Q3. A projectile has traveled a horizontal distance **x** and a vertical distance **y** at its peak position. Find the direction of its velocity as it leaves the ground.

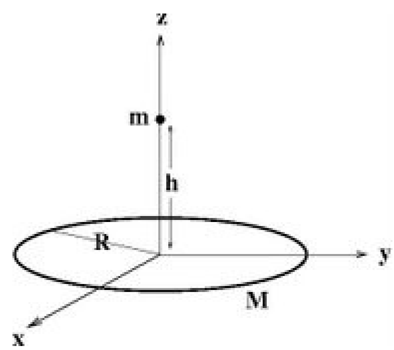
A. Let it reach its peak in time **t**. Then,

$$x = v\cos\theta\, t \qquad \text{and} \qquad t = v\sin\theta / g$$

$$y = g t^2 / 2$$

where θ is the angle between the horizontal and the initial velocity vector. Combining the two, we obtain $\theta = \arctan(2y/x)$.

Q6. Consider a small mass **m** initially located vertically **h** meters above the center of a uniform ring of mass **M** and radius **R**. Assume **h<<R**, so that $\mathbf{h^2+R^2 \sim R^2}$. Show that, when released, the mass **m** performs a harmonic oscillation. Find the period of oscillation.



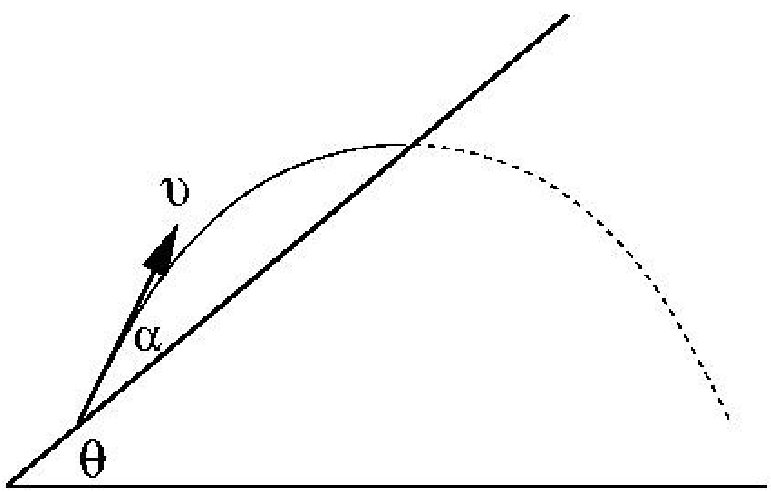
A. The vertical force on the mass **m** can be written easily as

$$F_y = -\frac{mM}{h^2+R^2}\frac{h}{\sqrt{h^2+R^2}} \cong -\frac{mM}{R^3}h \equiv -Kh$$

which is the spring equation. Therefore it behaves as a harmonic oscillator. The period is $T = 2\pi\sqrt{m/K} = 2\pi\sqrt{R^3/M}$.

MT-I. A projectile is thrown on at an angle α and initial speed υ from an inclined plane with an inclination angle θ. The projectile touches the inclined surface at the point where it reaches its maximum height.

(a) (5 pts) Find the time of flight, **t**, in terms of given variables, using the condition that the vertical speed is zero at the moment of contact.
(b) (5 pts) Express the horizontal distance, **x**, traveled in the air as a function of given variables.
(c) (5 pts) Express the vertical distance, **y**, traveled in the air as a function of given variables.
(d) (10 pts) Using the condition that the point of maximum height is also the point of contact, show that, independent of υ, there is only one angle α that satisfies the condition.



SOLUTION:

(a) $v\sin(\theta+\alpha)=gt \;\Rightarrow\; t=v\sin(\theta+\alpha)/g$

(b) $x=v\cos(\theta+\alpha)t=v^2\sin(\theta+\alpha)\cos(\theta+\alpha)/g$

(c) $y=gt^2/2 = v^2\sin^2(\theta+\alpha)/2g$

(d) $\tan(\theta)=\frac{y}{x}=\tan(\theta+\alpha)/2=\frac{1}{2}\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha} \;\Rightarrow\; \tan\alpha=\frac{\tan\theta}{1+2\tan^2\theta}$

MT-II.

(a) Calculate the gravitational force acting on the mass **m** due to the uniform circular ring of mass **M** and radius **R** a distance **h** below it, as shown in Fig.1.

(b) Use your result in part (a) to find the gravitational force on the mass **m** in Fig.2 due to a hemisphere of <u>surface charge density</u> σ and radius **R** immediately under it.

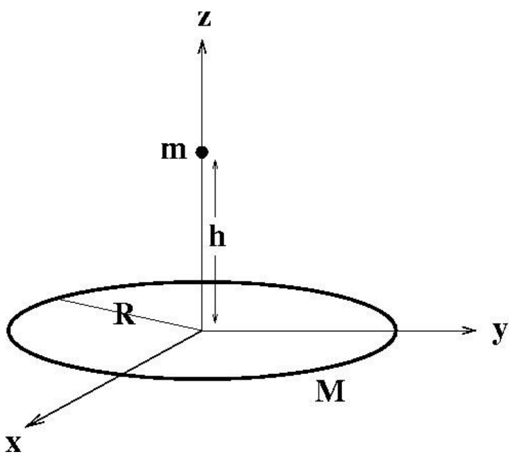


Fig.1

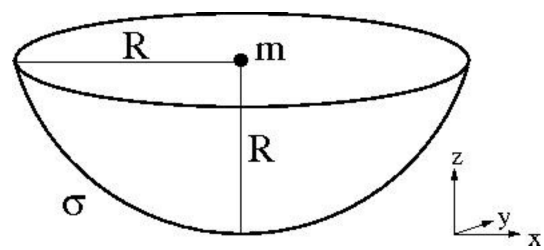


Fig.2

SOLUTION:

(a) By symmetry, the net gravitational force is in the z-direction. Since each part of the ring contributed equally to F_z, we can write down the force without integration as

$$\vec{F} = -\frac{GMm}{h^2+R^2}\frac{h}{\sqrt{h^2+R^2}}\hat{k}$$

(b) Consider the hemisphere as a collection of rings labeled by the angle θ from 0 to π/2. The radius, r(θ), and the distance from **m**, h(θ), of the ring located between [θ, θ+dθ] are

$$r(\theta) = R\cos\theta \quad \text{and} \quad h(\theta) = R\sin\theta.$$

Mass of the ring is

$$dM = \sigma dA = \sigma\, 2\pi\, R\cos\theta\, Rd\theta. \qquad \Rightarrow$$

$$\vec{F} = -\int_0^{\pi/2} G\frac{dM\, m}{h(\theta)^2+R(\theta)^2}\frac{h(\theta)}{\sqrt{h(\theta)^2+R(\theta)^2}}\,\hat{k} = -2\pi Gm\sigma\hat{k}\int_0^{\pi/2}\sin\theta\cos\theta\, d\theta = -\pi Gm\sigma\hat{k}$$

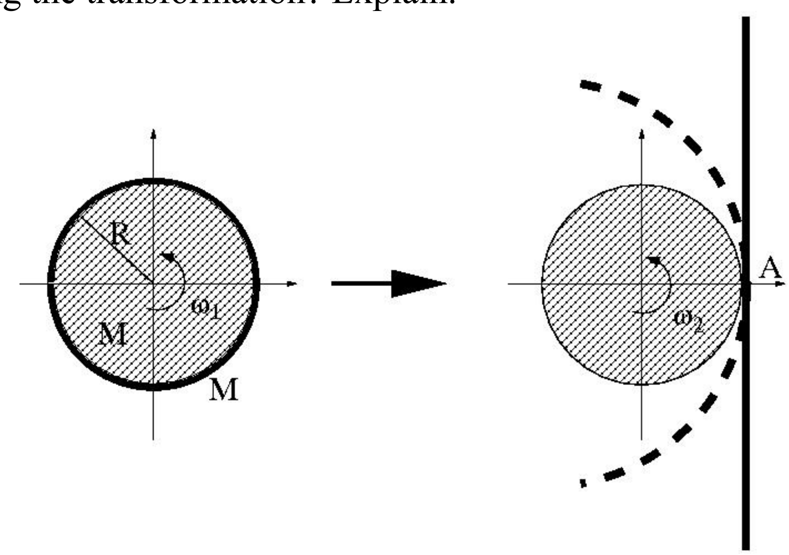
FINAL.

A flat, uniform disk of mass **M** which has an elastic metal strip of also mass **M** wrapped around it is fixed to the plane of the paper from its center. The disk and the strip rotate together with an angular velocity ω_1 around a central axis perpendicular to the paper. At a certain instant, the metal strip springs open symmetrically into a straight rod, while still attached to the disk at point A, as shown below.

(a) Find the angular momentum of the system while the strip is wrapped around the disk.

(b) Find the angular velocity ω_2 of the system after the strip opens up.

(c) Compare the kinetic energies before and after. Is the energy conserved during the transformation? Explain.



SOLUTION:

Moment of inertia of the system is

$I_1 = I_{disk}+I_{strip} = MR^2/2 + MR^2 = 3MR^2/2$, and

$I_2 = I_{disk}+I_{strip} = MR^2/2 + [\ M(2\pi R)^2/12 + MR^2\ ] = (9+2\pi^2)\ MR^2/6$, by the parallel axis theorem.

(a) $L_1 = I_1\omega_1 = 3MR^2\omega_1/2$.

(b) $L_2 = L_1$, therefore, $\omega_2 = L_1/I_2 = \omega_1/\ (1 + 2\pi^2/9)$.

(c) $K_1/K_2 = I_1\omega_1^2\ /\ [\ I_2\omega_2^2 + M(R\omega_2)^2\ ] = \omega_1\ /\ [\omega_2+2\omega_2^2/3\omega_1] = [1 + 2\pi^2/9]^2\ /\ [5/3 + 2\pi^2/9] > 1$. Decrease in kinetic energy is due to the inelastic deformation of the strip, which dissipates energy in the form of heat.