Name:	Signature:
Department:	Number:

2. (25 points) A ball is attached to a train moving at constant angular speed **ω** on a vertical circular track of radius **R** as shown in the figure. The position of the ball at **t=0** is given as **x(0)=R** and **y(0)=0**.

(a) Express **x(t)** and **y(t)** in terms of **R,ω,t**.

Constant angular velocity => $\theta(t) = \omega t$
$x(t) = R\cos(\omega t)$ (2 pts)
$y(t) = R\sin(\omega t)$ (2 pts)

(b) Express $\mathbf{v_x}$ and $\mathbf{v_y}$, the horizontal and vertical components of the ball's velocity, in terms of **x** and **y**.

$v_x = dx/dt = -\omega R\sin(\omega t) = -\omega y$ (3 pts)
$v_y = dy/dt = \omega R\cos(\omega t) = \omega x$ (3 pts)

At a certain point $\mathbf{(x_1,y_1)}$ between **(R,0)** and **(0,R)**, the ball is released. From this moment on, it performs a projectile motion which peaks at **x=0** and falls back on the track at $\mathbf{(-x_1,y_1)}$. The downwards gravitational acceleration is given by **g.**

(c) Using (b), calculate the time of flight of the ball in terms of $\mathbf{x_1}$, $\mathbf{y_1}$, **R, g, ω** (you may not need to use all).

$|v_y| = g(t_{flight}/2)$ (3 pts) => $t_{flight} = 2\omega x_1/g$ (2 pts)

Alternatively: $2x_1 = |v_x|\, t_{flight} = (\omega y_1)\, t_{flight}$ => $t_{flight} = 2x_1/\omega y_1$ (2 pts)

(d) Find $\mathbf{x_1}$, $\mathbf{y_1}$ in terms of **R, g, ω.** (Hint: Consider the distance traversed in the x-direction during the flight)

$2x_1 = |v_x|\, t_{flight} = (\omega y_1)(2\omega x_1/g)$ (3 pts) => $y_1 = g/\omega^2$ (2 pts)
$x_1 = \mathrm{sqrt}[R^2 - (g/\omega^2)^2]$ (2 pts)

Alternatively: $|v_y| = g(t_{flight}/2)$ => $\omega x_1 = gx_1/\omega y_1$ ...

(e) What is the minimum angular speed for which the above scenario is possible?

$y_1 = g/\omega^2 \leq R$ => $\omega \geq \mathrm{sqrt}\,[g/R]$ (3 pts)

Solution continues on the back side of the page **Yes** □ **No** □