

Closed book. No calculators are to be used for this quiz.

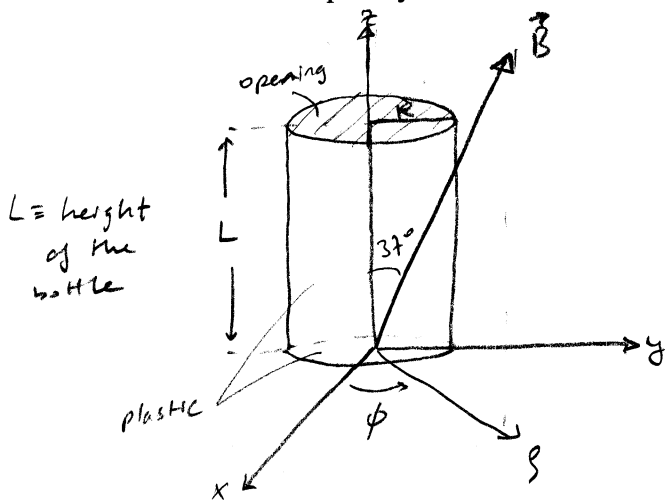
Quiz duration: 10 minutes

Name:

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Signature:

An open plastic bottle with a disk shaped opening radius of R is placed on a table. A uniform magnetic field with magnitude B , directed upward and oriented 37° from vertical encompasses the bottle. (a) What is the total magnetic flux through the opening of the bottle? (b) What is the total magnetic flux through the plastic of the bottle? Explain your answer.



$$\Phi_B = \int \vec{B} \cdot d\vec{A}, \quad \vec{B} = |\vec{B}| (\cos 37^\circ \hat{k} + \sin 37^\circ \hat{j})$$

$$= B (\cos 37^\circ \hat{k} + \sin 37^\circ \hat{j})$$

a) Φ_B through the opening

$$\Phi_B = \int_{\text{opening}} \vec{B} \cdot d\vec{A}, \quad d\vec{A} = |d\vec{A}| \hat{k}$$

$$= \rho d\rho d\phi \hat{k}$$

$$\Phi_B = \int_0^{2\pi} \int_0^R B (\cos 37^\circ \hat{k} + \sin 37^\circ \hat{j}) \rho d\rho d\phi \hat{k}$$

$$\Phi_B = \int_0^{2\pi} \int_0^R B \cos 37^\circ \rho d\rho d\phi \underbrace{\hat{k} \cdot \hat{k}}_1 + B \sin 37^\circ \rho d\rho d\phi \underbrace{\hat{j} \cdot \hat{k}}_0$$

$$= B \cos 37^\circ \int_0^{2\pi} \int_0^R \rho d\rho d\phi = B \cos 37^\circ \cdot \frac{R^2}{2} \cdot 2\pi = \boxed{\pi R^2 B \cos 37^\circ}$$

b) Φ_B through the plastic:

The bottle's geometry as a whole (i.e. opening + plastic) defines a closed surface. From Gauss' Law for magnetism, we know that Φ_B through any closed surface is zero. We can benefit this rule to find the flux through plastic part of the bottle:

$$\Phi_B^{\text{total}} = \Phi_B^{\text{opening}} + \Phi_B^{\text{plastic}} = 0 \Rightarrow \Phi_B^{\text{plastic}} = -\Phi_B^{\text{opening}}$$

then,

$$\boxed{\Phi_B^{\text{plastic}} = -\pi R^2 B \cos 37^\circ}$$

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A group of particles is traveling in a constant magnetic field of unknown magnitude and direction. You observe that a proton moving at 2 km/s in the $+x$ -direction experiences a force of $3.2 \times 10^{-16} \text{ N}$ in the $+y$ -direction, and an electron moving at 3 km/s in the $-z$ -direction experiences a force of $1.6 \times 10^{-16} \text{ N}$. What are the magnitude and direction of the magnetic field? (Take the charge of a proton as $e = 1.6 \times 10^{-19} \text{ C}$.)

- Constant magnetic field $\rightarrow \vec{B}$ is independent of position.

- Unknown magnitude & direction: $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

- Proton: $q = 1.6 \times 10^{-19} \text{ C}$

i) - If $v = 2 \times 10^3 \text{ m/s } \hat{i}$, $\vec{F} = 3.2 \times 10^{-16} \text{ N } \hat{j}$

ii) - If $v = -3 \times 10^3 \text{ m/s } \hat{k}$, $|\vec{F}| = 1.6 \times 10^{-16} \text{ N}$

- $\vec{B} = ?$

$\vec{F} = q \vec{v} \times \vec{B}$

i) $3.2 \times 10^{-16} \text{ N } \hat{j} = 1.6 \times 10^{-19} \text{ C} \cdot 2 \times 10^3 \text{ m/s } \hat{i} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \text{ T}$

$\hat{i} \times \hat{i} = 0$, \rightarrow Cannot determine B_x from above info. alone.

$\hat{i} \times \hat{j} = \hat{k}$, \rightarrow but no \hat{k} term on the lhs. \rightarrow $B_y = 0$

$\hat{i} \times \hat{k} = -\hat{j}$, $\rightarrow 3.2 \times 10^{-16} \text{ N} = -1.6 \times 10^{-19} \text{ C} \cdot 2 \times 10^3 \text{ m/s } B_z$

$B_z = -10^{-16+19-3} = -1 \text{ T}$

ii) $\vec{F} = 1.6 \times 10^{-19} \text{ C} (-3 \times 10^3 \text{ m/s}) \hat{k} \times (B_x \hat{i} + 0 \hat{j} - 1 \hat{k}) \text{ T}$

$= -4.8 \times 10^{-16} \text{ C m/s } (B_x \hat{j} - 0 \hat{i} - 1 \cdot 0) \text{ T}$

$= -4.8 \times 10^{-16} B_x \text{ N } \hat{j}$

$|\vec{F}| = 1.6 \times 10^{-16} \text{ N}$

$B_x = \frac{1}{3} \text{ T}$

$\vec{B} = \frac{1}{3} \text{ T } \hat{i} - 1 \text{ T } \hat{k}$

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A particle with charge q is moving with initial velocity $\vec{v} = v_y \hat{j}$. The magnetic force on the particle is measured to be $\vec{F} = F_x \hat{i} + F_z \hat{k}$. (a) In terms of the given quantities, calculate all the components of the magnetic field you can from this information. (b) Are there components of magnetic field that are not determined by the measurement of the force? Explain your answer.

- Particle w/ charge q .

- $\vec{v} = v_y \hat{j}$

- $\vec{F}_B = F_x \hat{i} + F_z \hat{k}$

a) $\vec{F}_B = q \vec{v} \times \vec{B}$. Suppose $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$. Substituting \vec{F}_B and \vec{v} , we get:

$$F_x \hat{i} + F_z \hat{k} = q v_y \hat{j} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$F_x \hat{i} + F_z \hat{k} = q v_y B_x (-\hat{k}) + q v_y B_y \cdot 0 + q v_y B_z \hat{i}$$

$$\boxed{B_x = -\frac{F_z}{q v_y}}, \quad \boxed{B_z = \frac{F_x}{q v_y}}, \quad \boxed{B_y \rightarrow \text{cannot be determined.}}$$

b) Component of the \vec{B} field parallel to \vec{v} cannot be determined from the force measurement alone.

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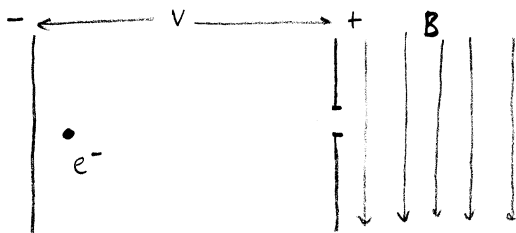
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An electron (charge $-e$, mass m) in the beam of a TV picture tube is accelerated by a potential difference V . Then it passes through a region of transverse magnetic field with magnitude B , where it moves in a circular arc. Derive an expression for the radius of the circular arc R , in terms of e , V , B , and m .



The e^- gains an energy of eV in the accelerator. This energy adds to the KE of the e^- . So at the entrance to the B field region we can write

$$\frac{1}{2} m v^2 = eV \rightarrow v = \sqrt{\frac{2eV}{m}}$$

Magnitude of the magnetic force, $F = qvB$

We set this equal to the centripetal force experienced by the particle:

$$qvB = \frac{m v^2}{R}, \text{ where } R \text{ is the radius of the circular arc of the resulting rotational motion.}$$

Organizing terms, and substituting $v = \sqrt{\frac{2eV}{m}}$, we get:

$$R = \frac{m}{qB} \sqrt{\frac{2eV}{m}}$$

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A particle with charge q and mass m moves under the influence of a magnetic field directed along the x-axis ($\vec{B} = B_0 \hat{i}$). At $t=0$, the velocity of the particle is $\vec{v} = v_x \hat{i} + v_y \hat{j}$. Derive an expression for the pitch (the distance traveled along the helix axis per revolution) of the helical path followed by the particle in terms of the given quantities.

- Particle of q, m under the influence of $\vec{B} = B_0 \hat{i}$

- $\vec{v} = v_x \hat{i} + v_y \hat{j}$ → initial velocity.

Parallel component of \vec{v} to \vec{B} (in this case the x-component) gives the translational motion, and perpendicular component (the y-component) gives the rotational motion.

We can find period of the rotational motion by equating the magnetic and centripetal forces along y-direction, and using $v_y \cdot T = 2\pi R$:

$$q v_y B_0 = m \frac{v_y^2}{R}, \quad v_y \cdot T = 2\pi R$$

$$q v_y B_0 = m \frac{v_y^2}{v_y T} \cdot 2\pi \rightarrow T = \frac{m 2\pi}{q B_0}$$

Pitch = Period \times Translational Speed

$$\text{Pitch} = \frac{m 2\pi}{q B_0} \cdot v_x$$