

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: ANWER

Student ID:

Signature:

The wire shown in the figure is infinitely long and carries a current I . Starting from the Biot-Savart law, find the magnetic field (both magnitude and direction) at point P.

$$(d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2})$$

$$d\vec{B} = 0$$

for horizontal part, so $\vec{B} = \vec{0}$

for vertical part

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} d\vec{l} \times \hat{r}$$

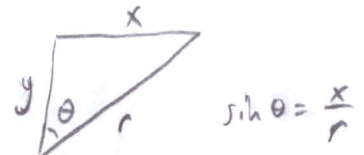
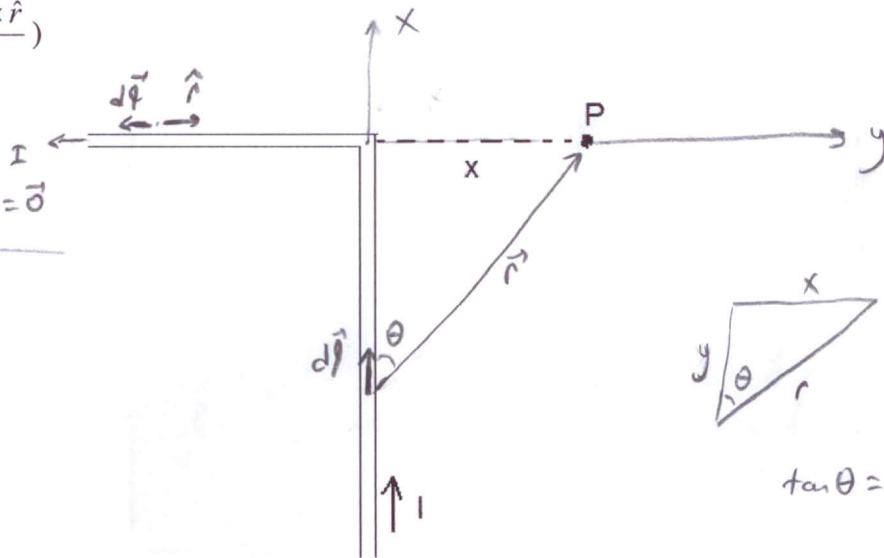
$$d\vec{B} = \hat{k} \frac{\mu_0 I}{4\pi r^2} dl \sin\theta$$

$$\vec{B} = \hat{k} \int dB = \hat{k} \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2} = \hat{k} \frac{\mu_0 I}{4\pi} \int \frac{\sin\theta dy}{r^2}$$

$$\frac{\mu_0 I}{4\pi} \int_{\pi/2}^0 \sin\theta \cdot \frac{\sin^2\theta}{x^2} \cdot \frac{(-x) d\theta}{\sin^2\theta}$$

$$= \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{\sin\theta}{x} d\theta = \frac{\mu_0 I}{4\pi x} [-\cos\theta]_0^{\pi/2}$$

$$\vec{B} = \hat{k} \frac{\mu_0 I}{4\pi x}$$



$$\sin\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

$$\frac{1}{r^2} = \frac{\sin^2\theta}{x^2}$$

$$y = \frac{x}{\tan\theta}$$

$$\frac{dy}{dy} = \frac{d}{dy} \left(\frac{x}{\tan\theta} \right) = 1$$

$$x \frac{d}{dy} \left(\frac{\cos\theta}{\sin\theta} \right) = x \frac{(-\sin^2\theta - \cos^2\theta)}{\sin^2\theta} \cdot \frac{d\theta}{dy} = 1$$

$$-\frac{x d\theta}{\sin^2\theta} = dy$$

↑ chain rule

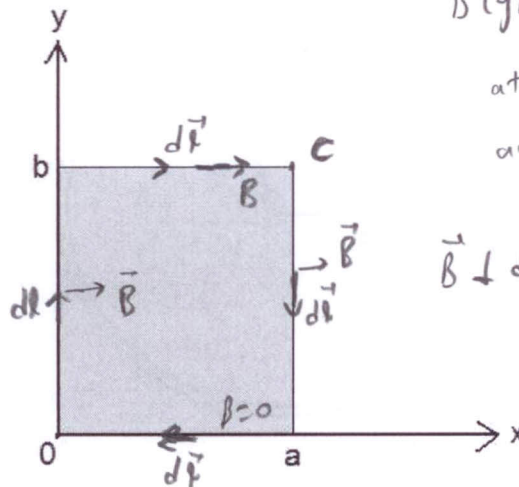
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Using Ampere's law, find the direction and magnitude of the net current passing from the rectangular region shown in the figure, if the resulting magnetic field is $\vec{B} = Ay \hat{i}$, where $A > 0$ is a constant. ($\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$)



$$\vec{B}(y) = Ay \hat{i}$$

$$\text{at } y=0 \quad \vec{B}(0) = A \cdot 0 \hat{i} = 0$$

$$\text{at } y=b \quad \vec{B}(b) = Ab \hat{i}$$

$\vec{B} \perp d\vec{l}$ means \vec{B} and $d\vec{l}$ are perpendicular so dot product give us zero.

$$\oint \vec{B} \cdot d\vec{l} = \int_0^b \underbrace{\vec{B} \cdot d\vec{l}}_{\vec{B} \perp d\vec{l}} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^a \underbrace{\vec{B} \cdot d\vec{l}}_{\vec{B} \perp d\vec{l}} + \int_a^0 \underbrace{\vec{B} \cdot d\vec{l}}_{\vec{B} \perp d\vec{l}} = 0 + \int_b^c \vec{B} \cdot d\vec{l} + 0 + 0$$

$$\oint \vec{B} \cdot d\vec{l} = \int_b^c \vec{B} \cdot d\vec{l} = \int_b^c (Ab \hat{i}) \cdot (dx \hat{i}) = Ab \int_b^c dx = Ab \underbrace{a}_{\text{length of } bc} = \mu_0 I_{enc}$$

$$I_{enc} = \frac{Ab a}{\mu_0} \quad (\text{direction is in to the page, } -\hat{k}, \text{ from right hand rule})$$

look at $d\vec{l}$

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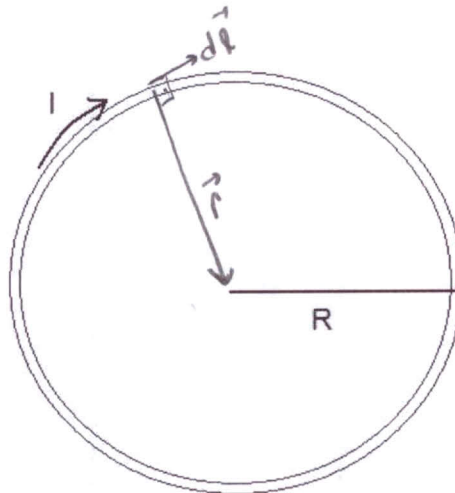
Name: ANSWER Student ID:

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A circular loop has radius R and carries a current I in a clockwise direction. Starting from the Biot-Savart law, find the magnetic field (both magnitude and direction) at its center.

$$(d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2})$$

from right hand rule
direction is in to the page.



$d\vec{l}$ and \vec{r} are perpendicular
to each other.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{r^2} = \frac{\mu_0 I}{4\pi} \frac{R d\theta}{R^2}$$

$$B = \int dB = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{d\theta}{R} = \frac{\mu_0 I}{4\pi R} 2\pi = \frac{\mu_0 I}{2R}$$

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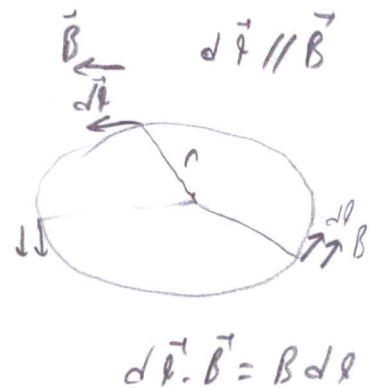
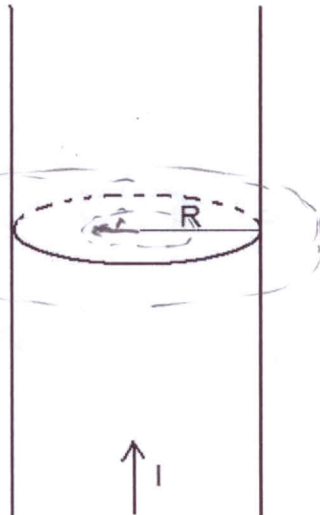
A cylindrical wire with radius R carries a current I . The current is uniformly distributed over the cross-sectional area of the conductor. Using Ampere's law, find the magnetic field (both magnitude and direction) everywhere (for $r < R$ and $r > R$),

$$(\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}).$$

draw a circle which has the same (axis) center as the cylinder has.

by using symmetry argument

i.e. any two point on this circle cannot be distinguished from other.



$$\oint \vec{B} \cdot d\vec{l} = \int B dl = B \int dl = B 2\pi r = \mu_0 I_{enc}$$

$$\text{for } r \leq R \quad I_{enc} = \pi r^2 \cdot \frac{I}{\pi R^2} = I \frac{r^2}{R^2} \quad B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I r^2}{2\pi r R^2} = \frac{\mu_0 I r}{2\pi R^2}$$

$$\text{for } r \geq R \quad I_{enc} = I \quad B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I}{2\pi r}$$

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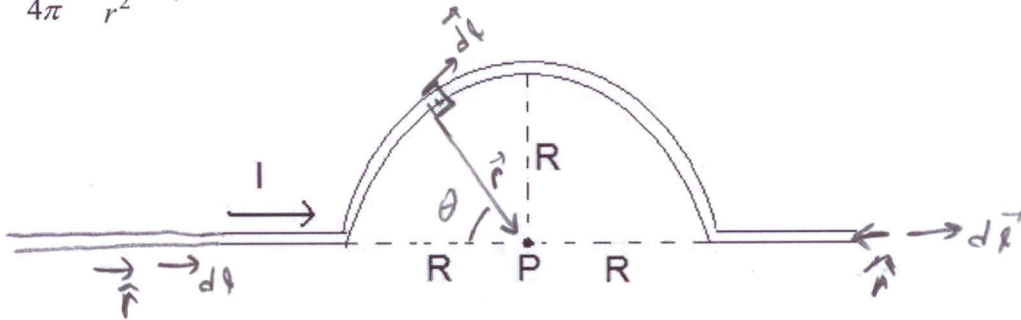
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The wire shown in the figure is infinitely long and carries a current I . Starting from the Biot-Savart law, find the magnetic field (both magnitude and direction) at point P .

$$(d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2})$$



for the straight part $d\vec{l} \times \hat{r} = \vec{0}$ since $\sin(0) = \sin(\pi) = 0$

for curvature

$$dB = \frac{\mu_0}{4\pi} \frac{I dl |\hat{l}| \cdot \sin(\pi/2)}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dl}{r^2} \quad \begin{array}{l} dl = R \cdot d\theta \\ r = R \end{array}$$

$$B = \int dB = \frac{\mu_0 I}{4\pi} \int \frac{R d\theta}{R^2} = \frac{\mu_0 I}{4\pi R} \int_0^\pi d\theta = \frac{\mu_0 I}{4R} \otimes$$

direction is in-to-the page, from right hand rule