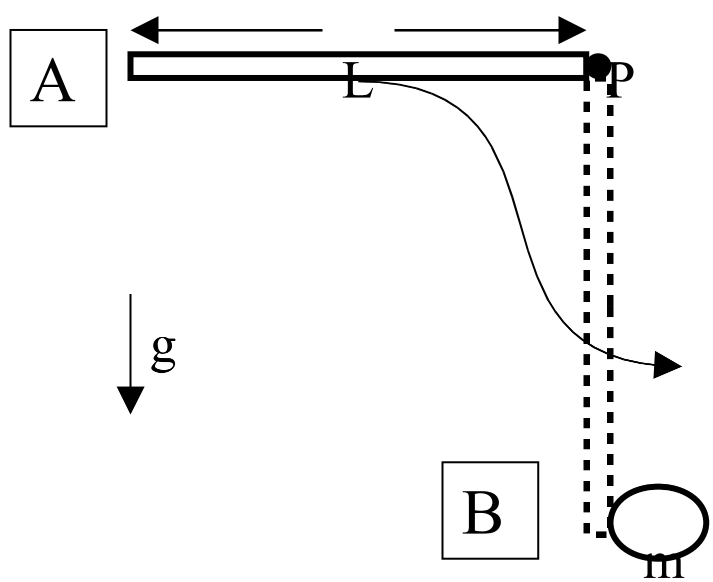
A thin, uniform rod of length **L** and mass **M** is attached from one end to a freely rotating pivot, P. The rod is released at a horizontal position, A, and swings downwards under the influence of gravity. At its lowest (vertical) position, B, it contacts at its other end with a sticky ball of mass **m** which is initially at rest and they move together thereafter. The moment of inertia of the rod w.r.t. its center of mass and w.r.t. P are $\mathbf{I_{cm}} = ML^2/12$ and $\mathbf{I_p} = ML^2/3$, respectively.

Answer the questions below in terms of given variables.

(a) (7 pts) Find the angular acceleration of rod immediately after it is released.
(b) (8 pts) Use the Work-Energy theorem to find the angular velocity of the rod immediately before it contacts the ball.
(c) (3 pts) Is the angular momentum with respect to the pivot P conserved through the collision? Why/Why not?
(d) (7 pts) Find the angular velocity of rod+ball immediately after the contact.



SOLUTION:

(a) It is more convenient to consider the rotational motion around P. The torque wrt P on the rod is solely due to the rod's weight downward from the center of mass, i.e., $\boldsymbol{\tau_P} = \mathbf{MgL/2}$. (Note that the net force on the rod is different since the pivot also exerts a force on the rod, although this second force does not contribute to the torque wrt P) The angular acceleration follows from

$$\boldsymbol{\tau_P = I_P\, \alpha \qquad \Rightarrow \qquad \alpha = (MgL/2) / (ML^2/3) = 3g/2L}$$

(b) Work-Energy Thm: $\mathbf{W = \Delta K}$. W is the work done on the bar by gravity: $\mathbf{W} = \int_0^{\pi/2} \tau\, d\theta$. θ is the angle the bar makes with the horizontal and $\boldsymbol{\tau = MgL\cos\theta/2}$, therefore $\mathbf{W = MgL/2 = I_P \omega_B^2/2} \quad \Rightarrow \quad \boldsymbol{\omega_B = \sqrt{3g/L}}$

Alternative 1: Conservation of mechanical energy, $\mathbf{-\Delta U = \Delta K}$ gives the same result, since $\mathbf{-\Delta U = U_A - U_B = MgL - MgL/2 = MgL/2}$ = Work done by gravity.

Alternative 2: Instead of P one can use the center of mass as reference, too. In this case, the bar's kinetic energy has both rotational and translational components: $\mathbf{\Delta K = K_B = I_{cm}\, \omega_B^2/2 + Mv_{cm}/2}$, where $\mathbf{v_{cm} = \omega_B L/2}$.

(c) During the collision (immediately before and after the rod contacts the ball), the weight is radial, therefore the net torque on the system (rod+ball) is zero (although the net force is not). Hence, the angular momentum is conserved, but <u>not the linear momentum, nor the mechanical energy</u>! Note that, the force between the rod and the ball during the collision is not an external force on the system and they cancel each other by the 3rd Law.

(d) By conservation of angular momentum, $\mathbf{I_P\, \omega_B = (I_P + mL^2)\, \omega'_B}$. After little algebra, we obtain $\mathbf{\omega'_B = M\, \omega_B / (M+3m)}$, where $\boldsymbol{\omega_B}$ was calculated earlier.