PHYS 102: General Physics 2 **KOÇ UNIVERSITY** **Spring Semester 2011**

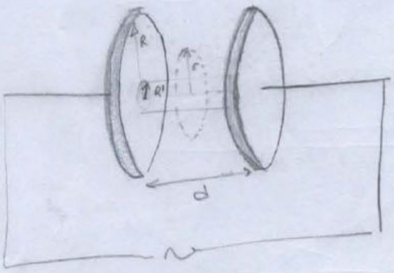
College of Sciences

Section 1 **Quiz 10** **05 April 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A parallel-plate capacitor has circular plates of radius R separated by distance d. A thin straight wire of length d, radius R' lies along the axis of the capacitor and connects the plates. The capacitor plates are connected to an emf source with $V = V_0 \sin(\omega t)$. What is the magnetic field between the capacitor plates at a distance $r < R$ from the capacitor axis?



$V = V_0 \sin \omega t$

$$i_c(t) = \frac{V(t)}{R}$$ where R is the resistance!

$$R = \frac{\rho d}{\pi R'^2}$$

$$\Rightarrow i_c = \frac{V_0 \sin \omega t}{\rho d / \pi R'^2} = \frac{\pi R'^2}{\rho d} V_0 \sin \omega t$$

$$i_D = \epsilon_0 \frac{d\Phi_E}{dt} \quad \& \quad \Phi_E = E \cdot \pi r^2 = \frac{V_0 \sin \omega t}{d} \pi r^2$$

$$\frac{d\Phi_E}{dt} = \pi r^2 \frac{V_0}{d} \omega \cos \omega t$$

$$i_D = \frac{\epsilon_0 V_0 \pi \omega}{d} r^2 \cos \omega t$$

assuming $r > R'$,

$$\underbrace{\oint \vec{B} \cdot d\vec{l}}_{B \cdot 2\cancel{\pi} r} = \mu_0 (i_c + i_D)_{encl} = \frac{\mu_0 V_0 \cancel{\pi}}{d}\left(\frac{R'^2}{\rho} \sin \omega t + \epsilon_0 \omega r^2 \cos \omega t\right)$$

$$B(r,t) = \frac{\mu_0 V_0}{2d}\left(\frac{1}{\rho}\frac{R'^2}{r} \sin \omega t + \epsilon_0 \omega r \cos \omega t\right)$$

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Section 2 **Quiz 10** **05 April 2011**

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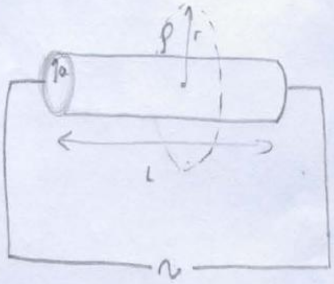
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A linear (ohmic) cylindrical resistor of radius a, length L, and resistivity ρ is connected to an alternating emf source with $V = V_0 \sin(\omega t)$.

a) Calculate the conduction current through the resistor.
b) What is the magnetic field outside the resistor at the instant when the conduction current is zero?



$V = V_0 \sin \omega t$

$$R = \frac{\rho L}{\pi a^2}$$

$$i_c(t) = \frac{V(t)}{R} = \frac{\pi a^2 V_0}{\rho L} \sin \omega t \rightarrow \boxed{\text{when } t = \frac{n\pi}{\omega} \Rightarrow i_c = 0}$$

for $r > a$:

$$i_D = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \pi a^2 \frac{dE}{dt}$$

From definition of resistivity we can find E field

$\rho = \frac{E}{J}$ where J is current density; $J = \frac{I}{A}$

Thus $E = \rho J = \rho \frac{I}{A} = \cancel{\rho} \frac{\frac{\pi a^2 V_0}{\cancel{\rho} L} \sin \omega t}{\pi a^2}$

$$E = \frac{V_0}{L} \sin \omega t$$

$$\frac{dE}{dt} = \frac{V_0 \omega}{L} \cos \omega t \longrightarrow i_D = \frac{\epsilon_0 \pi a^2 V_0 \omega}{L} \cos \omega t$$

$$\underbrace{\oint \vec{B} \cdot d\vec{l}}_{B \cdot 2\pi r} = \mu_0 (i_c + i_D)_{\text{encl.}} = \mu_0 \frac{\pi a^2 V_0}{L} \left(\frac{1}{\rho} \sin \omega t + \epsilon_0 \omega \cos \omega t \right)$$

$$B = \frac{\mu_0 a^2 V_0}{L r} \left(\frac{1}{\rho} \sin \omega t + \epsilon_0 \omega \cos \omega t \right)$$

when $t = n\pi/\omega \Leftrightarrow i_c = 0$

$$B = \frac{\mu_0 \epsilon_0 a^2}{L r} V_0 \omega$$

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Section 3 Quiz 10 05 April 2011

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Quiz duration: 10 minutes

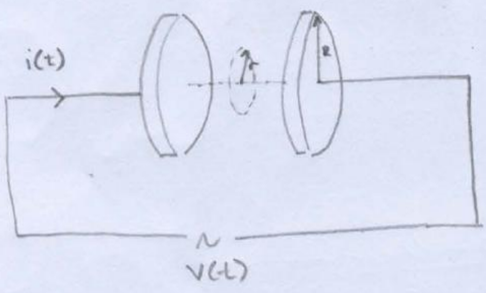
Name: Student ID: Signature:

A parallel-plate capacitor with circular plates of radius R is being charged with a time dependent current i.

a) Calculate the maximum induced magnetic field B_{max} between the plates.
b) What is the magnetic field at radius $r = R/2$, inside the capacitor, in terms of B_{max}?

$B_{max} = C \cdot \frac{d}{R} \cdot i(0)$

$B(r=R/2) = \frac{C}{2R} i(t)$



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (\cancel{i_c} + i_D) = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}$$

between the plates

for a parallel plate capacitor, $E = Q/\epsilon_0 A$

$$\Rightarrow \frac{dE}{dt} = \frac{1}{\epsilon_0 A}\frac{dQ}{dt} \quad \text{and} \quad \frac{dQ}{dt} \equiv i(t)$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \cancel{\epsilon_0} \cancel{\pi} r^2 \frac{1}{\cancel{\epsilon_0}\cancel{\pi} R^2} i(t)$$

$$B \cdot 2\pi \cancel{r} = \mu_0 \frac{r^2}{R^2} i(t)$$

$$B = \frac{\mu_0}{2\pi}\frac{r}{R^2} i(t)$$

Magnetic field attains its max value at $r=R$

$$B_{max} = \frac{\mu_0}{2\pi}\frac{1}{R} i(t)$$

$$B(r=R/2) = \frac{\mu_0}{4\pi}\frac{1}{R} i(t) = \frac{B_{max}}{2}$$

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Section 4 **Quiz 10** **05 April 2011**

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Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A sinusoidally varying voltage is applied across a capacitor with capacitance C = 1μF. The frequency and the amplitude of the applied voltage are f = 1kHz and V_0 = 10V, respectively. Calculate the displacement current in the capacitor.

$V(t) = V_0 \sin \omega t$ and

$Q(t) = C \cdot V(t) = V_0 C\, V(t)$

$i_c(t) = \frac{dQ}{dt} = V_0 C \omega \cos \omega t$

since $i_D = i_c$,

$i_D = V_0 C \omega \cos \omega t$

$= (10V) \times (1 \times 10^{-2} F) \times (2\pi 1 \times 10^3 Hz) \cos(2\pi \times 10^3 t)$

$$\boxed{i_D = 200\pi \cos(2\pi \times 10^3 t) \ A}$$

OR

$i_D = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 A \frac{dE}{dt}$ & $E = \frac{V}{d}$

$\Rightarrow i_D = \underbrace{\frac{\epsilon_0 A}{d}}_{C} V_0 \omega \cos \omega t$

Similarly

$$\boxed{i_D(t) = 200\pi \cos(2\pi \times 10^3 t) \ A}$$

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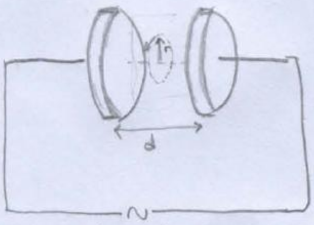
Section 5 Quiz 10 05 April 2011

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: Student ID: Signature:

A circular parallel-plate capacitor is connected to an alternating emf source with voltage $V = V_0 \sin(\omega t)$. The capacitor plates are separated by distance d.

a) Calculate the magnetic field inside the capacitor.
b) Calculate the electric field induced by the magnetic field determined in (a).



$V = V_0 \sin\omega t$

$V(t) = E(t)\cdot d$ for a parallel plate capacitor.

$\Rightarrow E(t) = \frac{V(t)}{d} = \frac{V_0 \sin\omega t}{d}$

$\Phi_E = E(t)\cdot A \rightarrow \frac{d\Phi_E}{dt} = \frac{dE}{dt}\cdot A = \frac{A V_0 \omega}{d}\cos\omega t$

a)

$i_D = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{\epsilon_0 A V_0 \omega}{d}\cos\omega t$

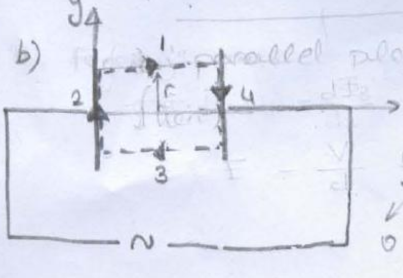
Ampere's Law:

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 (\cancel{i_C}^{\,0} + i_D)_{encl}$

$B\cdot 2\cancel{\pi}\cancel{r} = \frac{\mu_0 \epsilon_0 \cancel{\pi} r^{\cancel{2}} V_0 \omega}{d}\cos\omega t$ and $A = \pi r^2$

$\Rightarrow \boxed{B(r,t) = \frac{\epsilon_0 \mu_0 V_0 \omega}{2d}\, r \cos\omega t}$

b) for a parallel plate capacitor the E-field is uniform!



$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\underbrace{\int_1 \vec{E}\cdot d\vec{\ell}}_{0} + \int_2 \vec{E}\cdot d\vec{\ell} + \int_3 \vec{E}\cdot d\vec{\ell} + \underbrace{\int_4 \vec{E}\cdot d\vec{\ell}}_{0} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$

since $\vec{E} \perp d\vec{\ell}$ in these paths

$\Rightarrow \cancel{2}\, E(r)\cdot d = -\frac{d}{dt}\left(2\cdot\int_{y=0}^{r}\int_{x=0}^{d} \frac{\epsilon_0 \mu_0 V_0 \omega}{2d}\, y \cos\omega t \; dx\,dy\right) = \frac{\epsilon_0 \mu_0 V_0 \omega^2}{2d}\,\cancel{2}\,\frac{r^2}{2}\sin\omega t$

$E(r) = \frac{\mu_0 \epsilon_0 V_0 \omega^2}{4}\,\frac{r^2}{d^2}\sin\omega t$