by nearly all the industrial nations of the world. The following material is adapted from B. N. Taylor, ed., National Institute of Standards and Technology Spec. Pub. 811 (U.S. Govt. Printing Office, Washington, DC, 1995). See also http://physics_nist.gov/cuu

| Quantity | Name of unit | Symbol |  |
| :---: | :---: | :---: | :---: |
|  | SI base units |  |  |
| length | meter | m |  |
| mass | kilogram | kg |  |
| time | second | s |  |
| electric current | ampere | A |  |
| thermodynamic temperature | kelvin | K |  |
| amount of substance | mole | mol |  |
| luminous intensity | candela | cd |  |
|  | SI derived units |  | Equivalent units |
| area | square meter | $\mathrm{m}^{2}$ |  |
| volume | cubic meter | $\mathrm{m}^{3}$ |  |
| frequency | hertz | Hz | $\mathrm{s}^{-1}$ |
| mass density (density) | kilogram per cubic meter | $\mathrm{kg} / \mathrm{m}^{3}$ |  |
| speed, velocity | meter per second | $\mathrm{m} / \mathrm{s}$ |  |
| angular velocity | radian per second | $\mathrm{rad} / \mathrm{s}$ |  |
| acceleration | meter per second squared | $\mathrm{m} / \mathrm{s}^{2}$ |  |
| angular acceleration | radian per second squared | $\mathrm{rad} / \mathrm{s}^{2}$ |  |
| force | newton | N | kg $\cdot \mathrm{m} / \mathrm{s}^{2}$ |
| pressure (mechanical stress) | pascal | Pa | N/m ${ }^{2}$ |
| kinematic viscosity | square meter per second | $\mathrm{m}^{2} / \mathrm{s}$ |  |
| dynamic viscosity | newton-second per square meter | $\mathrm{N} \cdot \mathrm{~s} / \mathrm{m}^{2}$ |  |
| work, energy, quantity of heat | joule | J | $\mathrm{N} \cdot \mathrm{m}$ |
| power | watt | W | J/s |
| quantity of electricity | coulomb | C | A-s |
| potential difference, electromotive force | volt | V | J/C, W/A |
| electric field strength | volt per meter | V/m | N/C |
| electric resistance | ohm | $\boldsymbol{\Omega}$ | V/A |
| capacitance | farad | F | A $\cdot \mathrm{s} / \mathrm{V}$ |
| magnetic flux | weber | Wb | $\mathrm{V} \cdot \mathrm{s}$ |
| inductance | henry | H | V-s/A |
| magnetic flux density | tesla | T | $\mathrm{Wb} / \mathrm{m}^{2}$ |
| magnetic fleld strength | ampere per meter | A/m |  |
| magnetomotive force | ampere | A |  |
| luminous flux | lumen | 1 m | cd $\cdot \mathrm{sr}$ |
| luminance | candela per square meter | $\mathrm{cd} / \mathrm{m}^{2}$ |  |
| illuminance | lux | 1 x | $1 \mathrm{~m} / \mathrm{m}^{2}$ |
| wave number | 1 per meter | $\mathrm{m}^{-1}$ |  |
| entropy | joule per kelvin | J/K |  |
| specific heat capacity | joule per kilogram-kelvin | J/kg $\cdot \mathrm{K}$ |  |
| thermal conductivity | watt per meter-kelvin | W/m $\cdot \mathrm{K}$ |  |

*University Physics, Young and Freedman, $14^{\text {th }}$ ed.Addision-Wesley.

### 8.3 APPENDIX C. UNIT CONVERSION FACTORS*

```
Length
\(1 \mathrm{~m}=100 \mathrm{~cm}=1000 \mathrm{~mm}=10^{6} \mu \mathrm{~m}=10^{9} \mathrm{~nm}\)
\(1 \mathrm{~km}=1000 \mathrm{~m}=0.6214 \mathrm{mi}\)
\(1 \mathrm{~m}=3.281 \mathrm{ft}=39.37 \mathrm{in}\)
\(1 \mathrm{~cm}=0.3937 \mathrm{in}\).
1 in. \(=2.540 \mathrm{~cm}\)
\(1 \mathrm{ft}=30.48 \mathrm{~cm}\)
\(1 \mathrm{yd}=91.44 \mathrm{~cm}\)
\(1 \mathrm{mi}=5280 \mathrm{ft}=1.609 \mathrm{~km}\)
\(1 \AA=10^{-10} \mathrm{~m}=10^{-8} \mathrm{~cm}=10^{-1} \mathrm{~nm}\)
1 nautical mile \(=6080 \mathrm{ft}\)
1 light year \(=9.461 \times 10^{15} \mathrm{~m}\)
```


## Area

$1 \mathrm{~cm}^{2}=0.155 \mathrm{in}^{2}$
$1 \mathrm{~m}^{2}=10^{4} \mathrm{~cm}^{2}=10.76 \mathrm{ft}^{2}$
$1 \mathrm{in}^{2}=6.452 \mathrm{~cm}^{2}$
$1 \mathrm{ft}=144 \mathrm{in}^{2}{ }^{2}=0.0929 \mathrm{~m}^{2}$

Volume
1 liter $=1000 \mathrm{~cm}^{3}=10^{-3} \mathrm{~m}^{3}=0.03531 \mathrm{ft}^{3}=61.02 \mathrm{in}^{3}$ $1 \mathrm{ft}^{3}=0.02832 \mathrm{~m}^{3}=28.32$ liters $=7.477$ gallons
1 gallon $=3.788$ liters

Time
$1 \mathrm{~mm}=60 \mathrm{~s}$
$1 \mathrm{~h}=3600 \mathrm{~s}$
$1 \mathrm{~d}=86,400 \mathrm{~s}$
$1 \mathrm{y}=365.24 \mathrm{~d}=3.156 \times 10^{7} \mathrm{~s}$

Angle
$1 \mathrm{rad}=57.30^{\circ}=180^{\circ} / \pi$
$1^{\circ}=0.01745 \mathrm{rad}=\pi / 180 \mathrm{rad}$
1 revolution $=360^{\circ}=2 \pi \mathrm{rad}$
$1 \mathrm{rev} / \mathrm{min}(\mathrm{rpm})=0.1047 \mathrm{rad} / \mathrm{s}$

## Speed

$1 \mathrm{~m} / \mathrm{s}=3.281 \mathrm{ft} / \mathrm{s}$
$1 \mathrm{ft} / \mathrm{s}=0.3048 \mathrm{~m} / \mathrm{s}$
$1 \mathrm{mi} / \mathrm{min}=60 \mathrm{mi} / \mathrm{h}=88 \mathrm{ft} / \mathrm{s}$
$1 \mathrm{~km} / \mathrm{h}=0.2778 \mathrm{~m} / \mathrm{s}=0.6214 \mathrm{mi} / \mathrm{h}$
$1 \mathrm{mi} / \mathrm{h}=1.466 \mathrm{ft} / \mathrm{s}=0.4470 \mathrm{~m} / \mathrm{s}=1.609 \mathrm{~km} / \mathrm{h}$
1 furlong/fortnight $=1.662 \times 10^{-4} \mathrm{~m} / \mathrm{s}$

Acceleration
$1 \mathrm{~m} / \mathrm{s}^{2}=100 \mathrm{~cm} / \mathrm{s}^{2}=3.281 \mathrm{ft} / \mathrm{s}^{2}$
$1 \mathrm{~cm} / \mathrm{s}^{2}=0.01 \mathrm{~m} / \mathrm{s}^{2}=0.03281 \mathrm{ft} / \mathrm{s}^{2}$
$1 \mathrm{ft} / \mathrm{s}^{2}=0.3048 \mathrm{~m} / \mathrm{s}^{2}=30.48 \mathrm{~cm} / \mathrm{s}^{2}$
$1 \mathrm{mi} / \mathrm{h} \cdot \mathrm{s}=1.467 \mathrm{ft} / \mathrm{s}^{2}$

Mass
$1 \mathrm{~kg}=10^{3} \mathrm{~g}=0.0685$ slug
$1 \mathrm{~g}=6.85 \times 10^{-5}$ slug
1 slug $=14.59 \mathrm{~kg}$
$1 \mathrm{u}=1.661 \times 10^{-27} \mathrm{~kg}$
1 kg has a weight of 2.205 lb when $g=9.80 \mathrm{~m} / \mathrm{s}^{2}$

Force
$1 \mathrm{~N}=10^{5} \mathrm{dyn}=0.2248 \mathrm{lb}$
$1 \mathrm{lb}=4.448 \mathrm{~N}=4.448 \times 10^{5} \mathrm{dyn}$

## Pressure

$1 \mathrm{~Pa}=1 \mathrm{~N} / \mathrm{m}^{2}=1.450 \times 10^{-4} \mathrm{lb} / \mathrm{in.}^{2}=0.209 \mathrm{lb} / \mathrm{ft}^{2}$
$1 \mathrm{bar}=10^{5} \mathrm{~Pa}$
$1 \mathrm{lb} / \mathrm{in}^{2}=6895 \mathrm{~Pa}$
$1 \mathrm{lb} / \mathrm{ft}^{2}=47.88 \mathrm{~Pa}$
$1 \mathrm{~atm}=1.013 \times 10^{5} \mathrm{~Pa}=1.013 \mathrm{bar}$
$=14.7 \mathrm{lb} / \mathrm{in}^{2}=2117 \mathrm{lb} / \mathrm{ft}^{2}$
$1 \mathrm{~mm} \mathrm{Hg}=1$ torr $=133.3 \mathrm{~Pa}$

Energy
$1 \mathrm{~J}=10^{7} \mathrm{ergs}=0.239 \mathrm{cal}$
$1 \mathrm{cal}=4.186 \mathrm{~J}$ (based on $15^{\circ}$ calorie)
$1 \mathrm{ft} \cdot \mathrm{lb}=1.356 \mathrm{~J}$
$1 \mathrm{Btu}=1055 \mathrm{~J}=252 \mathrm{cal}=778 \mathrm{ft} \cdot \mathrm{lb}$
$1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$
$1 \mathrm{kWh}=3.600 \times 10^{6} \mathrm{~J}$

Mass-Energy Equivalence
$1 \mathrm{~kg} \leftrightarrow 8.988 \times 10^{16} \mathrm{~J}$
$1 \mathrm{u} \leftrightarrow 931.5 \mathrm{MeV}$
$1 \mathrm{eV} \leftrightarrow 1.074 \times 10^{-9} \mathrm{u}$

Power
$1 \mathrm{~W}=1 \mathrm{~J} / \mathrm{s}$
$1 \mathrm{hp}=746 \mathrm{~W}=550 \mathrm{ft} \cdot \mathrm{lb} / \mathrm{s}$
$1 \mathrm{Btu} / \mathrm{h}=0.293 \mathrm{~W}$
*University Physics, Young and Freedman, $14^{\text {th }}$ ed.Addision-Wesley.

### 8.4 APPENDIX D. FUNDAMENTAL PHYSICAL CONSTANTS and PREFIXES for POWERS of 10 *

| Name | Symbol | Value |
| :--- | :--- | :--- |
| Speed of light | $c$ | $2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| Magnitude of charge of electron | $e$ | $1.60217653(14) \times 10^{-19} \mathrm{C}$ |
| Gravitational constant | $G$ | $6.6742(10) \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}$ |
| Planck's constant | $h$ | $6.6260693(11) \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ |
| Boltzmann constant | $k$ | $1.3806505(24) \times 10^{-23} \mathrm{~J} / \mathrm{K}$ |
| Avogadro's number | $N_{A}$ | $6.0221415(10) \times 10^{23} \mathrm{molec}$ |
| Gas constant | $R$ | $8.314472(15) \mathrm{J} / \mathrm{mol} \cdot \mathrm{K}$ |
| Mass of electron | $m_{e}$ | $9.1093826(16) \times 10^{-31} \mathrm{~kg}$ |
| Mass of proton | $m_{p}$ | $1.67262171(29) \times 10^{-27} \mathrm{~kg}$ |
| Mass of neutron | $m_{n}$ | $1.67492728(29) \times 10^{-27} \mathrm{~kg}$ |
| Permeability of free space | $\mu_{0}$ | $4 \pi \times 10^{-7} \mathrm{~Wb} / \mathrm{A} \cdot \mathrm{m}$ |
| Permittivity of free space | $\epsilon_{0}=1 / \mu_{0} c^{2}$ | $8.854187817 \ldots \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{m}^{2}$ |
|  | $1 / 4 \pi \epsilon_{0}$ | $8.987551787 \ldots \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$ |


| Power of ten | Prefix | Abbreviation | Pronunciation |
| :--- | :--- | :--- | :--- |
| $10^{-24}$ | yocto- | y | yoc-toe |
| $10^{-21}$ | zepto- | z | zep-toe |
| $10^{-18}$ | atto- | a | at-toe |
| $10^{-15}$ | femto- | f | fem-toe |
| $10^{-12}$ | pico- | p | pee-koe |
| $10^{-9}$ | nano- | n | nan-oe |
| $10^{-6}$ | micro- | $\mu$ | my-crow |
| $10^{-3}$ | milli- | m | mil-i |
| $10^{-2}$ | centi- | c | cen-ti |
| $10^{3}$ | kilo- | k | kil-oe |
| $10^{6}$ | mega- | M | meg-a |
| $10^{9}$ | giga- | G | jig-a or gig-a |
| $10^{12}$ | tera- | T | ter-a |
| $10^{15}$ | peta- | P | pet-a |
| $10^{18}$ | exa- | E | ex-a |
| $10^{21}$ | zetta- | Z | zet-a |
| $10^{24}$ | yotta- | Y | yot-a |

## Examples:

1 femtometer $=1 \mathrm{fm}=10^{-15} \mathrm{~m}$
1 picosecond $=1 \mathrm{ps}=10^{-12} \mathrm{~s}$
1 millivolt $=1 \mathrm{mV}=10^{-3} \mathrm{~V}$

1 nanocoulomb $=1 \mathrm{nC}=10^{-9} \mathrm{C}$
1 kilopascal $=1 \mathrm{kPa}=10^{3} \mathrm{~Pa}$
1 megawatt $=1 \mathrm{MW}=10^{6} \mathrm{~W}$
1 gigahertz $=1 \mathrm{GHz}=10^{9} \mathrm{~Hz}$
*University Physics, Young and Freedman, $14^{\text {th }}$ ed.Addision-Wesley.

