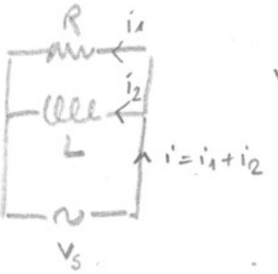
PHYS 102: General Physics KOÇ UNIVERSITY Spring Semester 2010
College of Arts and Sciences
Quiz 11 May 27, 2010

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Q. An AC-voltage source, a resistor R and an inductor L are all connected in parallel, so that $v_s(t) = v_R(t) = v_L(t) = V\cos(\omega t)$. Draw a single phasor diagram for all the voltages and the currents when $\omega t = \pi/4$. Assume $R > \omega L$.



$V_L(t) = L\frac{di_2}{dt} = V\cos\omega t$

$\frac{di_2}{dt} = \frac{V}{L}\cos\omega t \Rightarrow i_2 = \frac{V}{\omega L}\sin\omega t = \frac{V}{\omega L}\cos(\omega t - \pi/2)$ (1)

$V_R(t) = R\, i_1(t) = V\cos\omega t$

$i_1(t) = \frac{V}{R}\cos\omega t$ (2)

(1) + (2) $\Rightarrow$ $i = i_1 + i_2 = V\left(\frac{1}{R}\cos\omega t + \frac{1}{\omega L}\sin\omega t\right)$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\alpha = \omega t$
$\cos\beta = 1/R$
$\sin\beta = -1/\omega L$
$\Big\}\ \tan\beta = \frac{-\frac{1}{\omega L}}{\frac{1}{R}} = -\frac{R}{\omega L}$

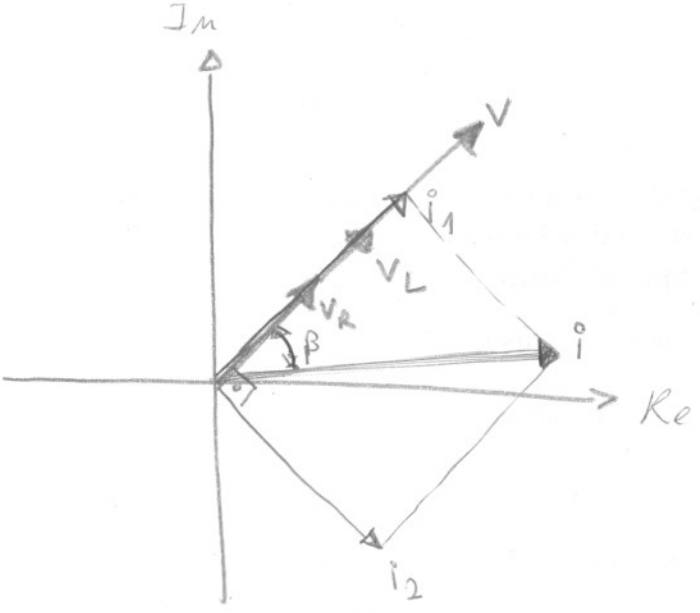
$\beta = \tan^{-1}(-R/\omega L)$

So we can write,

$\frac{R}{\omega L} > 1 \Rightarrow \frac{-R}{\omega L} < -1$

$i(t) = V\cos(\omega t + \beta)$ where $\beta = \tan^{-1}\left(\frac{-R}{\omega L}\right)$ $\Rightarrow \beta < 0$

$\longrightarrow$



$$\beta = \arctan\left(\frac{-R}{\omega L}\right)$$

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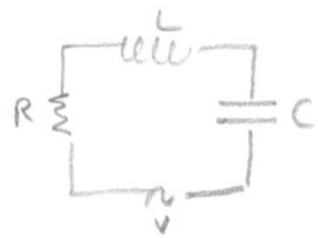
Q. An AC-current source, a resistor R, an inductor L and a capacitor C are all connected in series. The current supplied by the source is $i(t) = I\cos(\omega_0 t)$, oscillating at the resonant frequency $\omega_0 = 1/\sqrt{LC}$. Find the maximum power stored in the inductor.



Since the circuit is in resonance,

$X_L = X_C \Rightarrow \phi = 0 \quad \left(\tan\phi = \frac{X_L - X_C}{R}\right)$

Thus $v(t) = V\cos(\omega_0 t)$

$V_L(t) = L\frac{di}{dt} = -IL\omega_0 \sin\omega_0 t$

$P(t) = i(t)\,V_L(t) = (I\cos\omega_0 t)(-IL\omega_0\sin\omega_0 t)$

$= -I^2 L \frac{\sin(2\omega_0 t)}{2}$

the power is maximized when, $\quad -1 \leq \sin\theta \leq 1$

$\left.\frac{dP}{dt}\right|_{t=t_1} = 0 \qquad |P_{max}| = |-I^2 L| = I^2 L$

$\Rightarrow -\frac{I^2 L}{2} 2\omega_0 \cos(2\omega_0 t_1) = 0$

$2\omega_0 t_1 = \frac{\pi}{2} \rightarrow t_1 = \frac{1}{\omega_0}\frac{\pi}{4}$

and its maximum value is,

$|P(t=t_1)| = \left|-\frac{I^2 L}{2}\underbrace{\sin\left(2\omega_0 \cdot \frac{1}{\omega_0}\frac{\pi}{4}\right)}_{1}\right| = \frac{I^2 L}{2}$

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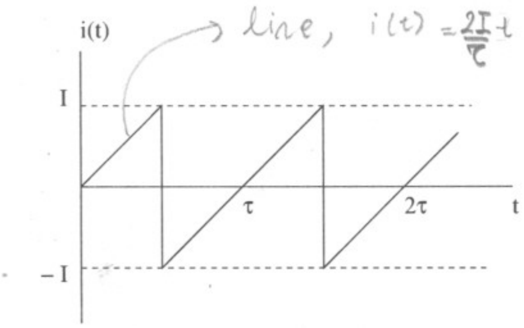
Q. The current in a circuit is given in the figure. Find the average current and the rms current.

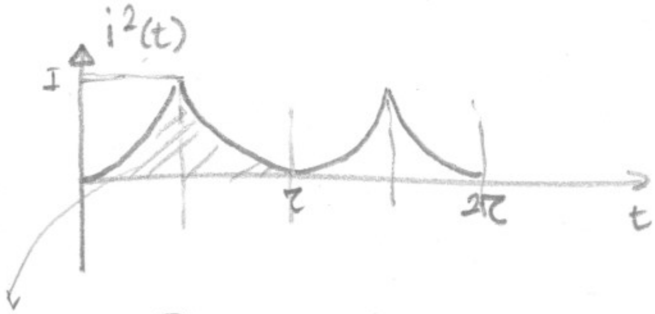


$I_{av} = \frac{1}{\tau}\int_0^{\tau} i(t)\,dt = 0$

$i_{rms} = \sqrt{\frac{1}{\tau}\int_0^{\tau} i^2(t)\,dt}$

$= \sqrt{\frac{1}{\tau}\cdot\frac{\tau I}{3}}$

$= \sqrt{\frac{I}{3}}$



$I = 2 \times \int_0^{\tau/2} \left[\frac{2I}{\tau} t\right]^2 dt = 2 \cdot \frac{4I}{\tau^2} \frac{\tau^3/8}{3} = \frac{\tau I}{3}$

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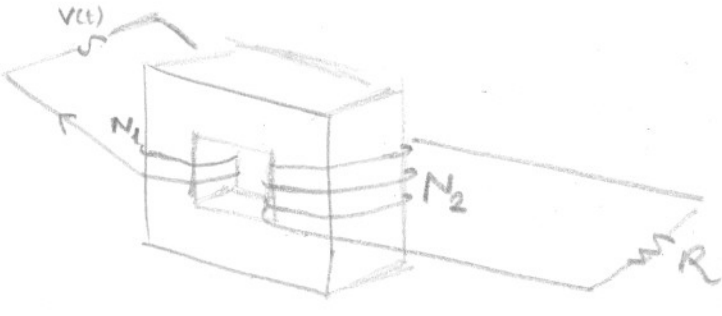
Q. You wish to step-down the 220 V rms house voltage for using your water heater (which is effectively a resistor), by placing a transformer between the heater and the wall socket. If the maximum voltage your heater can take is 30 V, what would you use for the winding number ratio N_2/N_1 in your transformer? Recall that the heater is connected to the secondary winding with N_2 turns. ($\sqrt{2} \simeq 1.4$.)

$V_{rms} = 220\,V = \frac{V}{\sqrt{2}}$ (for a sinusoidal source)

$V_R = V_R \cdot \cos(\omega t + \varnothing)$

$\Rightarrow V = \sqrt{2} \times 220\,V = 308\,V$



$\frac{V_2}{V_1} = \frac{N_2}{N_1}$

to step-down the voltage we need to choose $N_2 < N_1$ ($V_2 < V_1$).

$\frac{30\,V}{308\,V} = \frac{N_2}{N_1} \simeq 0.1$

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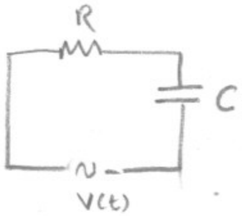
Q. You have a 200-Ω resistor and a 6.00-μF capacitor. You construct a series circuit with an AC-voltage source that has voltage amplitude $V = 30\,\text{V}$ and and angular frequency of 250 rad/s. Find the phase angle of the source voltage with respect to the current. Does the source voltage lag or lead the current?

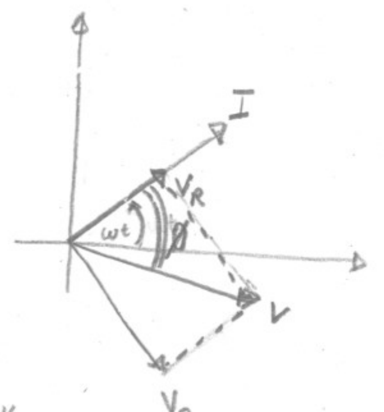


$V(t) = V\cos(\omega t + \phi)$, $V = 30\text{V}$, $\omega = 250\,\text{rad/s}$

$i(t) = I\cos\omega t$

$V_c = \frac{q(t)}{C}$ and $i = \frac{dq}{dt} \Rightarrow q = \int^t i(t')dt' = \frac{I}{\omega}\sin\omega t \Rightarrow V_c = \frac{I}{\omega C}\sin\omega t = \frac{I}{\omega C}\cos(\omega c - \pi/2)$

$V_R = i(t).R = IR\cos\omega t$



* V definitely lags the current.

$\tan\phi = \frac{V_c}{V_R} = \frac{\frac{I}{\omega C}}{IR} = \frac{1}{\omega RC}$

$= [250\,\text{rad/s} \times 200\,\Omega \times 6\times10^{-6}\,\text{F}]^{-1}$

$= [300000 \times 10^{-6}]^{-1}$

$= \frac{10}{3}$

$\phi = \arctan(10/3) \cong 1.28\ \text{rad}$