

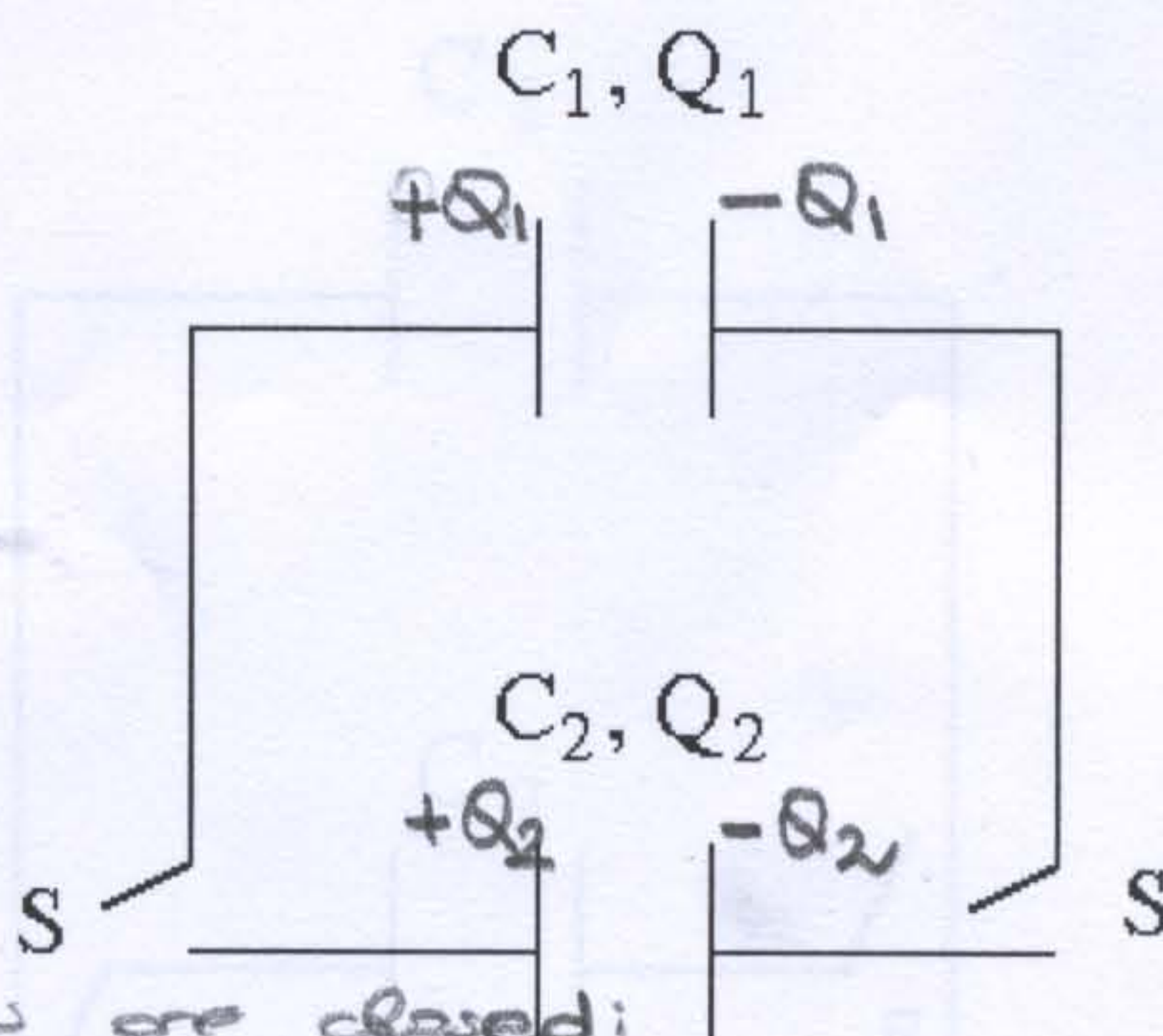
Closed book. No calculators are to be used for this quiz.  
 Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Initially the switches (labeled by S) are open, and both of the capacitors are charged as shown in the figure. After switches are closed, calculate the final energy stored in the second capacitor ( $C_2$ ) in terms of the capacitances and initial charges.



When the switches are closed:

\* Charge is conserved,  $Q = Q_1 + Q_2$

\* Potential difference for both capacitors is the same  
 $\Rightarrow$  Hence they are parallel connected!

$$C_{eq} = C_1 + C_2$$

\* Each share the total charge  $Q$  wrt ratio of their capacitance's

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1}{C_2}$$

$$\& \quad Q_{1f} + Q_{2f} = Q$$

$$\frac{C_1}{C_2} \cdot Q_{2f} + Q_{2f} = Q_1 + Q_2$$

$$Q_{2f} \left( \frac{C_1 + C_2}{C_2} \right) = Q_1 + Q_2$$

$$\Rightarrow \boxed{Q_{2f} = (Q_1 + Q_2) \cdot \frac{C_2}{C_1 + C_2}}$$

$\hookrightarrow$  \* Energy stored,

$$U = \frac{Q^2}{2C}$$

$$U_{2f} = \frac{Q_{2f}^2}{2C_2} = \frac{(Q_1 + Q_2)^2 C_2}{2(C_1 + C_2)}$$

Closed book. No calculators are to be used for this quiz.

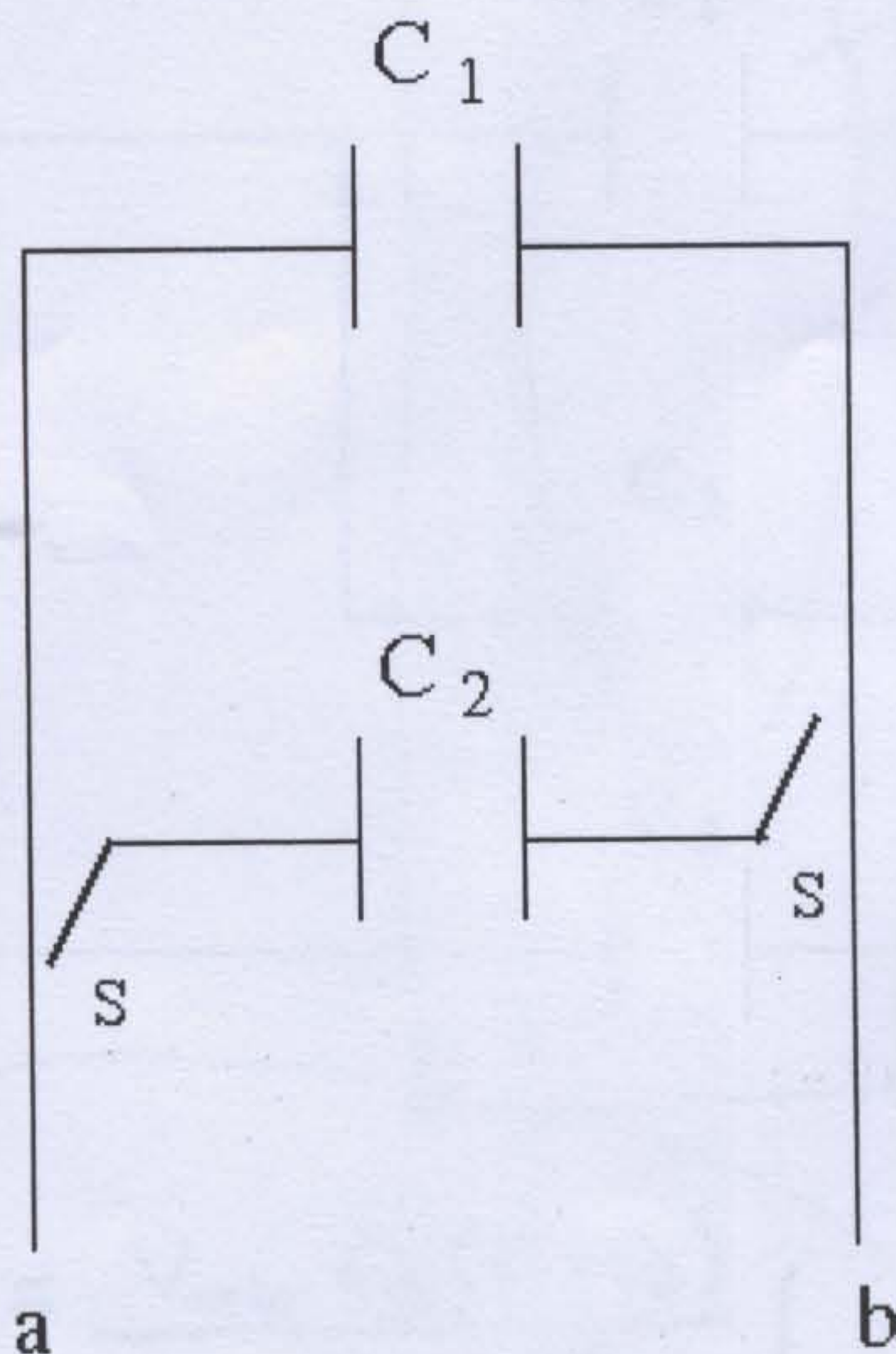
Quiz duration: 10 minutes

Name:

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Initially the switches (labeled by S) are open, and the electric potential difference between the "a" and "b" terminals is measured to be "V". After switches are closed an uncharged capacitor ( $C_2$ ) is connected to the circuit as shown in the figure. Calculate the energy stored in the second capacitor ( $C_2$ ) in terms of the capacitances and V.



\* Initially,  $C_1$  is charged and

$$Q_1 = C_1 V$$

\* When the switches are closed the total charge on the system is conserved.

$$Q = Q_1 = C_1 V$$

\* The potential difference for both capacitors is the same  
 $\Rightarrow$  They are parallel connected

$$C_{eq} = C_1 + C_2$$

\* Each share the total charge  $Q$  wrt ratio of their capacitances

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1}{C_2} \text{ \& } Q_{1f} + Q_{2f} = Q_1 = C_1 V \Rightarrow Q_{2f} = \frac{C_1 C_2 V}{C_1 + C_2}$$

\* Energy stored:  $U = Q^2 / 2C \Rightarrow U_{2f} = Q_{2f}^2 / 2C_2 = \frac{C_1^2 C_2 V^2}{2(C_1 + C_2)^2}$

Closed book. No calculators are to be used for this quiz.

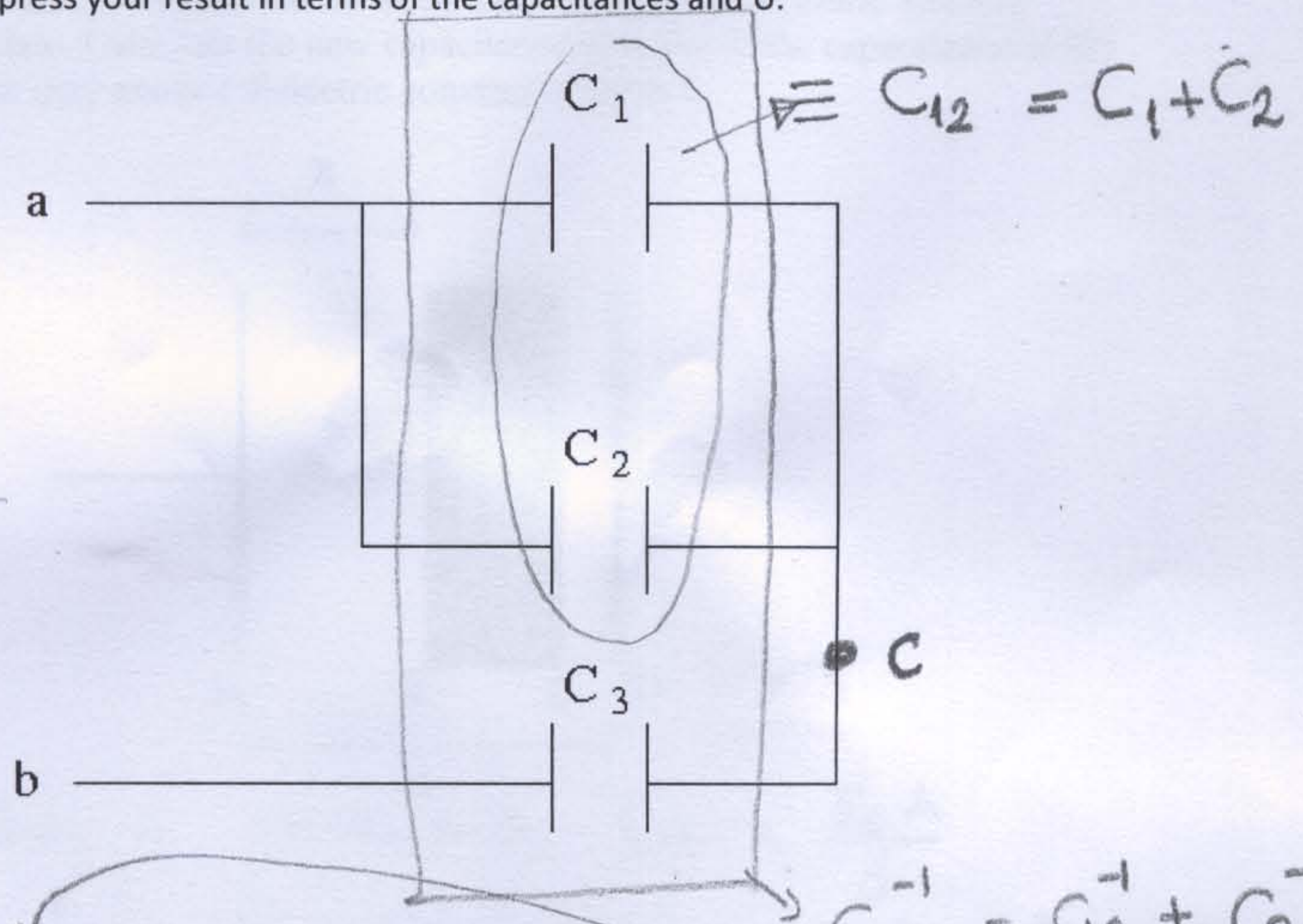
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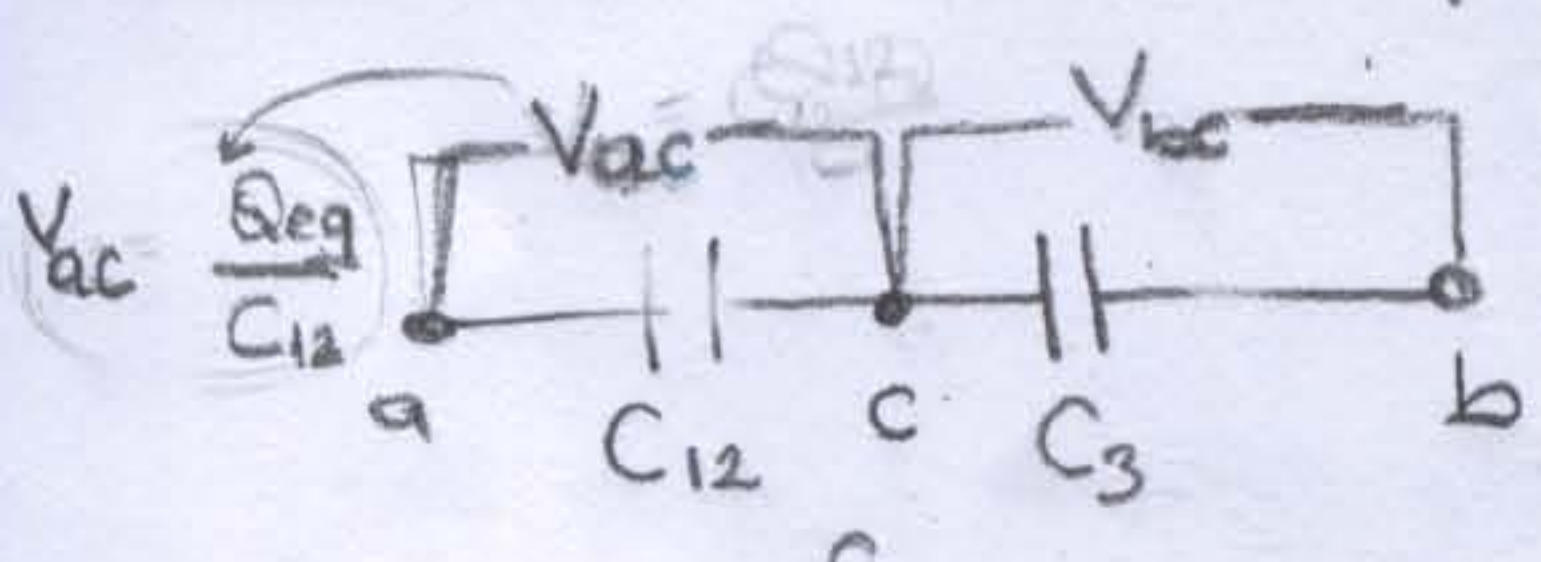
The terminals "a" and "b" are connected to a battery. After a long time if the energy stored in the first capacitor ( $C_1$ ) is found to be "U", how much energy is stored on the third capacitor ( $C_3$ )? Express your result in terms of the capacitances and U.



$$C = \frac{Q}{V}$$

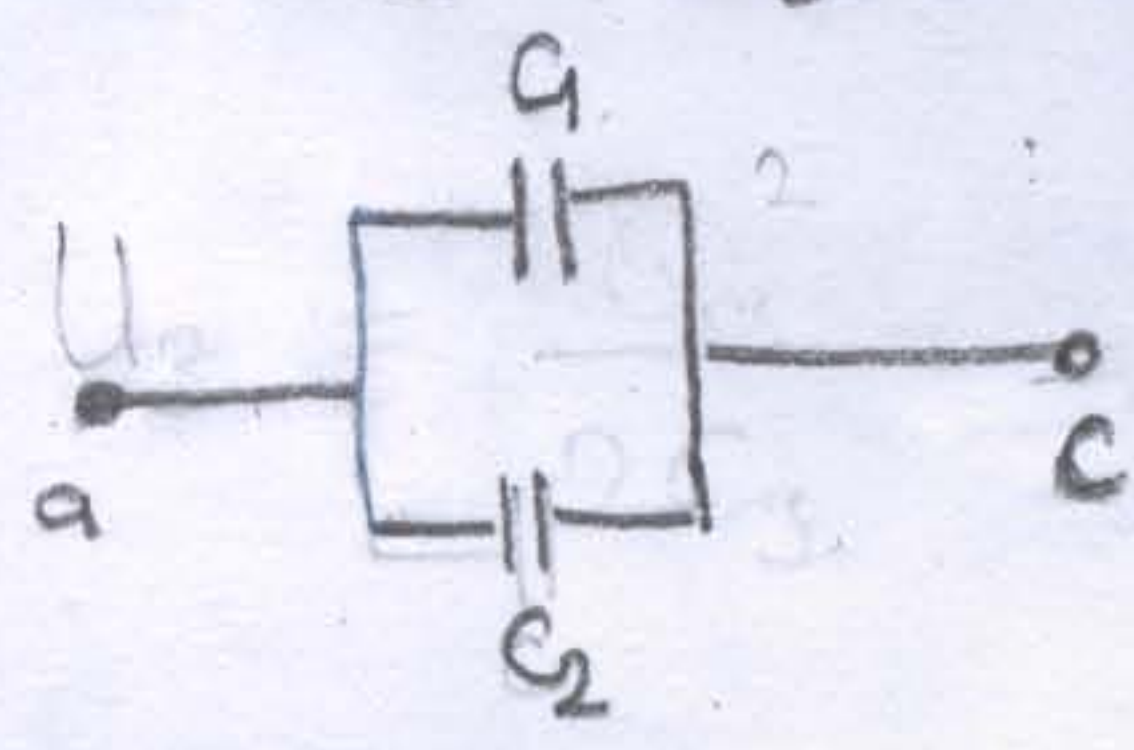
$$C_{eq}^{-1} = C_{12}^{-1} + C_3^{-1} = \frac{1}{C_1 + C_2} + \frac{1}{C_3}$$

$$* Q_{eq} = V_{ab} \cdot C_{eq} = \frac{V_{ab} C_3 (C_1 + C_2)}{C_1 + C_2 + C_3}$$



connected in series!  
each has the same charge;  $Q_{eq}$

$$C_{eq} = \frac{C_3 (C_1 + C_2)}{C_1 + C_2 + C_3}$$



connected in parallel  $\rightarrow$  kept in same potential ( $V_{ac}$ )

$$Q_1 = C_1 V_{ac} = C_1 \frac{Q_{eq}}{C_1 + C_2} = \frac{V_{ab} C_1 C_2 (C_1 + C_2)}{C_1 + C_2 + C_3}$$

$$U_1 = \frac{Q_1^2}{2C_1} = \frac{V_{ab}^2 C_2^2 C_1^2 (C_1 + C_2)^2}{2(C_1 + C_2 + C_3)^2} = U$$

\* for the 3rd capacitor:

$$U_3 = \frac{Q_3^2}{2C_3} = \frac{Q_{eq}^2}{2C_3} = \frac{V_{ab}^2 C_3^2 (C_1 + C_2)^2}{2(C_1 + C_2 + C_3)^2} = \frac{U}{C_1 C_2^2} C_3 (C_1 + C_2)^2$$

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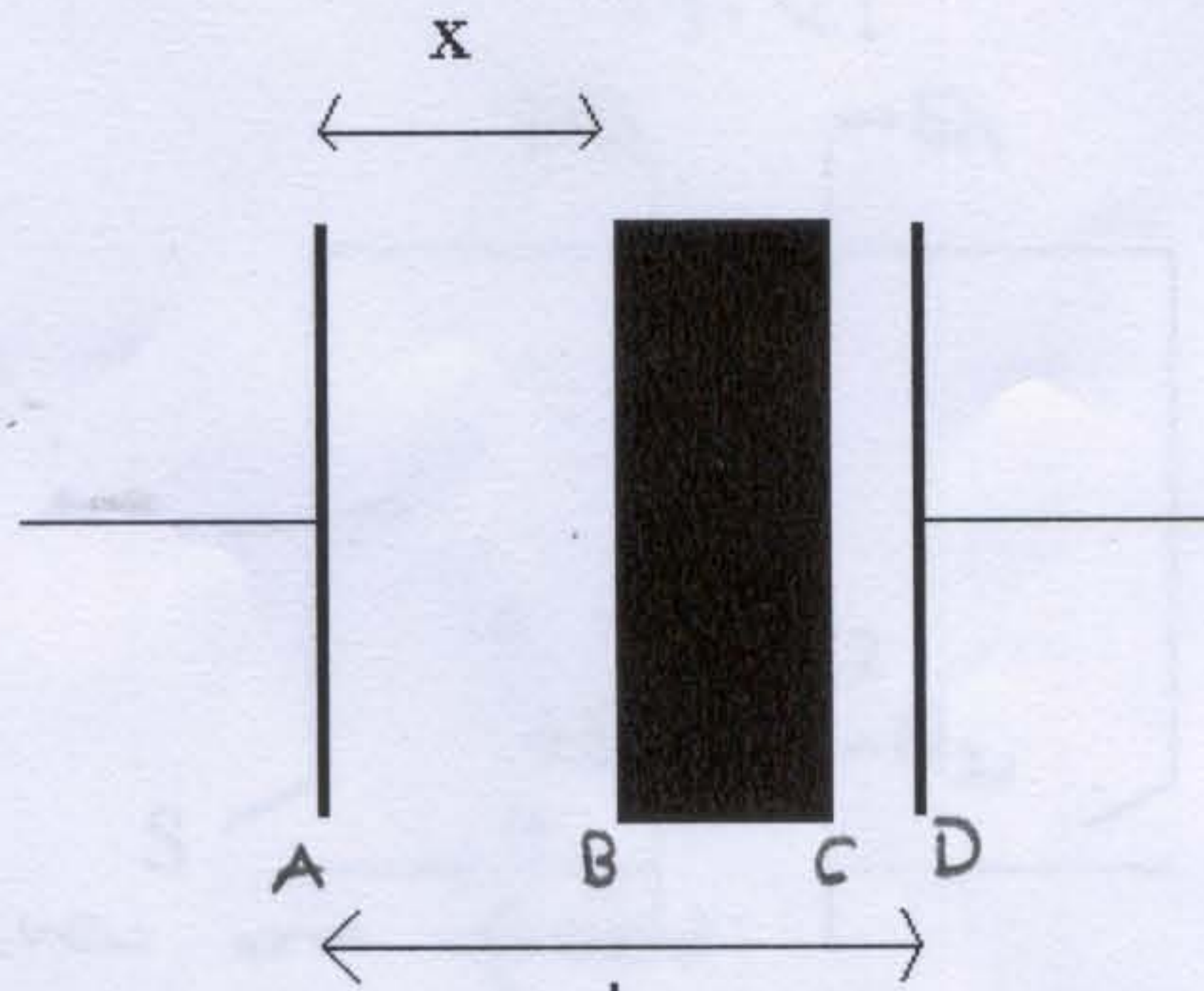
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Name:

Student ID: *with dielectric constant,  $K=1$  is*

Signature:

A parallel plate capacitor is made by using two flat conducting plates, each with area "A", separated by a distance "d". Then a metal slab having thickness "a" and the same shape and size as the plates inserted between them, parallel to the plates and not touching either plate. Calculate the new capacitance in terms of the capacitance of the air capacitor. You may assume dielectric constant of air is 1.



Before the slab is introduced,  $C_0 = \frac{\epsilon_0 A}{d}$

When the slab is introduced we need to calculate the new capacitance,

$$C = \frac{Q}{V_{AD}}$$

$$V_{AD} = V_{AB} + V_{BC} + V_{CD} = \int_A^B \vec{E} \cdot d\vec{l} + \int_B^C \vec{E} \cdot d\vec{l} + \int_C^D \vec{E} \cdot d\vec{l}$$

$$= \frac{\sigma}{\epsilon_0} \cdot d_{AB} + \frac{\sigma}{\epsilon_0} d_{CD} = \frac{\sigma}{\epsilon_0} (d_{AB} + d_{CD})$$

*since the field inside the conductor is zero*

$= d - a$

$$\Rightarrow C = \frac{Q}{\frac{\sigma}{\epsilon_0} (d-a)} \quad \& \quad \sigma = Q/A \quad \Rightarrow C = \frac{\epsilon_0 A}{d-a} = \left(\frac{d}{d-a}\right) \cdot C_0$$

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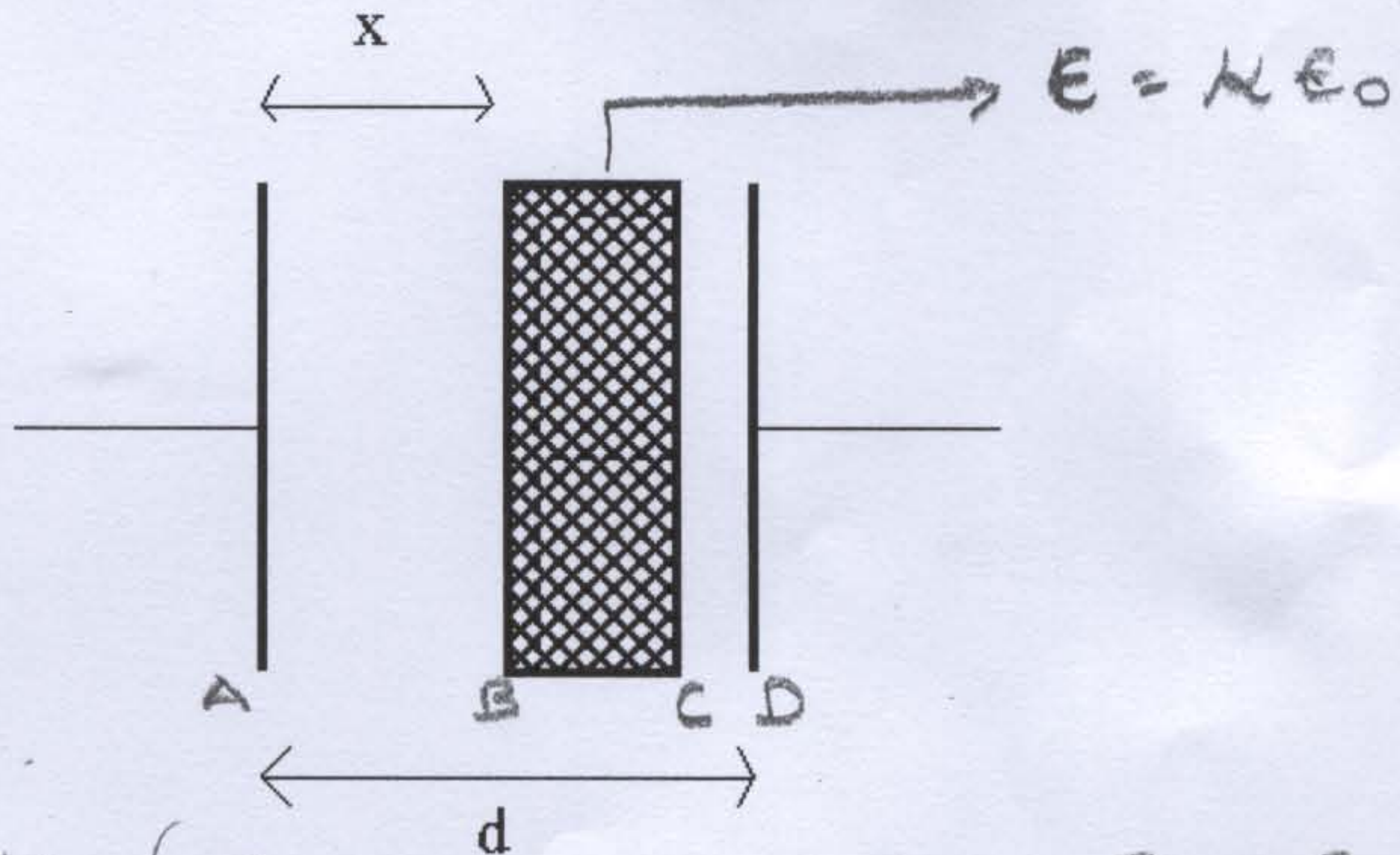
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Name:

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Signature:

A parallel air capacitor is made by using two flat conducting plates, each with area "A", separated by a distance "d". Then a dielectric slab having thickness "a" and the same shape and size as the plates inserted between them, parallel to the plates and not touching either plate. Calculate the new capacitance in terms of the capacitance of the air capacitor. You may assume dielectric constant of air is 1.



Before the dielectric  $C_{lab}$  is introduced,  $C_0 = \frac{\epsilon_0 A}{d}$

After the  $C_{lab}$  is introduced,

new capacitance:  $C = Q/V_{AD}$

$$V_{AD} = V_{AB} + V_{BC} + V_{CD} = \int_A^B \vec{E} \cdot d\vec{l} + \int_B^C \vec{E} \cdot d\vec{l} + \int_C^D \vec{E} \cdot d\vec{l}$$

$$= \frac{Q}{\epsilon_0} d_{AB} + \frac{Q}{\kappa \epsilon_0} a + \frac{Q}{\epsilon_0} d_{CD}$$

$$= \frac{Q}{\epsilon_0} \left( \underbrace{d_{AB} + d_{CD}}_{=d-a} + \frac{a}{\kappa} \right)$$

$$= \frac{Q}{\epsilon_0} \left( d + \left( \frac{1}{\kappa} - 1 \right) a \right) \quad \& \quad \sigma = Q/A$$

$$V_{AD} = \frac{Q}{\epsilon_0 A} \left( d + \left( \frac{1}{\kappa} - 1 \right) a \right)$$

$$C = \frac{Q}{\frac{Q}{\epsilon_0 A} \left[ d + \left( \frac{1}{\kappa} - 1 \right) a \right]} = \frac{\epsilon_0 A}{\left[ d - \left( \frac{\kappa - 1}{\kappa} \right) a \right]} = \frac{C_0 d}{d - \left( \frac{\kappa - 1}{\kappa} \right) a} = \frac{C_0}{1 - \left( \frac{\kappa - 1}{\kappa} \right) \frac{a}{d}}$$