PHYS 102: General Physics — KOÇ UNIVERSITY — Spring Semester 2010

College of Arts and Sciences

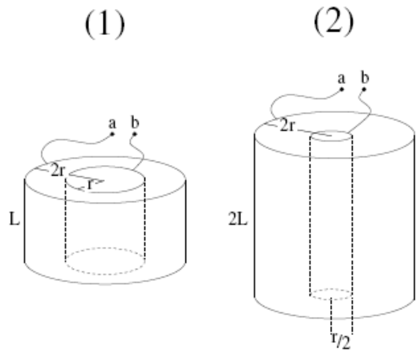
Quiz 5 April 1, 2010

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

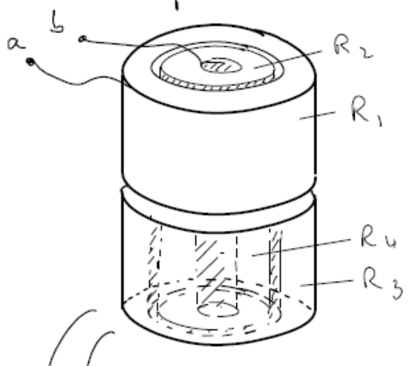
Name: **Student ID:** **Signature:**

Q. The inner and outer walls of the hollow cylindrical resistors below are coated with a metallic paint and between the walls is the same uniform material with a constant resistivity. The resistance measured between the terminals **a** and **b** in (1) is R, which is a function of L and <u>the ratio of the inner and outer radii</u>. Find the resistance measured between the terminals **a** and **b** in (2) by considering it as a collection of resistors connected in series and/or in parallel.



Solution:

Decompose (2) into four resistors:



R_1 and R_3 are identical to (1) with $\frac{r_{in}}{r_{out}} = \frac{r}{2r} = 1/2$.

R_2 and R_4 have the same length L and $\frac{r_{in}}{r_{out}} = \frac{r/2}{r} = 1/2$.

Therefore, $R_1 = R_2 = R_3 = R_4 = R$.

$$R_{ab} = (R_1 // R_3) + (R_2 // R_4) = \frac{R}{2} + \frac{R}{2} = \boxed{R.}$$

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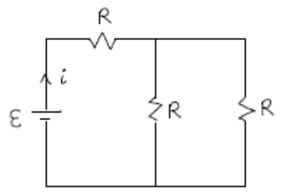
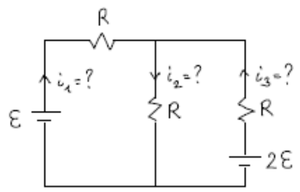
Q. The current flowing across the emf source in the resistor circuit in (a) is **i**. Calculate the currents in the circuit (b) in terms of **i** only.(*Hint:* You can consider each emf source separately first and then use the superposition principle to obtain the currents.)



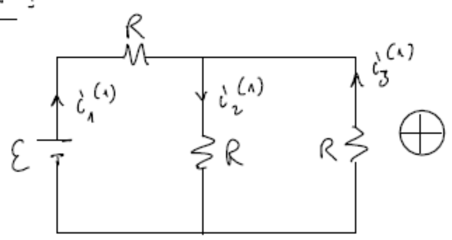
(a)



(b)

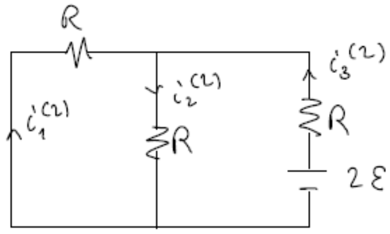
Solution:

(b) $\equiv$



(1)

$\oplus$



(2)

(1) is the same as (a) $\Rightarrow$ $\boxed{i_1^{(1)} = i\,,\quad i_2^{(1)} = i/2 \quad i_3^{(1)} = -i/2}$

(2) is the mirror image of (a), except the currents are doubled since $\varepsilon \to 2\varepsilon$. $\Rightarrow$ $\boxed{i_1^{(2)} = -i\,,\quad i_2^{(2)} = i\,,\quad i_3^{(2)} = 2i}$

$\Rightarrow$ Currents in (b) follow from superposition:

$$i_1 = i_1^{(1)} + i_1^{(2)} = 0$$
$$i_2 = i_2^{(1)} + i_2^{(2)} = 3i/2$$
$$i_3 = i_3^{(1)} + i_3^{(2)} = 3i/2 .$$

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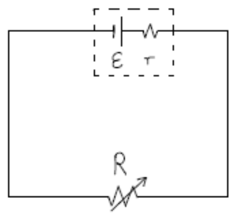
Q. The emf source in the circuit below has an internal resistance r as shown and is connected to a variable "load" resistor R. Find the value of R for which the power dissipated on the load has the maximum possible value.



Solution:

$$i = \frac{\varepsilon}{r+R} \Rightarrow \text{Power on the load} = i^2 R = \frac{\varepsilon R}{(r+R)^2}$$

Maximize w.r.t. R: $\frac{d}{dR}\left[\frac{\varepsilon R}{(r+R)^2}\right] = 0 \Rightarrow$

$$(r+R)^2 - 2R(r+R) = 0 \quad \text{or} \quad \boxed{R = r}$$

Make sure that $R=r$ gives a maximum, not a minimum:

$$\frac{d^2}{dR^2}\left[\frac{\varepsilon R}{(r+R)^2}\right]\Bigg|_{R=r} \overset{?}{<} 0$$

$$= \frac{d}{dR}\left[\frac{r-R}{(r+R)^3}\right]\Bigg|_{R=r} = -\frac{1}{2R^3} < 0 \ \checkmark.$$

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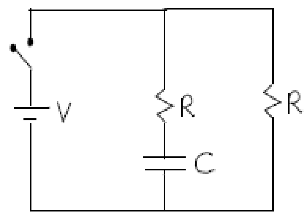
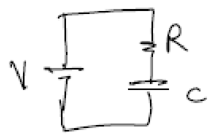
Q. The capacitor below is initially uncharged. The switch is closed at time $t = 0$. It is opened at a later time t_1, when the capacitor has a charge $Q_f/2$, half of its maximal value $Q_f = CV$. Find the charge on the capacitor at $t = 2t_1$ in terms of Q_f.

Hint:
Charging: $Q(t) = Q_f(1 - e^{-t/\tau})$
Discharging: $Q(t) = Q_i\, e^{-t/\tau}$



Solution:

The charging circuit is effectively



$\Rightarrow \tau_1 = RC$

$$\Rightarrow \left(1 - e^{-t_1/\tau_1}\right) = 1/2 \Rightarrow t_1 = \tau_1 \cdot \ln 2 = \ln 2\, RC$$

After the switch is opened, C discharges over a resistance $2R \Rightarrow \tau_2 = 2RC$

duration of discharge

$$\Rightarrow Q(2t_1) = \frac{Q_f}{2} \cdot \left(1 - e^{-t_1/\tau_2}\right)$$

$$= \frac{Q_f}{2} \cdot \left(1 - e^{-(\ln 2\, RC/2RC)}\right)$$

$$= \frac{Q_f}{2}\left(1 - e^{-\frac{\ln 2}{2}}\right) = \frac{Q_f}{2}\left(1 - \frac{\sqrt{2}}{2}\right) \approx \boxed{\frac{Q_f}{7}}$$