| Name: | Signature: |
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| Department: | Number: |

2. (25 points) A point mass **m** is located a distance **h** above a thin, uniform, circular ring of mass **M** and radius **R**.

(a) (5 pts) Find the mass density **λ** of the ring.

Uniform ring => **λ = M / 2πR (5pts)**

(b) (10 pts) Find the gravitational force on **m** due to the ring.

By symmetry, the total force is in z-direction, i.e., $F_x = F_y = 0$ and $\vec{F} = F_z \hat{k}$ (2 pts)
Every infinitesimal mass **dM** along the ring contributes equally to $F_z$ :

$$dF_z = -G \frac{m dM}{[R^2+h^2]} \frac{h}{\sqrt{R^2+h^2}} \quad => \quad F_z = -G \frac{Mmh}{[R^2+h^2]^{3/2}} \text{ (8 pts)}$$

(c) (10 pts) Find the gravitational force on **m** due to the half of the ring with x>0.

Again, $F_y = 0$ from symmetry.
The new $F_z$ is half of the result above, since both halves contribute equally to part (b).
However, the right half (x>0) also exerts a force in the +x direction. Assign an angle θ (0< θ< π) to each point on the half-ring. Then

$$dF_x = G \frac{m \lambda R}{[R^2+h^2]} \frac{R}{\sqrt{R^2+h^2}} \sin\theta \, d\theta \quad => \quad F_x = \frac{1}{pi} G \frac{mMR}{[R^2+h^2]^{3/2}}$$

Finally,

$$\vec{F} = G \frac{m M}{[R^2+h^2]^{3/2}} [\frac{R}{pi} \hat{i} - \frac{h}{2} \hat{k}]$$

 Solution continues on the back side of the page **Yes** □ **No** □