PHYS 102: General Physics 2 **KOÇ UNIVERSITY** **Spring Semester 2011**

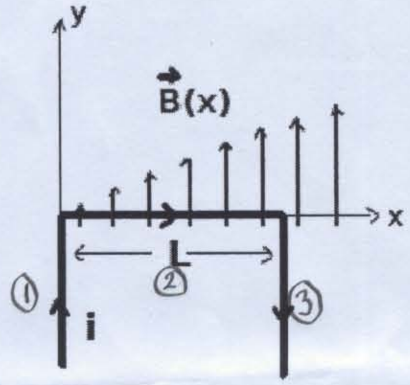
College of Sciences

Section 1 **Quiz 7** **14 April 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

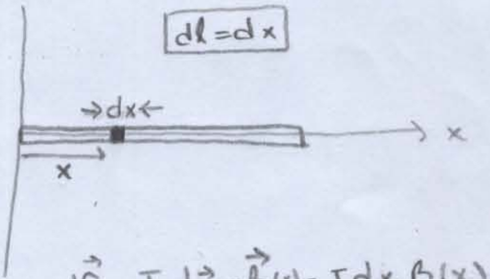
Calculate the magnitude and direction of the force acting on the wire in the figure in terms of L, i, and the variable magnetic field $\vec{B}(x) = B_0(x/L)\hat{j}$.



$d\vec{F} = I\,d\vec{l} \times \vec{B}$

There is no force on side ① and ③, because ($d\vec{l} \times \vec{B} = 0$) current and magnetic field lines are parallel.

$\boxed{dl = dx}$

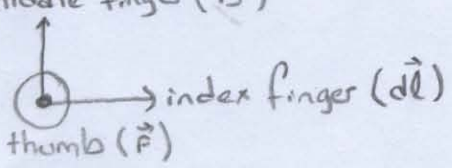


$d\vec{F} = I\,d\vec{x} \times \vec{B}(x) = I\,dx.\,B(x)\,\sin 90^\circ$

$\sin 90^\circ = 1$

$B(x) = B_0 . \frac{x}{L}$

The direction of $\vec{F}$ on side ② can be found by right-hand-rule.



$\vec{F} = \left(\frac{1}{2} I B_0 L\right)\hat{z}$

$dF = \frac{I B_0}{L} . x\,dx$

$F = \int_0^L dF = \int_0^L \left(\frac{I B_0}{L}\right) x\,dx$

$F = \frac{I B_0}{L}\left[\frac{x^2}{2}\right]_{x=0}^{x=L} = \boxed{\frac{1}{2} I B_0 L = F}$

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Section 2 **Quiz 7** **14 April 2011**

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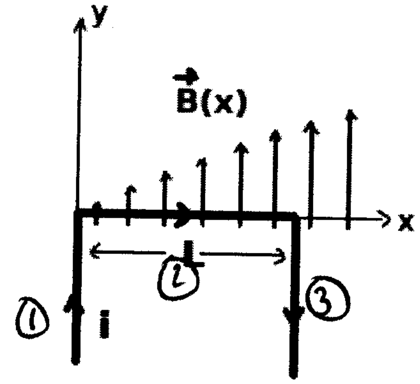
Quiz duration: 10 minutes

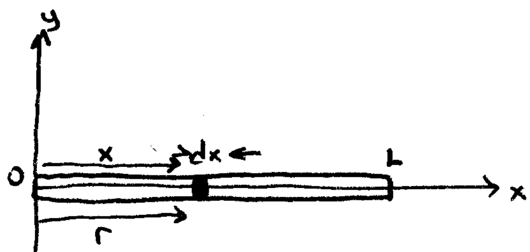
Name: **Student ID:** **Signature:**

Using $d\vec{\tau} = \vec{r} \times d\vec{F}$, calculate the magnitude and direction of the torque (with respect to the y-axis) acting on the wire in the figure in terms of L, i, and the variable magnetic field $\vec{B}(x) = B_0(x/L)\hat{j}$.



There is no force on side 1 and 3, because current and magnetic field lines are parallel.

On side 2 :



$$d\vec{F} = I(d\vec{x} \times \vec{B}(x)) = I\,dx\,B(x)\underbrace{\cancel{\sin 90}}_{1}$$

$$B(x) = \frac{B_0 . x}{L} \Rightarrow dF = \frac{IB_0}{L} . x\,dx$$

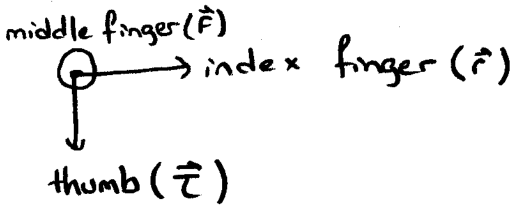
$$d\vec{\tau} = \vec{r} \times d\vec{F} \,, \; (\vec{r} = \vec{x})$$

$$= r\,dF \underbrace{\cancel{\sin 90}}_{1} = x\,dF$$

$$d\tau = \frac{IB_0}{L} x^2 dx \qquad \tau = \int_0^L d\tau = \frac{IB_0}{L}\int_0^L x^2 dx = \frac{IB_0}{L}\left(\frac{x^3}{3}\right|_{x=0}^{x=L}$$

$$\boxed{\tau = \frac{1}{3} IB_0L^2}$$

The direction of $\vec{F}$ on side 2 is $(\hat{z})$ by right-hand-rule.
The direction of $\vec{\tau}$ is $(-\hat{y})$



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Section 3 **Quiz 7** **14 April 2011**

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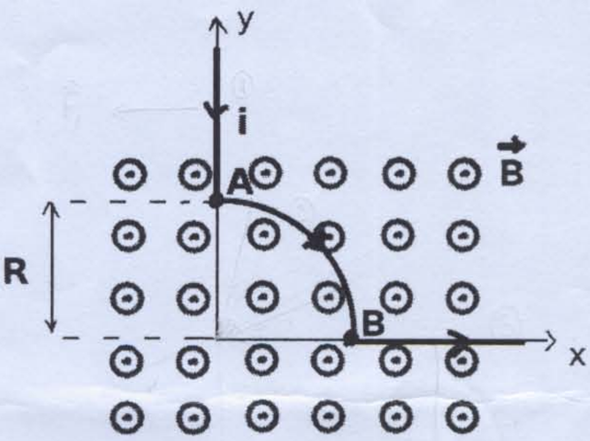
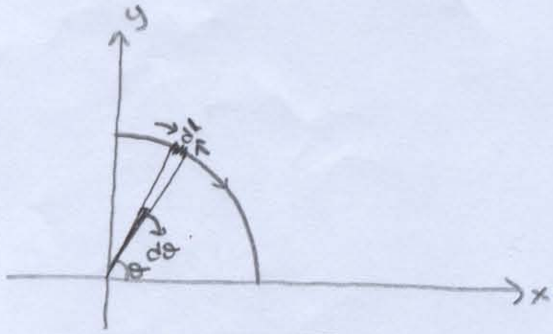
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Calculate the magnitude and the direction of the force acting on the circular portion A-B of the shown wire in terms of i, R, and the uniform magnetic field $\vec{B} = B\hat{k}$.





$\boxed{dl = R.d\theta}$

$d\vec{F} = I\,d\vec{l} \times \vec{B} = I.\,dl.B.\sin\cancel{90^\circ}$ $\underset{1}{}$

$dF = I.B.Rd\theta$

$d\vec{F} = -dF_x - dF_y$

$dF_x = dF\cos\theta$

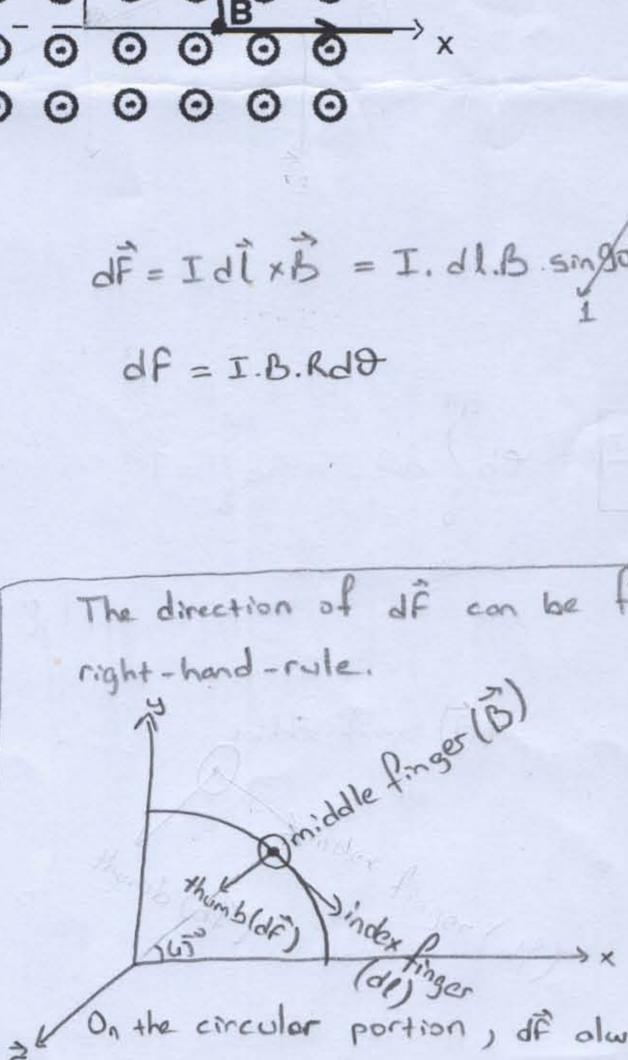
$dF_y = dF\sin\theta$

$F_x = \int_0^{\pi/2} IBR\cos\theta\, d\theta = IBR$

$F_y = \int_0^{\pi/2} IBR\sin\theta\, d\theta = IBR$

$\vec{F} = F_x + F_y = \boxed{-IBR\,\hat{i} - IBR\,\hat{j} = \vec{F}}$

The direction of $d\vec{F}$ can be found by right-hand-rule.



On the circular portion, $d\vec{F}$ always points to the origin, so by symmetry $\vec{F}$ points to origin with an angle 45°.

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Section 4 **Quiz 7** **14 April 2011**

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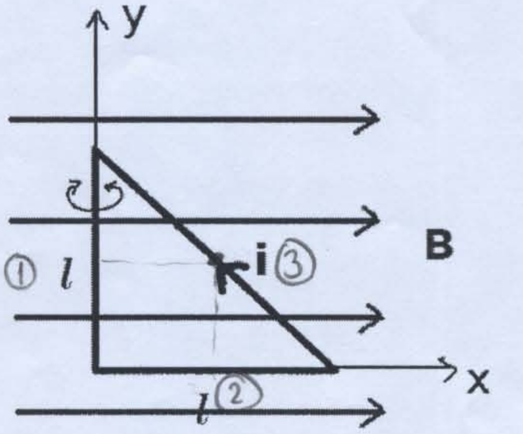
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

The triangular current loop in the figure is attached to the y-axis and is free to rotate around it. Using $d\vec{\tau} = \vec{r} \times d\vec{F}$, calculate the magnitude and the direction of the net torque on the loop with respect to the y-axis in terms of B, i, and *l*.



No force on side ② since ($d\vec{l} \parallel \vec{B}$)

No torque on side ① since side-1 is the rotation axis.

On side ③:

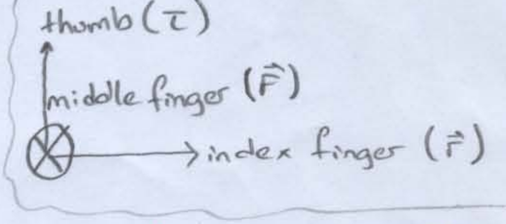
$F_3 = i(l\sqrt{2}).B.\sin 45°$

$\boxed{\vec{F}_3 = -ilB\,\hat{z}}$

We can think that this force is applied on the gravitational center of side-3.

$\tau = \left(\frac{l}{2}\right).(ilB).\underbrace{\sin 90}_{1}$

$\boxed{\vec{\tau} = \frac{iBl^2}{2}\hat{y}}$



(Note: This solution would be incorrect if $\vec{B}$ were nonuniform. See below)

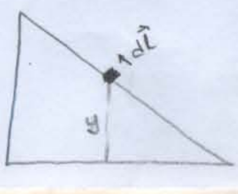
More general solution:

$d\vec{\tau} = \vec{r} \times d\vec{F} = r\,dF\sin 90 = r\,dF$

$\boxed{\vec{r} = \vec{y}}$

$d\vec{F} = i\,d\vec{l} \times \vec{B} = i\,dl\,B\sin 45$

$d\tau = y.\,dF = y.\,i(dy\sqrt{2})B\left(\frac{\sqrt{2}}{2}\right) = iBy\,dy$



$dl = \sqrt{dx^2 + dy^2}$

$dx = dy$

$dl = dy\sqrt{2}$

$\tau = \int_0^l d\tau = iB\int_0^l y\,dy = \frac{iBl^2}{2}$

$\boxed{\vec{\tau} = \frac{iBl^2}{2}\hat{y}}$

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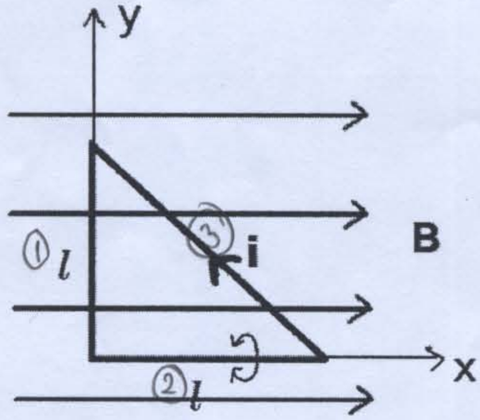
Section 5 **Quiz 7** **14 April 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

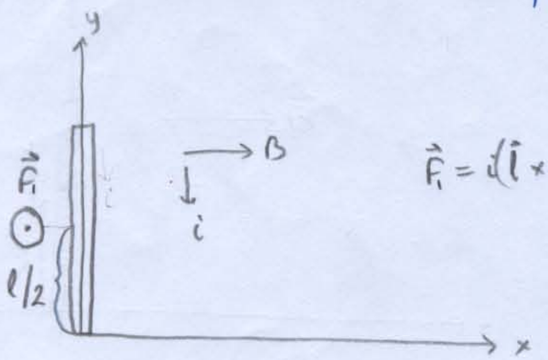
The triangular current loop in the figure is attached to the x-axis and is free to rotate around it. Using $d\vec{\tau} = \vec{r} \times d\vec{F}$, calculate the magnitude and the direction of the net torque on the loop with respect to the x-axis in terms of B, i, and *l*.



No torque on side ② since it is the rotation axis.

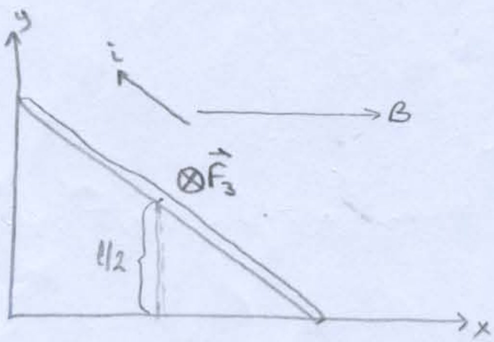
$\boxed{\tau_2 = 0}$

Since $\vec{B}$ is uniform,



$\vec{F}_1 = i(\vec{l} \times \vec{B}) = ilB \underbrace{\cancel{\sin 90}}_{1}$

$\tau_1 = \frac{1}{2} \cdot F_1 \cdot \underbrace{\cancel{\sin 90}}_{1}$ $\boxed{\tau_1 = \frac{1}{2} i l^2 B}$



$\vec{F}_3 = i(\vec{l_3} \times \vec{B}) = i l \sqrt{2} B \sin 45°$

$\tau_3 = \frac{1}{2} \cdot F_3 \cdot \underbrace{\sin 90}_{1} = \frac{1}{2} \cdot \underbrace{\left(i l \sqrt{2} B \frac{\sqrt{2}}{2}\right)}_{F_3}$

$\boxed{\tau_3 = \frac{i l^2 B}{2}}$

$\boxed{\tau_{net} = \tau_1 - \tau_3 = 0}$ (Note: This method would not work if $\vec{B}$ were nonuniform. See previous question)