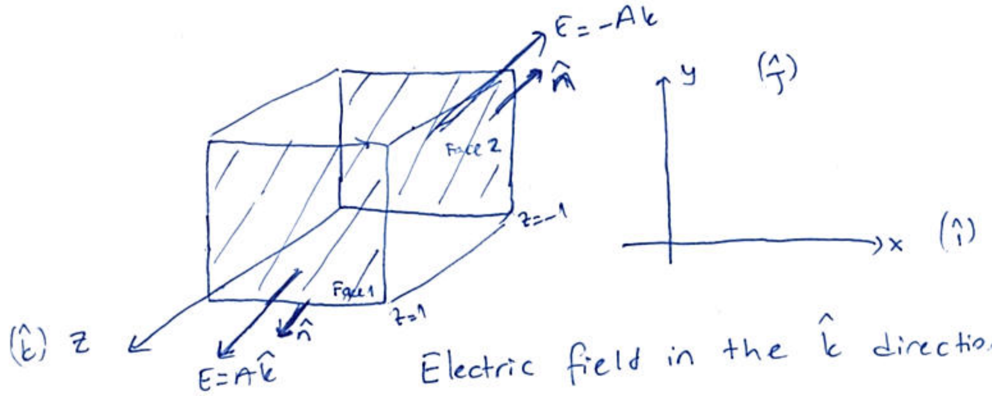
PHYS 102: General Physics **KOÇ UNIVERSITY** **Spring Semester 2011**

College of Arts and Sciences

Quiz 2 Feb 24, 2011

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes
Name: Solution **Student ID:** **Signature:**

Q. The electric field on the faces of the cube with corners $(\pm1, \pm1, \pm1)$ is found to be $\vec{E}(x, y, z) = Az\hat{k}$, where A is a constant with proper units. Calculate the total charge inside the cube by using Gauss's law.



4 = Area (Face 1) = Area (Face 2)

Electric field in the $\hat{k}$ direction, therefore we will be only concerned w/ the cube faces 1 and 2 in flux calculation. (which lies on the x-y plane)

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int \vec{E}\cdot d\vec{A}}_{\text{front face}} + \underbrace{\int \vec{E}\cdot d\vec{A}}_{\text{back face}}$$

$$= (A\hat{k}).4.(\hat{k}) + (-A\hat{k}).4(-\hat{k})$$

$$= 8A$$

$$\oint E\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\boxed{Q = 8A\epsilon_0}$$

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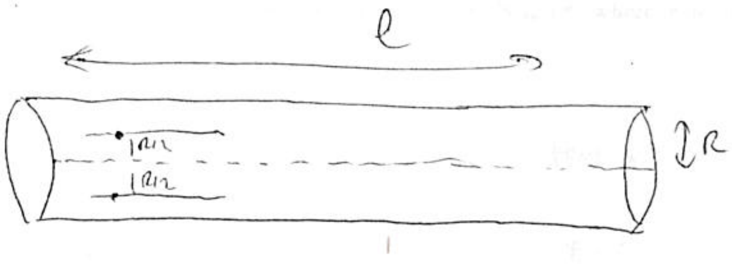
Q. Consider an infinite cylinder with radius R and a uniform charge density ρ (C/m^3). Find the electric field magnitude at a distance $R/2$ from the cylinder's central axis.



$\ell >> R$

$$E \cdot A = \frac{Q}{\epsilon_0}, \quad Q = \left[\pi\left(\frac{R}{2}\right)^2 \cdot \ell\right] \cdot \rho = \frac{\pi R^2 \ell \rho}{4}$$

$$E \cdot 2\pi\left(\frac{R}{2}\right)\ell = \frac{\pi R^2 \ell \rho}{4\epsilon_0}$$

$$E = \frac{\pi R^2 \ell \rho}{4\epsilon_0} \cdot \frac{1}{\pi R \ell} = \boxed{\frac{R\rho}{4\epsilon_0}}$$

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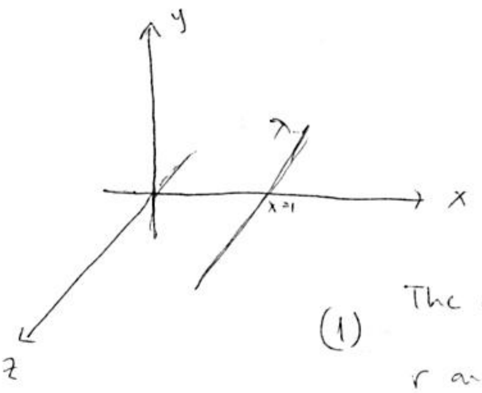
Q. Find the electric field $\vec{E}(x, y, z)$ due to a straight wire with charge density λ if the line is parallel to the z-axis and passes through the point $(1, 0, 0)$. Answer in terms of x, y, z, λ and constants.

Hint: For a line charge, Gauss's law gives $\vec{E}(\vec{r}) = (\lambda/2\pi\epsilon_0 r)\,\hat{r}$, where r is the distance from the line.



Hint provides useful info here:

$$E(\vec{r}) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

(1) The only thing we need to do is to write r and $\hat{r}$ in terms of x, y, z, λ.

(2) Additionally, we need to be careful using this transformation of the coordinate system, since our line charge passes through $x=1$ (not from $x=0$)!

(3) It is parallel to the $\hat{k}$ (z-coordinate), therefore $\vec{E}$ will have no $\hat{k}$ component!

$$\Rightarrow \quad \hat{r} = \frac{x\hat{x} + y\hat{y}}{\sqrt{x^2+y^2}}, \quad r = \sqrt{x^2+y^2}$$

keeping in mind $x=1$ coordinate,

$$\vec{E}(x, y, z, \lambda) = \lambda \frac{(x-1)\hat{x} + y\hat{y}}{\sqrt{(x-1)^2+y^2}} \cdot \frac{1}{2\pi\epsilon_0\sqrt{(x-1)^2+y^2}}$$

$$= \frac{\lambda\,\left((x-1)\hat{x} + y\hat{y}\right)}{2\pi\epsilon_0\left((x-1)^2+y^2\right)}$$

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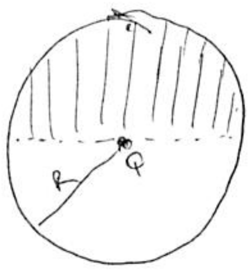
Q. A charge Q is located at the center of a sphere with radius R. Calculate the electric flux through the surface of the northern hemisphere ($x^2 + y^2 + z^2 = R^2,\ x, y, z > 0$).



Electric field is same everywhere on the surface of the sphere (and perpendicular to the surface (or parallel to surface normal)) with value of

$$E(R) = \frac{Q}{4\pi\epsilon_0 R^2}$$

Flux $= \Phi_E = E_\perp A = E A$, where A is the area of the hemisphere

$$\Phi_E = \frac{Q}{4\pi\epsilon_0 R^2} \cdot \frac{4\pi R^2}{2} = \frac{Q}{2\epsilon_0}$$

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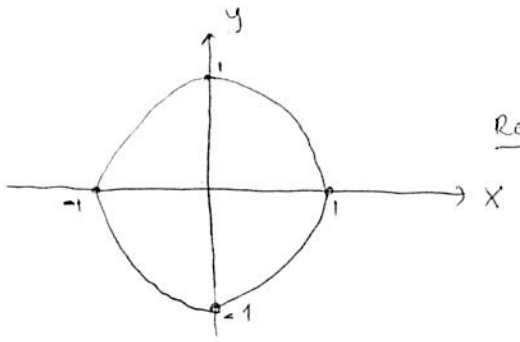
Solution

Q. Calculate the magnitude of the electric flux through the unit disc on the xy-plane ($z = 0,\ x^2 + y^2 \leq 1$) due to the electric field $\vec{E}(x, y, z) = Ay^2\hat{i} + Bx^2\hat{j} + C(x^2 + y^2)\hat{k}$. ($A, B, C$ are constant with proper units.)



$\vec{E}$ field through disk

Remember: only $\hat{k}$ component will add nonzero flux.

Flux $= \Phi_E = \oint \vec{E} \cdot d\vec{A}$, where $dA = r\,dr\,d\theta$

and $\vec{E}_{effective} = C(x^2+y^2)\hat{k}$

Also, $x^2+y^2 = r^2$

$$\Phi = \int_{r=0}^{1} \int_{\theta=0}^{2\pi} C r^2 \cdot r\,dr\,d\theta = 2\pi C \frac{r^4}{4}\Big|_0^1 = \frac{\pi C}{2}$$