Gauss' Law for Gravitation

- It states that the accelwation g due to gravity of mass m is given by

$$\oint_S \vec{g}\cdot\vec{n}\, dA = -4\pi G m$$

→ The $\vec{g}$ vector always points toward the mass

→ The $\hat{n}$ unit vector perpendicular to the surface S, outward to S.

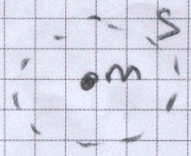
→ m ← total mass inside the surface

→ * most useful for finding the acceleration to gravity g due to a highly symmetrical mass distribution

POINT MASS:

→ Use Gauss' law to find the accelwation g due to the gravity of a point mass m

1) construct an imaginary closed surface S



S simple choose sphere of radius r

g perpendicular to S everywhere

g has same value everywhere on S

apply Gauss' law for gravity

$$\oint_S \vec{g}\cdot\vec{n}\cdot dA = -4\pi G m$$

$\vec{g}\cdot\vec{n} = -g$ ← because $\vec{g}$ and $\vec{n}$ are antiparallel

$$-g \oint_S dA = -4\pi G m$$

$dA \rightarrow$ integrated over the surface of a sphere

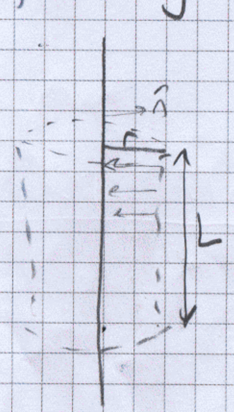
$$\oint dA = 4\pi r^2$$

$$-g\,(\cancel{4\pi} r^2) = \cancel{-4\pi} G m$$

$$\boxed{g = \frac{Gm}{r^2}}$$

Line of Mass:

infinitely long line of mass, having linear mass density λ



choose gaussian surface $\rightarrow$ cylinder

$$\oint_S \vec{g}\cdot\vec{n}\, dA = -4\pi G m$$

$$\vec{g}\cdot\hat{n} = -g$$

$$-g \oint_S dA = -4\pi G m$$

$$\oint dA = 2\pi r L$$

$$-g\,(2\pi r L) = -4\pi G\,(\overbrace{\lambda L}^{m})$$

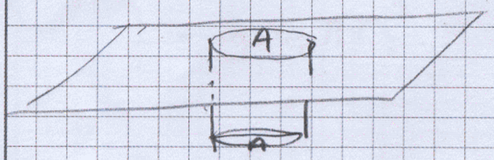
$$\boxed{g = \frac{2G\lambda}{r}}$$

Plane of Mass:

infinite plane → area mass density σ

→ calculate the acceleration g due to gravity of the plane at a distance r

choose gaussian surface → pillbox



$$\vec{g} \cdot \hat{n} = -g$$

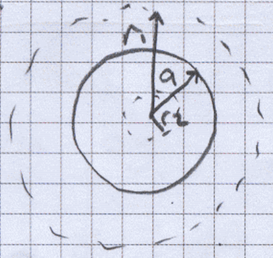
$$-g \oint dA = -4\pi G m$$

$$-g(2A) = -4\pi G m$$

$$-g(2A) = -4\pi G \sigma A$$

$$\boxed{g = 2\pi G \sigma}$$ independent of r

Küre:



1) Kürenin dışı:

$$-g\,\cancel{4\pi}r_1^2 = -\cancel{4\pi}Gm$$

$$\boxed{g = \frac{Gm}{r_1^2}}$$

2) Kürenin içi:

$$-g\,4\pi r_2^2 = -4\pi G M_{iç}$$

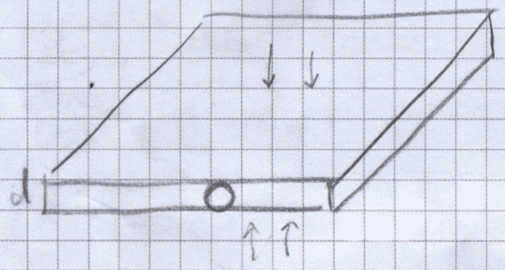
$$M_{iç} = \frac{4}{3}\pi r_2^{3} = M\frac{r_2^3}{a^3}$$

$$-g\,\cancel{4\pi}\cancel{r_2^2} = -\cancel{4\pi}G\,m\,\frac{\cancel{r_2^3}r_2}{a^3}$$

$$\boxed{g = \frac{Gmr_2}{a^3}}$$

at $r=a$ eşitler

Kalın Tabaka içinde küresel boşluk



$$\vec{g}_{tabaka} = \mp 2\pi G\sigma d\,\hat{z}$$

$$\vec{g}_{küre} = -\frac{4\pi G\frac{4}{3}\pi a^3\rho}{4\pi r^2} = -\frac{4G\pi a^3}{3r^2}\rho\,\hat{r}$$

$$\vec{g} = \vec{g}_{tab} - \vec{g}_{küre} = \mp 2\pi G\sigma d\,\hat{z} + \frac{4\pi G a^3}{3r^2}\rho\hat{r}$$

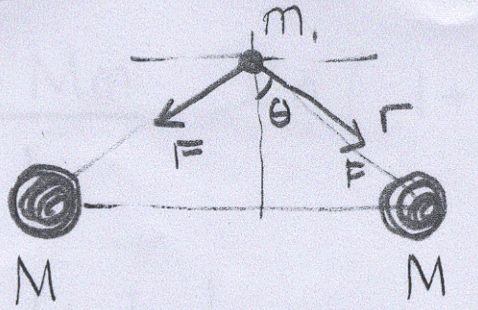
$$= \mp 2\pi G\sigma d\,\hat{z} + \frac{G\rho\,4\pi a^3(x\hat{x}+y\hat{y}+z\hat{z})}{3(x^2+y^2+z^2)^{3/2}}$$

$$= -2\pi G\sigma d + G\rho\frac{4\pi a^3}{3z^2}$$

$$= -2\pi G\sigma\left[d - \frac{2a^3}{3z^2}\right]$$

Gravitational Force

12.13



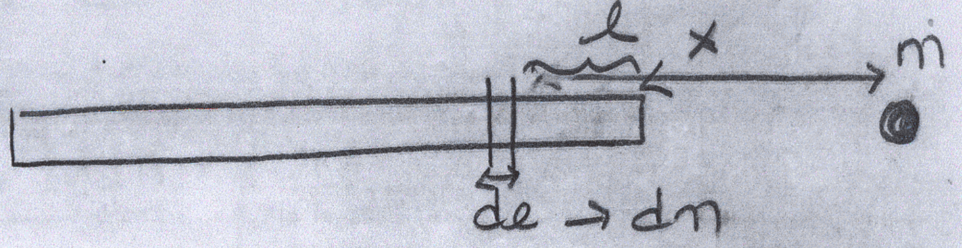
$\cos\theta = \frac{6}{10}$

$$F_{net} = 2F_y \cos\theta = 2\frac{GMm}{r^2}\cos\theta$$

$$a = \frac{F_{net}}{m} = \frac{2GM}{r^2}\cos\theta$$

12.40

(a)



$$\frac{dm}{dl} = \frac{M}{L} \Rightarrow dm = dl\left(\frac{M}{L}\right)$$

$$dU = -G\frac{m\,dm}{l+x} = -G\frac{mM}{L}\frac{dl}{l+x}$$

$$U = -\frac{GmM}{L}\int_0^L \frac{dl}{l+x}$$

$$= -\frac{GmM}{L}\ln(l+x)\Big|_0^L$$

$$= -\frac{GmM}{L}\ln(L+x) - \ln x$$

$$= -\frac{GMm}{L} \ln\left(1+\frac{L}{x}\right) = \boxed{-\frac{GMm}{x}}$$

$$\ln\left(1+\frac{L}{x}\right) = \frac{L}{x} - \frac{1}{2}\left(\frac{L}{x}\right)^2 + \cdots \quad \left(\frac{L}{x}\right) < 1$$

$\boxed{x \gg L}$

(b) $$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[-\frac{GMm}{L}\ln\left(1+\frac{L}{x}\right)\right]$$

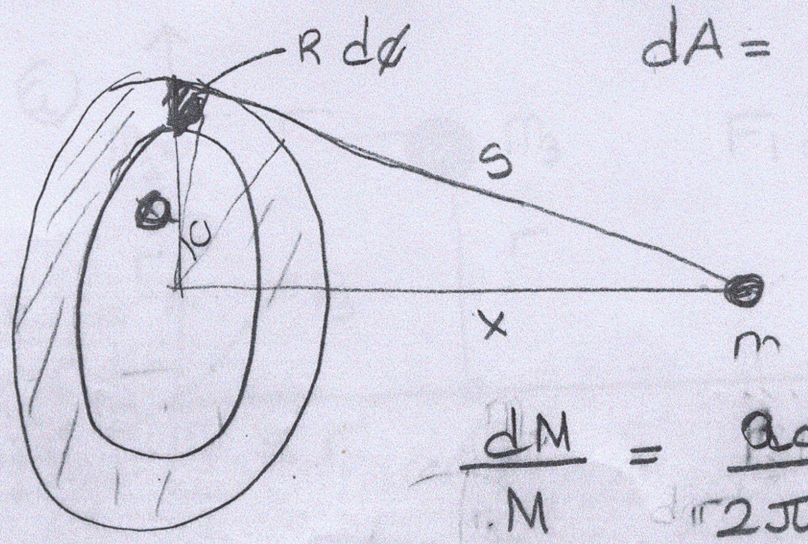
$$= \frac{GMm}{L}\left(\frac{-L}{x^2}\right)\frac{1}{1+\frac{L}{x}}$$

$$= -\frac{GMm}{x^2}\,\frac{x}{x+L} = -\frac{GMm}{x(x+L)}$$

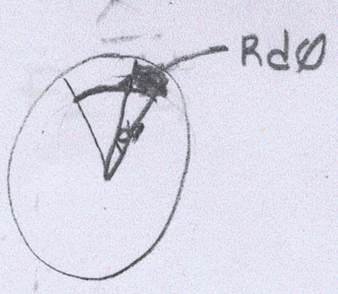
$(-)$: attractive force

$$x \gg L \Rightarrow -\frac{GMm}{x^2}$$

(12.41)



dA =



$$\frac{dM}{M} = \frac{a\,d\phi}{2\pi a}$$

$$dM = \frac{M a\, d\phi}{2\pi a}$$

$$dU = \frac{-G\, m\, dM}{s}$$

$$dU = -\frac{G m M a}{2\pi a s}\int_0^{2\pi} d\phi$$

$$U = -\frac{G m M}{\sqrt{x^2+a^2}}$$

(b) when $x \gg a$ $\quad U = -\frac{G m M}{x}$

(c) $F_x = -\frac{dU}{dx} = \frac{-G M m\, x}{(x^2+a^2)^{3/2}}$

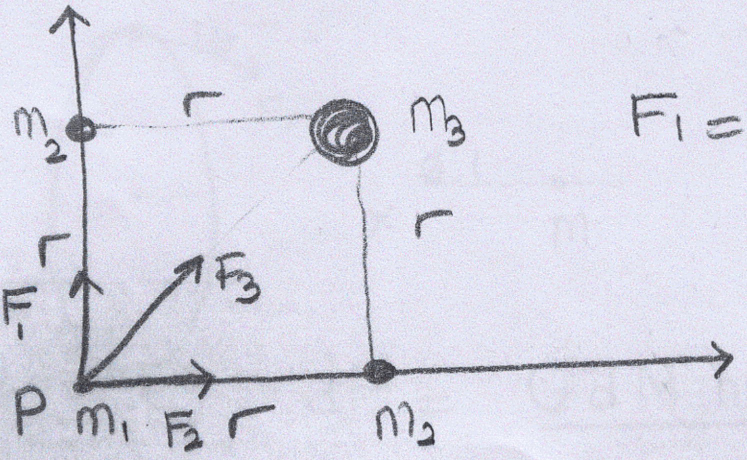
$GMm(x^2+a^2)^{-1/2}$

$2x \cdot -\frac{1}{2}(x^2+a^2)^{-3/2}$

(d) $x \gg a$ $\quad F_x = \frac{-G M m}{x^2}$

12.49

(a)

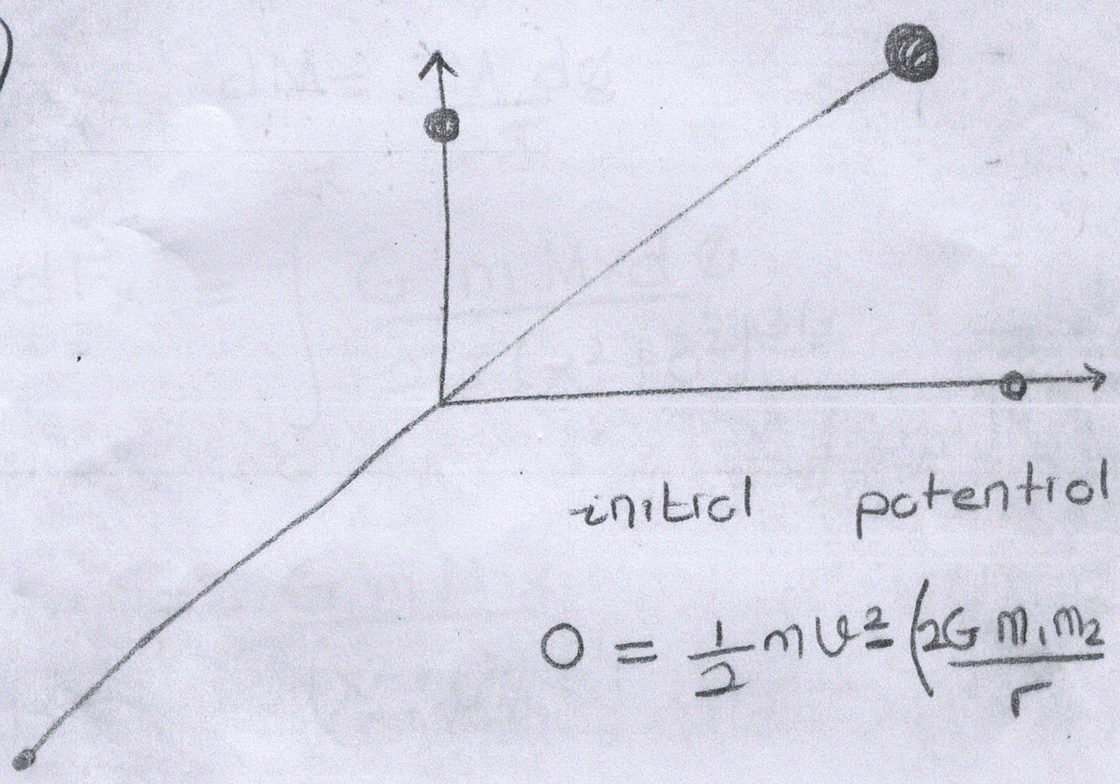


$F_1 = F_2 \rightarrow$

a) $F_{net} = \sqrt{2}\ F_1 + F_3$

$$F_{net} = \sqrt{2}\ G\frac{m_1 m_2}{r^2} + \frac{G\ m_1 m_3}{(r\sqrt{2})^2}$$

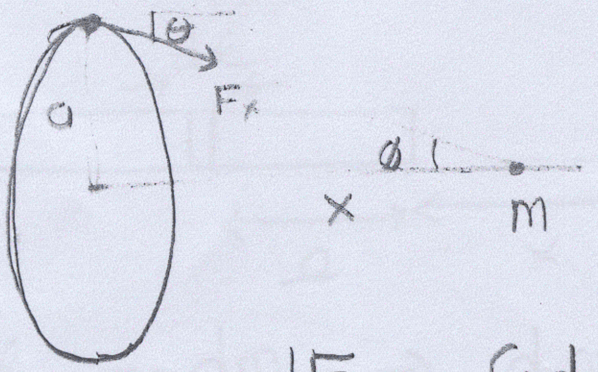
(b)



initial potential $= 0$

$$0 = \frac{1}{2}mv^2 \left(\frac{2G\ m_1 m_2}{r} + \frac{G\ m_1 m_3}{r\sqrt{2}}\right)$$

12.83



$$\cos\theta = \frac{x}{(x^2+a^2)}$$

$$dF_x = \frac{G\,dM\,m}{x^2+a^2}\,\frac{x}{(x^2+a^2)^{1/2}}$$

$$dF_x = \frac{G\,m\,dM\,x}{(x^2+a^2)^{3/2}}$$

$$\frac{dM}{M} = \frac{a\,d\phi}{2\pi a}$$

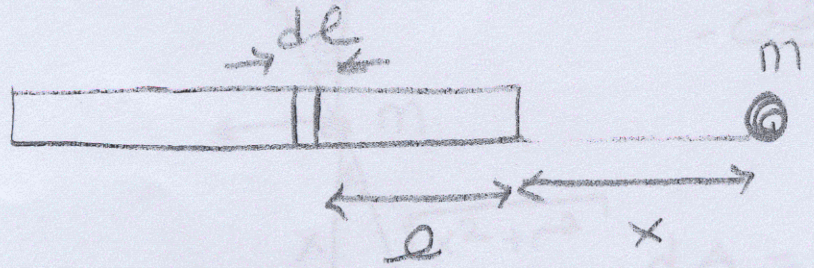
$$\therefore\ dM = \frac{M\,d\phi}{2\pi}$$

$$\int dF_x = \int_0^{2\pi} \frac{G\,m\,M\,x\,d\phi}{2\pi(x^2+a^2)^{3/2}}$$

$$F_x = \frac{G\,m\,M\,x}{(x^2+a^2)^{3/2}}$$

$$x \gg a \rightarrow F_x = \frac{G\,m\,M}{x^2}$$

(12.84)



$$\frac{dl}{L} = \frac{dm}{M} \Rightarrow dm = \frac{M\,dl}{L}$$

$$dF = -\frac{G\,m\,dm}{(l+x)^2}$$

$$F = -\frac{G\,m\,M}{L}\int_0^L \frac{dl}{(l+x)^2}$$

$\frac{1}{u}\Big|$

$du = dl$

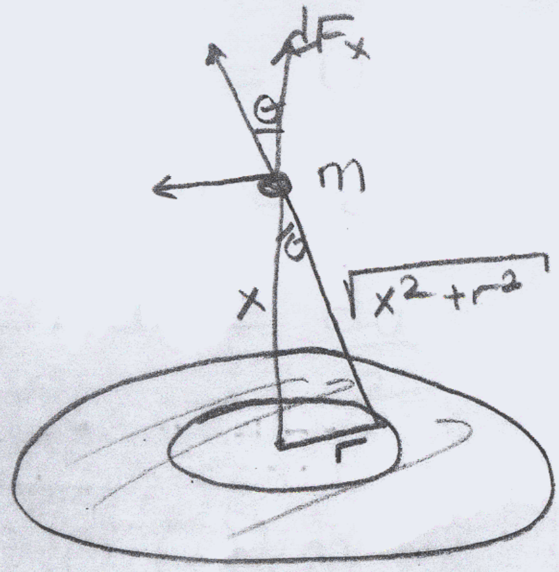
$\frac{1}{l+x}\Big|_0^L$

$l + x = u$

$$F = -\frac{G\,m\,M}{L}\int_x^{x+L} \frac{du}{u^2}$$

$$F = \frac{G\,m\,M}{L}\left(\frac{1}{x+L} - \frac{1}{x}\right)$$

12.89



$$-\cos\theta = \frac{x}{\sqrt{x^2+r^2}}$$

$$dA = 2\pi r\,dr$$

$$\frac{M}{A} = \frac{dM}{dA} \Rightarrow dM = \frac{M\,dA}{A}$$

$$dM = \frac{M\,2\pi r\,dr}{\pi a^2}$$

$$dF = \frac{G\,m\,dM}{x^2+r^2}\;\frac{x}{\sqrt{x^2+r^2}}$$

$$dF = \frac{2G\,Mmr}{a^2}\;\frac{x}{(x^2+r^2)^{3/2}}\,dr$$

$$\int dF = \int_0^a \frac{2G\,M\,m\,x}{a^2}\;\frac{r}{(x^2+r^2)^{3/2}}\,dx$$

$$x^2 + r^2 = u$$

$$2r\,dr = du$$

$$r\,dr = \frac{du}{2}$$

$$F = \frac{2GMm}{a^2}\,\frac{x}{2}\int_{x^2}^{a^2+r^2} \frac{du}{u^{3/2}}$$

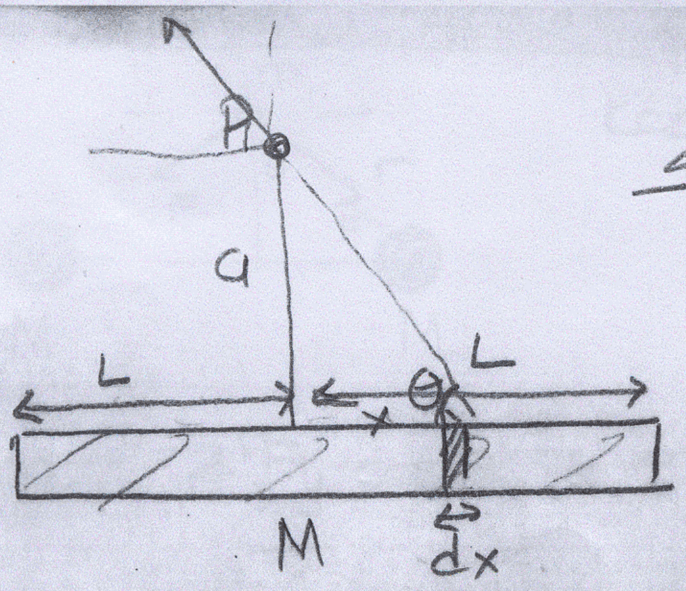
$$F = \frac{GMm\,x}{a^2}\,\frac{u^{-1/2}}{\left(-1/2\right)}\Bigg|_{x^2}^{a^2+x^2}$$

$$F = -\frac{2GMm\,x}{a^2}\left(\frac{1}{\sqrt{a^2+x^2}} - \frac{1}{\sqrt{x^2}}\right)$$

$$F = \frac{2GMm}{a^2}\left(1 - \frac{x}{\sqrt{a^2+x^2}}\right)$$

$$\left(\sqrt{a^2+x^2}\right)^{-1} = \left(x\sqrt{1+\left(\frac{a}{x}\right)^2}\right)^{-1} = \frac{1}{x}\left(1 - \frac{1}{2}\left(\frac{a}{x}\right.\right.$$

(12.90)



$$\sin\theta = \frac{a}{\sqrt{a^2+x^2}}$$

$$\frac{dx}{L} = \frac{dm}{M} \Rightarrow dm = \frac{M}{L}\,dx$$

$$dF_y = \frac{G\,m\,dM}{a^2+x^2}\sin\theta$$

$$dF_y = \frac{Gm\,dM}{a^2+x^2}\,\frac{a}{\sqrt{a^2+x^2}}$$

$$dF_y = \frac{Gm\,a\,M}{L}\,\frac{dx}{(a^2+x^2)^{3/2}}$$

$\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$\frac{\cos\theta}{\sin\theta}$

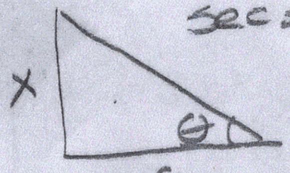
$\tan\theta \cdot d\theta$

$\frac{1}{\sin\theta}$

$\frac{1}{\cos\theta}$

$\frac{\cos\theta}{\sin\theta}$

$x = a\tan\theta$



$\sec = \frac{1}{\cos} =$

$dx = a\sec^2\theta\,d\theta$

$a^2 + x^2 = a^2(1+\tan^2\theta) = a^2\sec^2\theta$

$$\int \frac{a\sec^2\theta\,d\theta}{a^3\sec^3\theta} = \frac{1}{a^2}\int \frac{1}{\sec\theta}\,d\theta$$

$\cos\theta$

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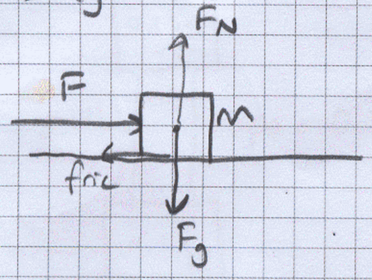
NEWTON'S LAW

1) $m = 2kg$



$F = 12N$

$F_N = F_g = mg$

$F_N = F_g = 20 \, kg\,m/s^2 = 20N$

$F_{Net} = ma$

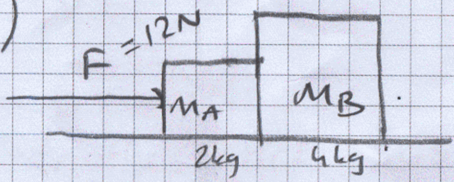
$12N = 2kg\, a \implies a = 6 m/s^2$

if there is friction $f_{fric} = 4N$

$F_{Net} = F - f_{fric} = ma$

$8N = 2kg\, a \implies a = 4 m/s^2$

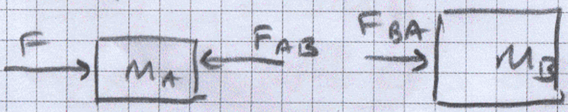
2)



$F_{net} = (2+4)\, a$

$a = 2 \ m/s^2$

F_{AB} : force of B on A

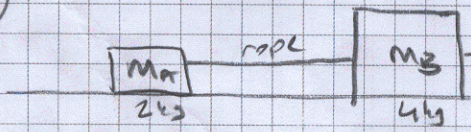


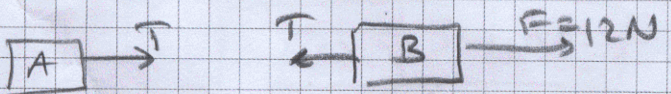
A	B
$F - F_{AB} = m_A a$	$F_{BA} = m_B a$
$12 - F_{AB} = 2.2$	$F_{BA} = 4.2$
$F_{AB} = 8N$	$F_{BA} = 8N$

if there is friction

$F_{net} = F - f_{friction}$

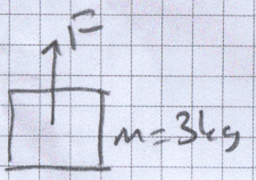
3)





B	A
$12N - T = 4.2$	$T = 2.2$
$T = 4N$	$= 4N$

4)



a) for constant velocity $= 3 m/s$

$F = mg = 30N$

b) for constant acceleration $3 m/s^2$

$F - 30N = ma$

$= 3kg \; 3 m/s^2$

$F - 30 = 9N \implies \boxed{F = 39N}$

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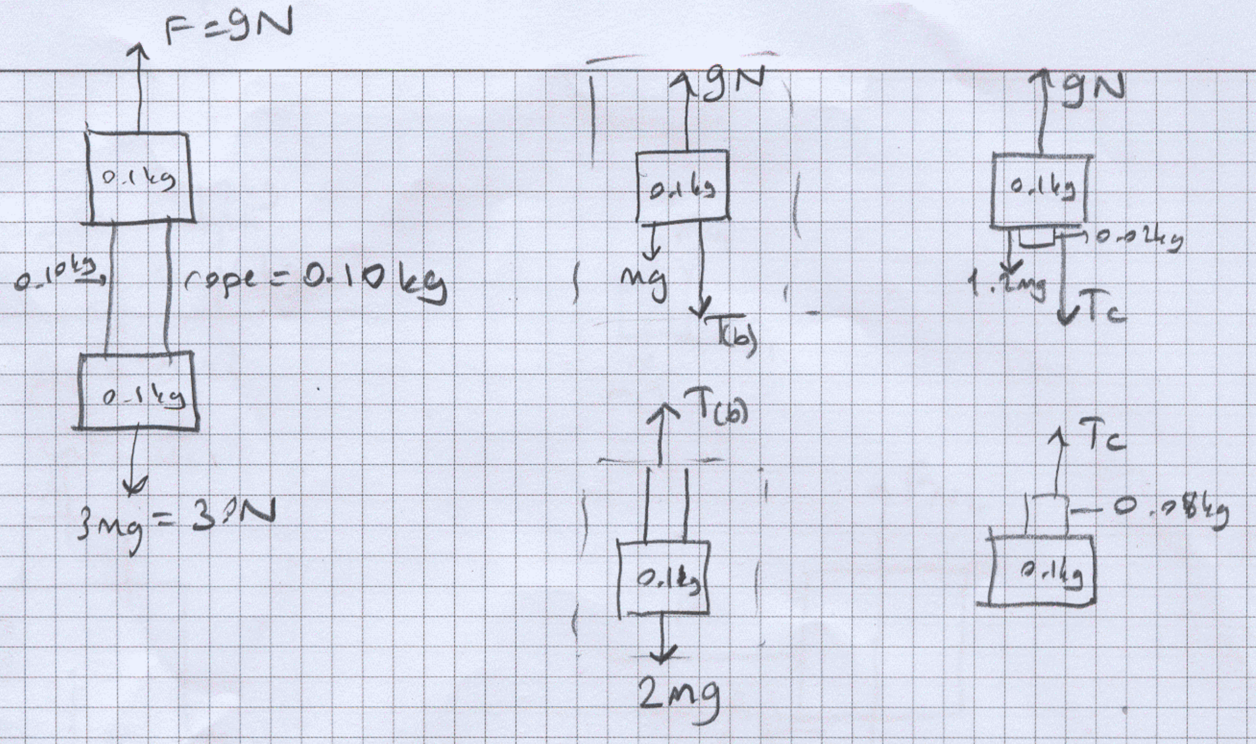
5)



a) For the entire system

$F_{Net} = Ma$

$F - 3mg = 3ma$

$9N - 3N = 0.3\ a$

$\boxed{a = 20 m/s^2}$

b) at the top of the rope tension?

$F_{net} = ma$

$T_{(b)} - 2mg = 2m\ a$

$T_{(b)} - 2N = 0.2\ 20 m/s^2$

$\boxed{T_{(b)} = 6N}$

c) the bottom of one fift of the rope tension?

$F_{net} = Ma$

$9N - 1.2mg - T_{(c)} = 0.12kg\ 20\ m/s^2 = 2.4N$

$\boxed{T_{(c)} = 5.4N}$

$T_{(c)} - 1.8\ mg = T_{(c)} - 1.8N = 0.18kg\ 20 m/s^2$

$\boxed{T_c = 5.4N}$

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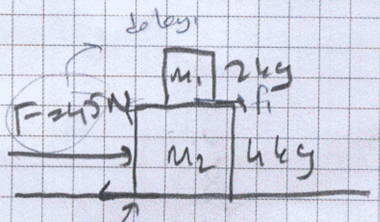
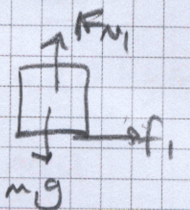
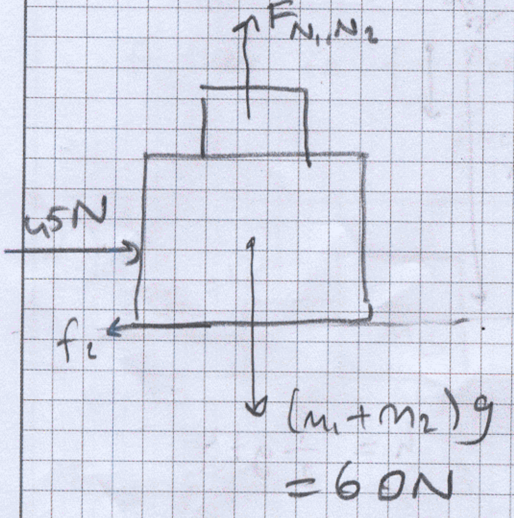
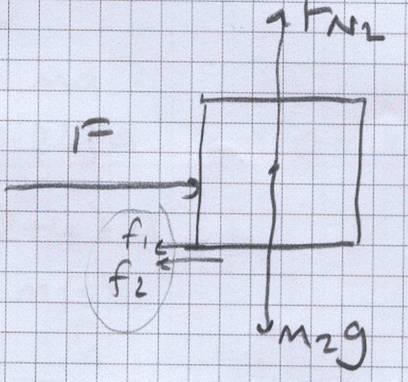
6)



$\mu_k = 0.25$







$F_{net_x} = M a_x$

$F_{net_y} = M a_y$

$F_{N_{1,2}} - 60 = 0$

$45 - f_2 = 6 \cdot a$

$f_2 = \mu_k F_N = 0.25 \cdot 60N = 15N$

$45 - 15 = 6 \cdot a$

$\boxed{a = 5 m/s^2}$

$\cancel{F_{net_x} = m_1 a_x}$

$\cancel{f_1 = 2kg \cdot 5}$

$45 - f_2 - f_1 = 4kg \; 5 m/s^2$

$45 - 15 - f_1 = 20N$

$f_1 = 10N$

$\mu_s 20N = 10N$

$\mu_s = \frac{1}{2}$

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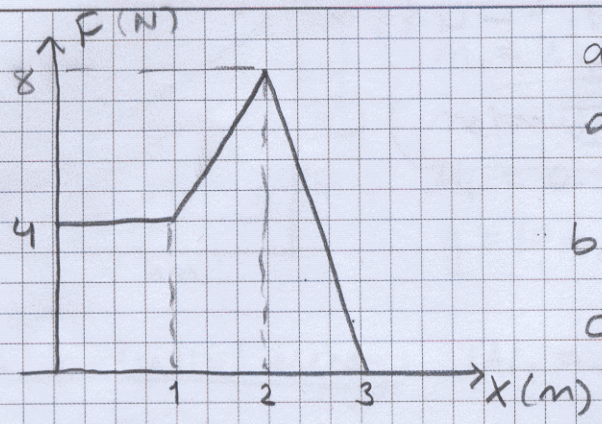
Work & Energy

3)



a) Find the work done on the object as it moves from $x=0$ to $x=1$

b) $x=1m$ to $x=2m$

c) $x=2$ to $x=3m$

$m=2kg$

its initial velocity $V_0=0$

d) use work energy theorem to find V_1 at $X=1m$.

e) V_2 at $X=2m$

f) V_3 at $X=3m$

a) work done $= \int F_x \, dx$ = area under curve

$x=0$ to $x=1$

$4N \times 1m = 4J$

b) $x=1$ to $x=2$

$\left.\begin{array}{l} \frac{1}{2} \, 1m \, 4N = 2J \\ 4N \, 1m = 4J \end{array}\right\} 6J$

c) $x=2$ to $x=3$

$\frac{1}{2} \, 1m \, 8N = 4J$

d) from $x=0$ to $x=1$ work done $= 4J = \frac{1}{2} m v_1^2 - V_0$

$4J = \frac{1}{2} \, 2kg \, V_1^2$

$V_1 = 2m/s$

e) from $x=1$ to $x=2$

$$6J = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} 2kg\, v_2^2 - \frac{1}{2} 2kg\, 4m^2/s^2$$

$$6J = v_2^2 - 4\, kg\, m^2/s^2$$

$$\boxed{v_2 = \sqrt{10}\ m/s}$$

f) $x=2$ to $x=3$

$$4J = \frac{1}{2} m v_3^2 - \frac{1}{2} m v_2^2$$

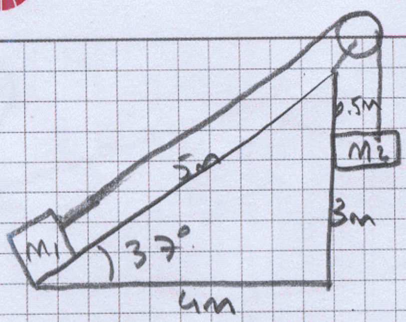
$$4J = \frac{1}{2} 2kg\, v_3^2 - 10J$$

$$14J = v_3^2$$

$$\boxed{v_3 = \sqrt{14}\ m/s}$$

same speed

1)

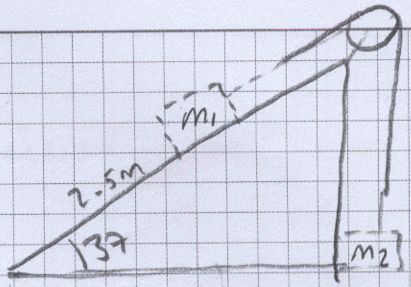


$m_1 = 2\,kg$

$m_2 = 4\,kg$

$\mu_k = 0.55$

$g = 10\,m/s^2$



initial potential energy: $U_{i_1} = m_1 g h = 0$

$U_{i_2} = m_2 g h = m_2 g\,(2.5m) = 100J$

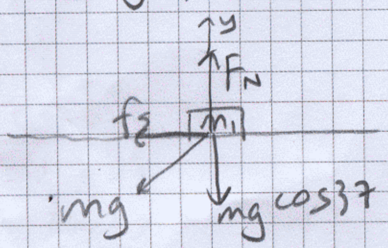
initial kinetic energy: $K_i = 0$

final potential energy: $U_{f1} = m_1 g \sin 37 \cdot (2.5m) = 30J$

$U_{f2} = 0$

final kinetic energy: $K_f = \frac{1}{2}(m_1 + m_2)\, v_f^2 = 3\,kg\, V_f^2$

work done by friction:



$F_{net_y} = m_1 a_y$

$F_N - mg\cos 37 = m_1 \cdot 0$

$F_N = m_1 g \frac{4}{5}$

$f_s = \mu F_N = 0.55\,(2\,kg)\,10\,m/s^2\,\frac{4}{5}$

$f_s = 8.8N$

$W = -8.8N\,(2.5m) \implies \boxed{W = -22J}$

work done by friction $= (U_f + K_f) - (U_i + K_i)$

$-22J = (30J + 3\,kg\,V_f^2) - (100J + 0)$

$-22 - 30 + 100 = 48J = 48\,kg\,m^2/s^2 = 3\,kg\,V_f^2$

$\boxed{V_f = 4\,m/s}$

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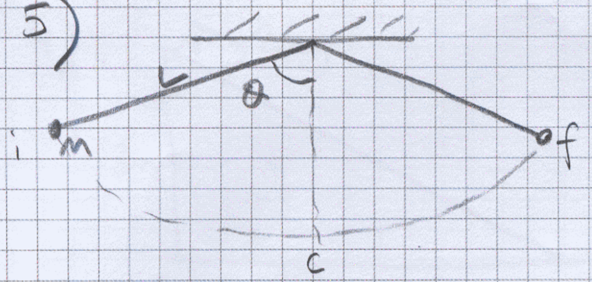
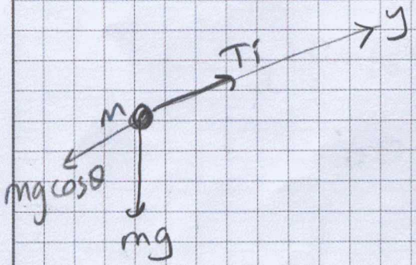
5)



find the tension in the string in i, c, f positions

at i, f

$F_{net\,y} = ma_y$



$$T_i - mg\cos\theta = \frac{m v_i^2}{L} = \frac{m\,(0)}{\cancel{L}}$$

because the object is at rest in i

$T_i = mg\cos\theta$

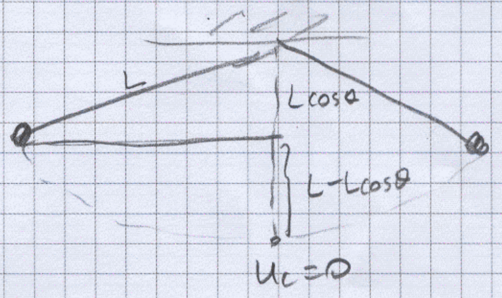
$T_f = mg\cos\theta$

at c :

$F_{net\,x} = ma$

$$T_c - mg = \frac{m v_c^2}{L}$$

$$T_c = mg + \frac{m v_c^2}{L}$$



$U_i + K_i = U_c + K_c$

$mgL(1-\cos\theta) + 0 = 0 + \frac{1}{2} m v_c^2$

$$\frac{m v_c^2}{L} = 2mg(1-\cos\theta)$$

$$T_c = mg + 2mg(1-\cos\theta) = \underline{mg(3 - 2\cos\theta)}$$

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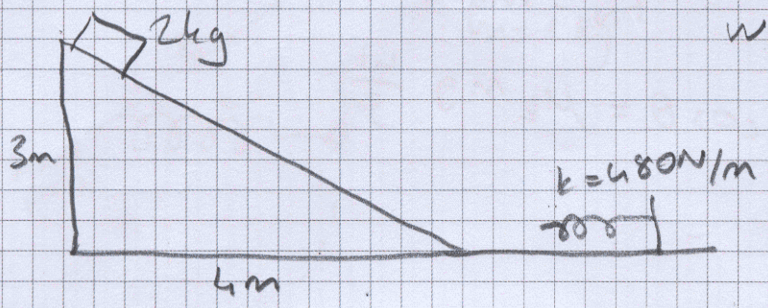
4)



what is the maximum compression of the spring

energy is conserved, no friction

$$U_i + \underset{0}{K_i} = \underset{0}{U_f} + K_f$$

$$2kg\,10m/s^2\,3m = \frac{1}{2}\,2kg\,V_f^2$$

$$60\,\cancel{kg}m^2/s^2 = \cancel{kg}\,V_f^2$$

$$V_f = 7.7\,m/s$$

for the spring part

$U_i = 0$, $K_i = 60J$

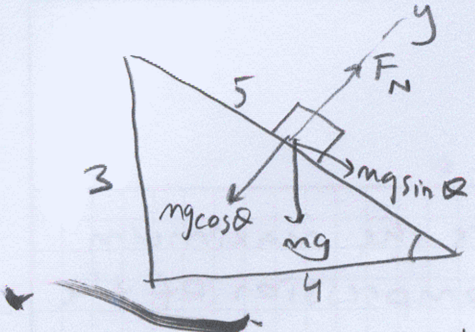
$U_f = \frac{1}{2}kx^2$, $K_f = 0$

$$60J = \frac{1}{2}kx^2 + 0$$

$$60J = \frac{1}{\cancel{2}}\,\overset{240}{\cancel{480}}\frac{N}{m}\,x^2$$

$$\frac{60}{240_4} = x^2$$

$$x = \frac{1}{2}m$$



$F_{net\,y} = ma_y = m0$

$F_N = mg\cos\theta$

$-f_k = \mu_k F_N = \mu_k mg\cos\theta = \mu_k mg \frac{4}{5}$

the work by friction

$W_f = -mg\cos\theta\,\mu_k \cdot S$

$= -\frac{1}{4} 2kg \; 10m/s^2 \; \frac{4}{5} \; 5m = -20J$

work done by friction $= (U_f + K_f) - (U_i + K_i)$

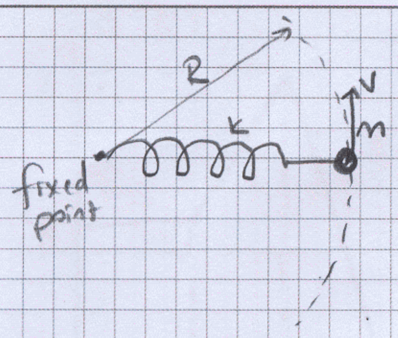
$-20J = (0 + \frac{1}{2} m v_f^2) - (mgh + 0)$

$-20J = \frac{1}{2} 2kg\, v_f^2 - 2kg\, 10m/s^2\, 3m$

$40 = v_f^2$

$v_f = 6.3\, m/s$

2)



$m = 1\,kg$

a) if $R = 1m$, $v = 1 m/s$
what is the tension in the spring at the point where it attaches to m?

b) If the relaxed length of the spring is 0.90m, what is the spring constant k?

c) if the ball and spring rotate with $v = 2m/s$, what is the new radius of the ball's path

d) How much work is done on the mass and on the spring

a) $F_{net} = ma$

$$T = \frac{mv^2}{r} = \frac{1kg\,(1m/s)^2}{1m} = 1N$$

b) The tension equals the force due to spring $= kx = 1N$

where $x = R -$ relaxed length $= 1m - 0.9m = 0.1m$

$$k = \frac{1.0N}{x} = \frac{1}{0.1} = 10 N/m$$

d)

c) $kx' = \frac{mv'^2}{R'}$

$$10\frac{N}{m}(R' - 0.9m) = \frac{1kg\,(2m/s)^2}{R'}$$

$$10\frac{N}{m}R' - 9N = \frac{4kg\,m^2/s^2}{R'}$$

$$10R'^2 - 9R' = 4 \Longrightarrow R' = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+9 \pm \sqrt{81 + 160}}{20}$$

$$\boxed{R' = 1.23m}$$

d) Work to increase the kinetic energy of mass:

$$= \frac{1}{2} m (V'^2 - V^2)$$

$$= \frac{1}{2} 1kg (4-1) m^2/s^2$$

$$= 1.5 J$$

$$x' = R' - R = 1.23 - 1m = 0.23m$$

work to stretch spring

$$= \frac{1}{2} k (x'^2 - x^2)$$

$$= \frac{1}{2} 10 \frac{N}{m} (0.053 - 0.010) = 0.21 J$$

Total work done $= 1.5 + 0.2 = 1.7 J$

SIMPLE HARMONIC MOTION

→ occurs when $\vec{F}$ is in the opposite dir. to the displacement x and proportional to x.

$$F = -kx$$

→ Force is called restoring force ⟹ acts on the object to return it to its eq. position

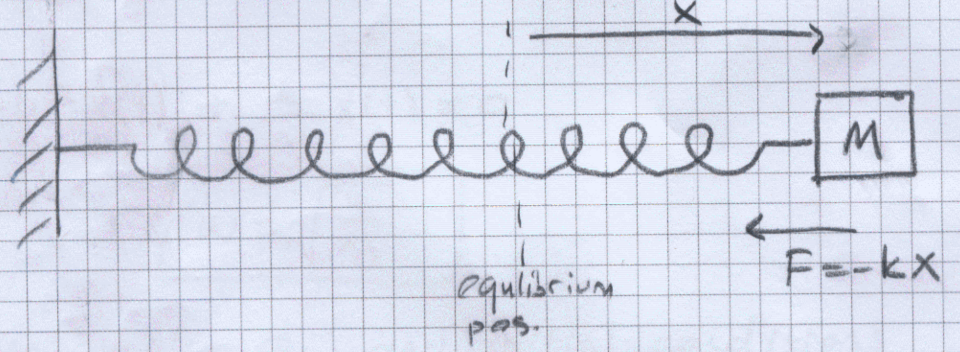
Amplitude A: max. displacement from the eq. position

Period T: time for one complete oscillation

$$x(t) = x(t+T)$$

⟹ harmonic motion is not motion in a circle. but we use angular freq.

$$\omega = 2\pi f = \frac{2\pi}{T}$$



✶ $F_{net} = ma$

$-kx = ma$

✶ $v = \frac{dx}{dt}$ and $a = \frac{dv}{dt}$

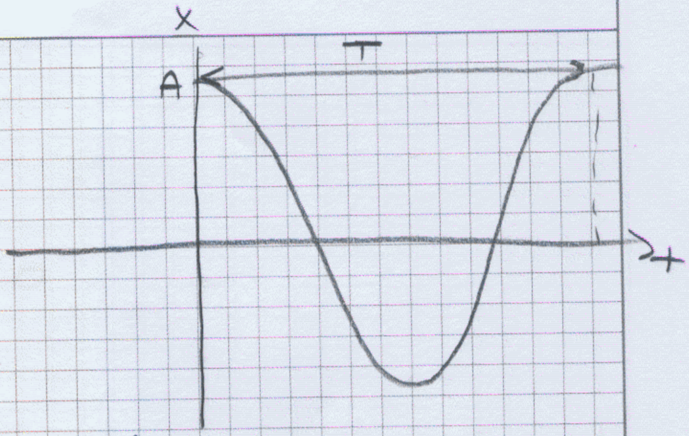
$a = \frac{d^2x}{dt^2}$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0} \quad eq\,1$$

solution of eq 1 gives x as a function of t

$$x(t) = A\cos\left(\frac{2\pi t}{T} + \phi\right)$$
$$= A\cos(2\pi f t + \phi)$$
$$\boxed{x(t) = A\cos(\omega t + \phi)}$$



max of $\cos(\omega t + \phi) = 1$ so $x = A$

$$\Rightarrow v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2\left[A\cos(\omega t + \phi)\right]$$

$$a(t) = -\omega^2 x(t)$$

$\dfrac{d^2x}{dt^2} = -\omega^2 x(t)$ put this into eq 1

$$-\omega^2 x(t) + \frac{k}{m}x(t) = 0$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Phase angle ϕ and initial conditions

$$x(t) = A\cos(\omega t + \phi)$$
$$x(0) = x_0 = A\cos\phi \Rightarrow \cos\phi = \frac{x_0}{A}$$
$$v(t) = -\omega A \sin(\omega t + \phi)$$
$$v(0) = v_0 = -\omega A \sin\phi \Rightarrow \sin\phi = -\frac{v_0}{\omega A}$$
$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{(-v_0/\omega A)}{x_0/A} = -\frac{v_0}{\omega x_0}$$
$$\sin^2\phi + \cos^2\phi = (-v_0/\omega A)^2 + (x_0/A)^2 = 1$$
$$A = \left[x_0^2 + \left(-\frac{v_0}{\omega}\right)^2\right]^{1/2}$$

Special Cases :

for $\omega = 0.350\ Hz = 0.35\ s^{-1}$

1) when $x_0 = 0.20\,m$ and $V_0 = 0$

$$A = \left[(x_0)^2 + \left(-\frac{v_0}{\omega}\right)^2 \right]^{1/2}$$

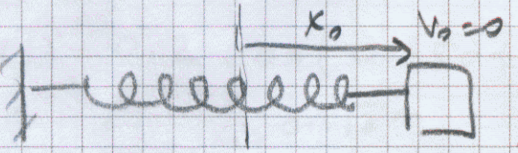
$$= \left[(0.20)^2 + 0 \right]^{1/2} = 0.20\,m$$

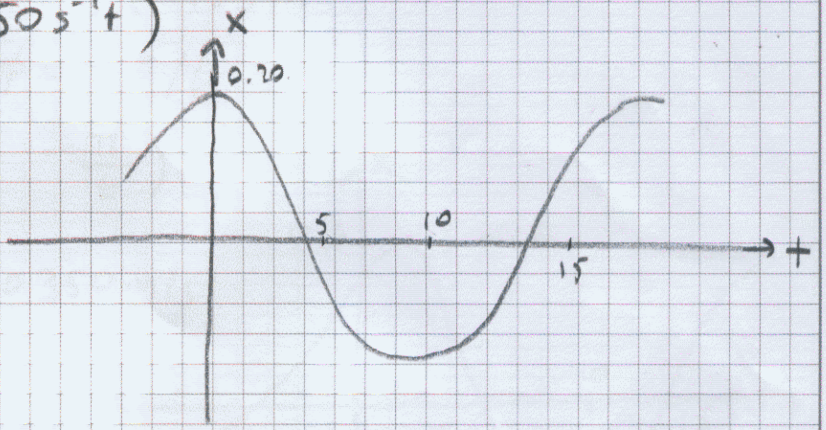
$$\cos\phi = \frac{x_0}{A} = \frac{0.20}{0.20} = 1$$

$$\sin\phi = -\frac{v_0}{\omega A} = 0 \qquad \phi = 0$$

$2\pi\ rad = 1\ rad$

$$x(t) = (0.20)\cos(0.350\,s^{-1}t)$$





2) when $x_0 = 0$ $\quad V_0 = 0.070\ m/s$

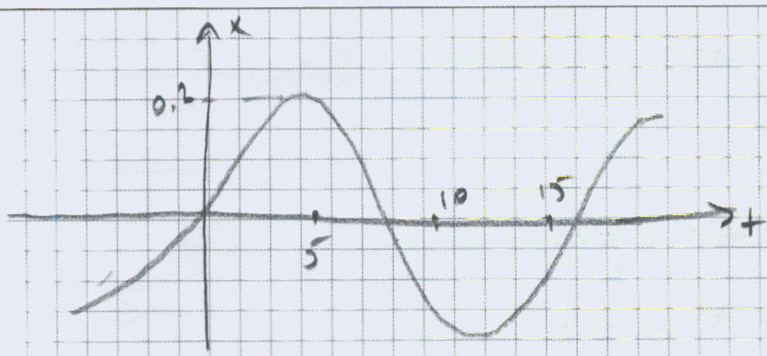
$$A = \left[(x_0)^2 + \left(-\frac{v_0}{\omega}\right)^2 \right]^{1/2} = \left[0 + \left[(-0.07/0.35)^2\right] \right]^{1/2}$$

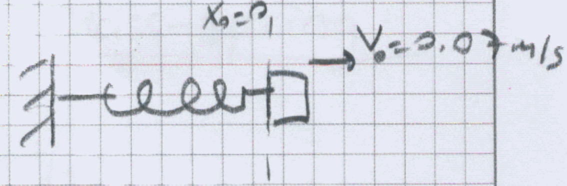
$A = 0.20\,m$

$$\cos\phi = \frac{x_0}{A} = \frac{0}{0.20} = 0$$

$$\sin\phi = -\frac{V_0}{\omega A} = \frac{-0.070}{(0.35)(0.20)} = -1 \qquad \phi = -\frac{\pi}{2}$$

$$x(t) = 0.20\cos\left(0.350t - \frac{\pi}{2}\right)$$





3) when $x_0 = 0.1\sqrt{3}$ m $\quad v_0 = 0.035$ m/s

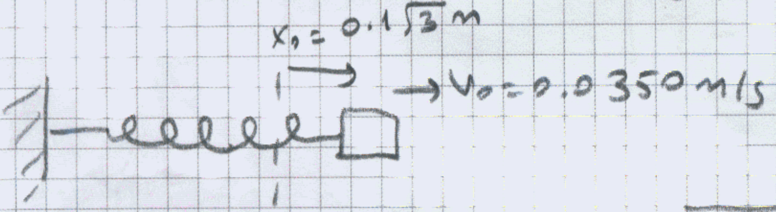
$$A = \left[(x_0)^2 + \left(-\frac{v_0}{\omega}\right)^2 \right]^{1/2} = \left\{ (0.03) + \left[\left(-0.035/0.35\right)^2\right] \right\}^{1/2}$$

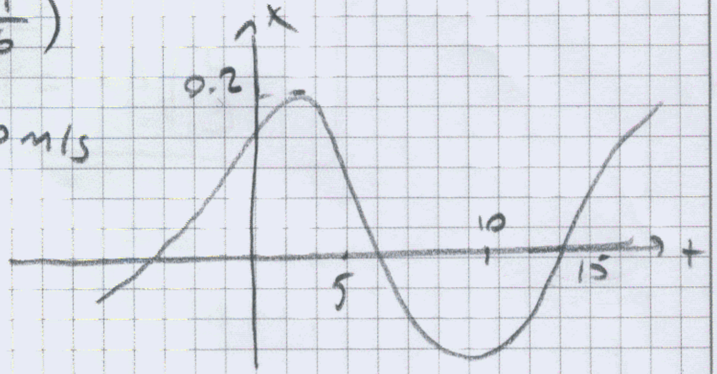
$A = 0.20$ m

$$\tan\phi = -\frac{v_0}{\omega x_0} = -\frac{0.070 \text{ m/s}}{(0.350 s^{-1})(0.1\sqrt{3} \text{ m}}$$

$\tan\phi = -0.577 \longrightarrow \phi = -\pi/6$

$$x(t) = 0.2\cos\left(0.350t - \frac{\pi}{6}\right)$$





★ find t when $x = 0.10$ m

$0.10\text{m} = 0.2\cos(0.35t - \pi/6)$

$1/2 = \cos(0.350t - \pi/6)$

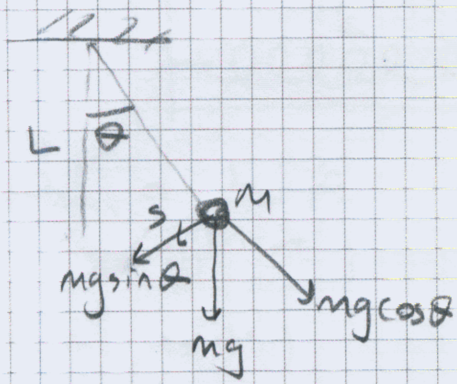
$\cos 60 = \cos\frac{\pi}{3} = \frac{1}{2}$ so $\frac{\pi}{3} = 0.35t - \pi/6$

$0.35t = \pi/2$

$t = 4.49$ s

Simple Harmonic Motion other than mass-spring

pendulum:



$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

$$s = L\theta \qquad \sin\theta \approx \theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0}$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\omega = \sqrt{\frac{g}{L}} = 2\pi f = \frac{2\pi}{T}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Energy: for a spring with $F = -kx$

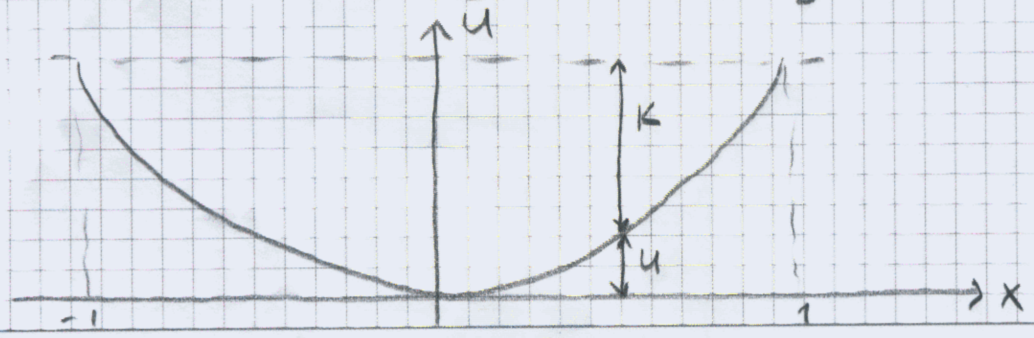
$$U = \frac{1}{2}kx^2$$

total $E = U + K$

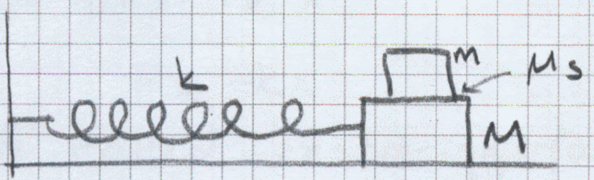
$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

for $x = \pm A$, $v = 0$

for $x = \pm A$, $E = \frac{1}{2}kA^2 + 0 = \frac{1}{2}kA^2$



$\boxed{13.68}$



Find the max amp. of oscillation such that the top block will not slip

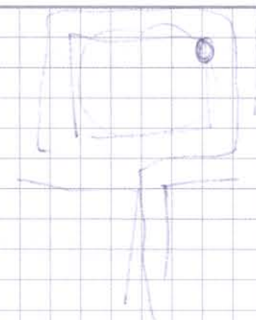
$$a_{max} = \frac{k}{m_{TOT}} A$$

$$f_s = \mu_s m g$$

$$F_{tot} = ma \implies \mu_s m g = m a \implies \boxed{a = \mu_s g}$$

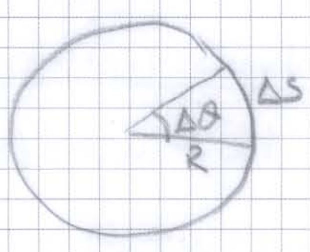
$$a_{max} = \left(\frac{k}{m+M}\right) A = \mu_s g \implies \boxed{A = \frac{\mu_s g \,(M+m)}{k}}$$

if A is larger than this, the spring gives a larger a so, top block slips



ANGULAR MOTION

Angle in Radians:



$$\underset{\text{meter}}{\Delta S} = \Delta\theta \underset{\text{meter}}{R}$$

angle has no unit

Period: T = time for one complete rotation $= \dfrac{2\pi r}{v}$

frequency: $f = \dfrac{1}{T} = \dfrac{v}{2\pi r}$

angular velocity: $\omega = \dfrac{d\theta}{dt}$

$$\Delta\theta = \frac{1}{R}\Delta S$$

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{R}\frac{\Delta S}{\Delta t} \implies \frac{d\theta}{dt} = \frac{1}{R}\frac{ds}{dt}$$

$$\omega = \frac{v}{R}$$

$$f = \frac{v}{2\pi R} \longrightarrow v = 2\pi R f$$

$$\omega = \frac{2\pi R f}{R} = 2\pi f$$

radial acceleration: $a_r = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R} = \dfrac{4\pi^2 R}{T^2}$

$$= (2\pi R f)^2 / R = (2\pi f)^2 R$$

$$\boxed{a_r = \omega^2 R}$$

Angular acceleration: $a_a = \frac{d\omega}{dt}$ (rate of change of angular velocity)

For constant angular acc.

$\omega(t) = \omega_0 + at$

$\theta(t) = \theta_0 + \omega t + \frac{1}{2}at^2$

$\omega^2(\theta) = \omega_0^2 + 2a(\theta - \theta_0)$

Linear analogy

$v(t) = v_0 + at$

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

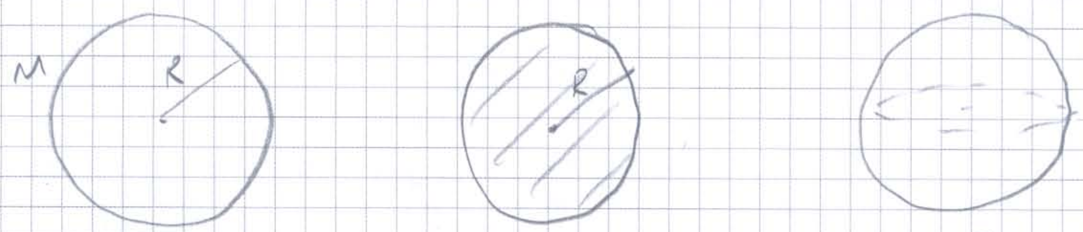
$v^2(x) = v_0^2 + 2a(x - x_0)$

Rigid Body:

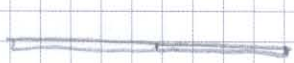
All points on the object rotate with same ω angular velocity

moment of inertia:

$$\boxed{K = \frac{1}{2}I\omega^2}$$



$I = MR^2$ $\qquad I = \frac{1}{2}MR^2$ $\qquad I = \frac{2}{5}MR^2$



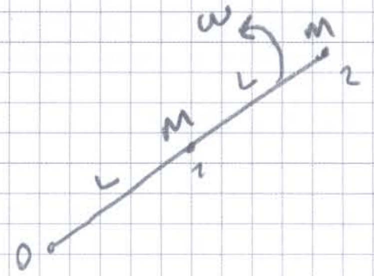
$I_{cm} = \frac{1}{12}ML^2$ $\qquad I_{end} = \frac{1}{3}ML^2$

Parallel axis theorem: $I_{parallel} = I_{cm} + Md^2$

d = distance between the axis

$$I_{end} = \frac{1}{12}ML^2 + M\frac{L^2}{4} = \frac{ML^2}{3}$$

Example:



a) moment of inertia about 0

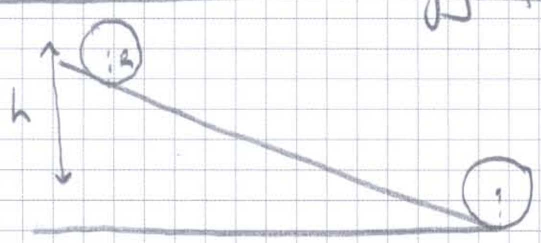
b) K.E of rotation

a) $I = mL^2 + m(4L^2) = 5mL^2$

b) $K = \frac{1}{2} I \omega^2$

$\left.\begin{array}{l} K_1 = \frac{1}{2} m L^2 \omega^2 \\ K_2 = \frac{1}{2} 4 m L^2 \omega^2 \end{array}\right\} \quad K_{tot} = \frac{1}{2} L^2 \omega^2 (1+4) = \frac{5}{2} L^2 \omega^2$

Conservation of energy for Rotation



$E_{top} = U + K = Mgh + 0$

$E_{bottom} = \frac{1}{2} M v^2 + \frac{1}{2} I_{cm} \omega^2$

$\omega = \frac{v}{R} \longrightarrow E_{bot} = \frac{1}{2} M v^2 + \frac{1}{2} I_{cm} \frac{v^2}{R^2}$

$\boxed{Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I_{cm} \frac{v^2}{R^2}} \longrightarrow v^2 = \frac{2gh}{(1 + I_{cm}/MR^2)}$

Special cases:

1) hoop: $I_{cm} = MR^2$, $v = (gh)^{1/2}$

2) disk: $I_{cm} = \frac{1}{2} MR^2$, $v = \left(\frac{4gh}{3}\right)^{1/2}$

3) Sphere: $I_{cm} = \frac{2MR^2}{5}$, $v = \left(\frac{10gh}{7}\right)^{1/2}$

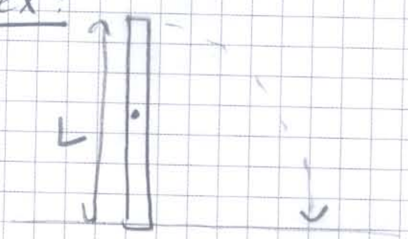
Work done on Rotating Rigid body:

$$w^2 = w_0^2 + 2a(\theta - \theta_0)$$

$$\frac{1}{2} I_{cm} w^2 - \frac{1}{2} I_{cm} w_0^2 = I_{cm} a (\theta - \theta_0)$$

$$\boxed{W = I_{cm} a (\theta - \theta_0)}$$

ex:



$$U_i + K_i = U_f + K_f$$

$$\frac{mgL}{2} = \frac{1}{2} I w^2$$

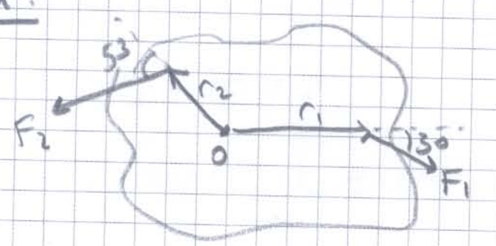
$$w = \left(\frac{mgL}{I}\right)^{1/2} = \left(\frac{mgL}{\frac{1}{3} m L^2}\right)^{1/2}$$

$$w = \left(\frac{3g}{L}\right)^{1/2}$$

$$V_{top} = wL = (3gL)^{1/2}$$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$ ← elini $\vec{r}$ yönünde uzat, f yönünde kıvır, baş parmağın torque'un yönü

ex:



find the magnitude and direction of the net torque on the object about O.

$r_1 = 2m$, $F_1 = 1.5N$, $F_2 = 1N$
$r_2 = 1m$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\tau_1 = r_1 F_1 \sin 30 = 2 . 1.5 . 0.5 = 1.5 Nm$ into the page

$\tau_2 = r_2 F_2 \sin 53 = 1 . 1 . 0.8 = 0.5 Nm$ out of the page

$\tau = 1.5 - 0.8 = 0.7 Nm$ into the page

Angular Momentum: $\vec{L}$ ← elini $\vec{r}$ yönünde uzat, v yönünde kıvır, başparmak L'in yönü

$$\vec{L} = \vec{r} \times m\vec{v}$$

rotating body:

$$\vec{L} = I\vec{\omega}$$

→ direction of $\vec{\omega}$ is ⇒ parmaklarını rotation yönünde kıvır başparmak yönü verir

$$\tau = \frac{dL}{dt} = \frac{d\,I\omega}{dt} = I\alpha$$

$$\text{work} = W = I\alpha\theta = \tau\theta$$

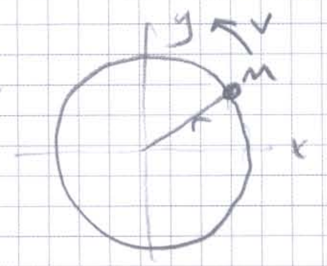
Conservation of angular momentum:

if no external torque acts on a system

→ L_{tot} → constant

$$\tau = 0 \rightarrow \tau = \frac{dL}{dt} = 0$$

ex:



a) Find mag. and dir. of L

b) alternative exp. for L in terms of ω

v ve r dik olmalı

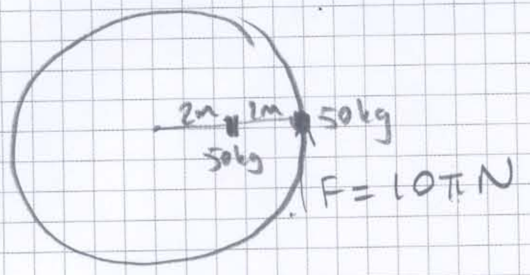
a) L is out of page

$$L = mvr$$

b) $v = \omega r$, $L = mr^2\omega = I\omega$

KOÇ ÜNİVERSİTESİ

example:



a) I ?

$I = 50kg(2m)^2 + 50kg(4m)^2 = 1000\ kgm^2$

b) angular acceleration?

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = 4m\ 10\pi N \sin 90 = 40\pi\ N\cdot m = I a$

$a = 0.04\pi\ s^{-1}$

c) one complete revolution $t = ?$

$2\pi = \frac{1}{2} a T^2$

$T = \left(\frac{4\pi}{a}\right)^{1/2} = \left[\frac{4\pi}{4\pi \times 10^{-2} s^{-2}}\right]^{1/2} = 10s$

d) ω? KE?

$\omega = \omega_0 + at = 0 + (4\pi \times 10^{-2} s^{-2})(10s) = 0.4\pi\ s^{-1}$

$KE = \frac{1}{2} I \omega^2 = \frac{1}{2}(1000\ kgm^2)(0.4\pi\ s^{-1})^2 = 80\pi^2 J$

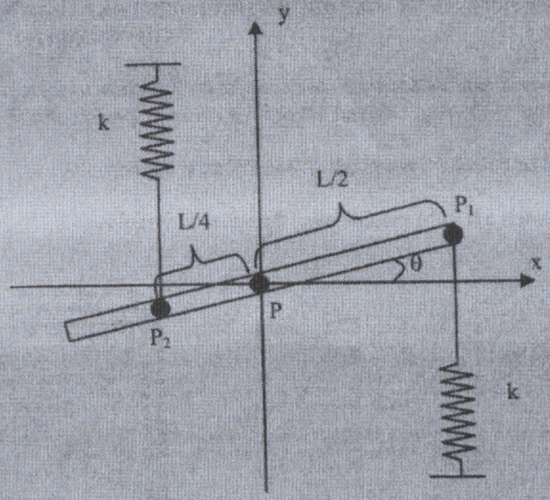
e) Work done ?

$W = \tau\theta = (40\pi\ Nm)(2\pi) = 80\pi^2 J = K_f - K_i$

$K_f = 80\pi^2 J$

ExamRoom:	
Student ID Num	

3. (25 Points) A thin, uniform rod of length L and mass M is pivoted about an axis through point P at its center and perpendicular to the xy plane as shown in figure below. Two ideal springs with spring constants k are attached to the rod from the points P_1 and P_2. Both of the springs are at equilibrium when the rod is aligned along the x-axis ($\theta = 0\ rad$). ($I_{cm} = ML^2/12$ for a thin, uniform rod of length L and mass M)



(a) (7 pts) Consider that the rod is at an angle θ with respect to the x-axis. What is the net torque about the rotation axis ($\sum \tau_z = ?$)? Express your answer as a function of θ, k, M, and L.

(b) (8 pts) For very small θ, the rod will undergo oscillations around the equilibrium point. Find out the frequency of the oscillations as a function of θ, k, M, and L?

(c) (8 pts) Consider that the maximum angular displacement during the oscillations is $\pi/10\ rad$. Find out the magnitudes of the tangential and radial components of acceleration (a_{tan}, a_{rad}) at point P_2 for $\theta = 0\ rad$ and $\theta = \pi/10\ rad$. Express your answers as functions of k, M, and L.

(a) $\sum \tau_z = -\left(k\frac{L}{2}\sin\theta\right)\cos\theta\,\frac{L}{2} - \left(k\frac{L}{4}\sin\theta\right)\cos\theta\,\frac{L}{4} = \boxed{-\frac{5}{16}kL^2\sin\theta\cos\theta}$

(b) for $\theta \approx 0$ $\sum\tau_z \approx -\frac{5}{16}kL^2\theta = I_{cm}\alpha_z = \frac{ML^2}{12}\frac{d^2\theta}{dt^2} \Rightarrow -\frac{15}{4}\frac{k}{M}\theta = \frac{d^2\theta}{dt^2}$

$\Rightarrow \omega = \sqrt{\frac{15}{4}\frac{k}{M}} \Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{15}{4}\frac{k}{M}}}$

(c) In general $a_{rad}(t) = \omega_z^2(t)\,r$, $a_{tan}(t) = r\,\alpha_z(t)$

at $\theta = 0$ rad $\Rightarrow \alpha_z = 0 \Rightarrow \boxed{a_{tan} = 0}$, $\omega_z = \frac{\pi}{10}\omega \Rightarrow a_{rad} = \frac{\pi^2}{100}\left(\sqrt{\frac{15}{4}\frac{k}{M}}\right)^2\frac{L}{4} = \boxed{\frac{15\pi^2}{1600}\frac{kL}{M}}$

at $\theta = \frac{\pi}{10}$ rad $\Rightarrow \omega_z = 0 \Rightarrow \boxed{a_{rad} = 0}$, $|a_{tan}| = +\frac{L}{4}\cdot\frac{\pi}{10}\left(\frac{15}{4}\frac{k}{M}\right) = \boxed{+\frac{15\pi}{160}\frac{kL}{M}}$

$|\alpha_z| = |-\omega^2\frac{\pi}{10}|$

Solution continues on the back side of the page Yes □ No □