PHYS 102: General Physics 2 **KOÇ UNIVERSITY** **Spring Semester 2011**

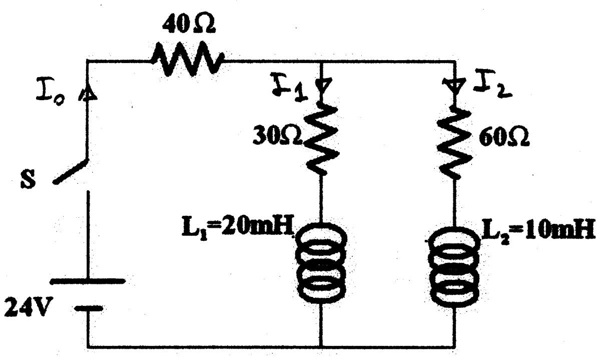
College of Sciences

Section 1 **Quiz 11** **12 May 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

In the circuit shown below, the switch S is closed at time $t=0$. Find the total energies stored in the inductors L_1 and L_2 after the switch has been closed a long time.



- Energy stored in an inductor : $U = \frac{1}{2} L I^2$
- after a long time : currents are settled to their steady state values.
- $R_{eq} = 40\Omega + \left(\frac{1}{30\Omega} + \frac{1}{60\Omega} \right)^{-1} = 40\Omega + 20\Omega = 60\Omega$
- I_0, I_1 and I_2 are as shown on the figure.

$$I_0 = \frac{24V}{R_{eq}} = \frac{24V}{60\Omega} = 0{,}4\,A \Rightarrow I_1 = \frac{2}{3} \times 0{,}4\,A, \quad I_2 = \frac{1}{3} \times 0{,}4\,A$$

$$U_1 = \frac{1}{2}\, 20 \times 10^{-3}\,H \times \frac{4}{9} \times 0{,}16\,A = \underline{\underline{0{,}711\,mJ}}$$

$$U_2 = \frac{1}{2}\, 10 \times 10^{-3}\,H \times \frac{1}{9} \times 0{,}16\,A = \underline{\underline{0{,}089\,mJ}}$$

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Section 2 **Quiz 11** **12 May 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Show that the quantity $\sqrt{\frac{L}{C}}$ has units of resistance (ohms).

- Energy of an inductor is: $U = \frac{1}{2} L I^2$

Dimensional analysis of this equation gives:

$$[\text{energy}] = [\text{inductance}][\text{current}]^2 \rightarrow [\text{inductance}] = \frac{[\text{energy}]}{[\text{current}]^2}$$

- Energy of a capacitor: $U = \frac{1}{2} C V^2$

Dimensional analysis of this equation gives:

$$[\text{energy}] = [\text{capacitance}][\text{potential}]^2 \rightarrow [\text{capacitance}] = \frac{[\text{energy}]}{[\text{potential}]^2}$$

The quantity $\sqrt{\frac{L}{C}}$ has units of $\sqrt{\frac{[\text{inductance}]}{[\text{capacitance}]}} = \sqrt{\frac{\frac{[\text{energy}]}{[\text{current}]^2}}{\frac{[\text{energy}]}{[\text{potential}]^2}}}$

$$= \sqrt{\frac{[\text{potential}]^2}{[\text{current}]^2}} = [\text{resistance}]$$

$[\text{resistance}] = \Omega$ in the SI system.

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Section 3 **Quiz 11** **12 May 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A toroidal solenoid with cross-sectional area A and mean radius r is closely wound with N turns of wire. The toroid is wound on a nonmagnetic core. Determine its self-inductance L. Assume that B is uniform across a cross section.

- Toroidal solenoid
- Cross-sectional area: A
- mean radius: r → length $= 2\pi r$
- N turns
- assume uniform B.

$$L = \frac{N\Phi_B}{i} = \frac{N B.A}{i}$$

B of a solenoid $= \mu_0 n i$, where n is the number of turns per length. For the case of a toroidal solenoid length is the circumference of the toroid, and $n = \frac{N}{2\pi r}$. With this n, B of a toroidal solenoid becomes $\mu_0 \frac{N}{2\pi r} i$. Substituting this into L, we obtain:

$$L = \frac{N.A}{i} \cdot \mu_0 \frac{N}{2\pi r} i = \mu_0 \frac{N^2}{2\pi r} A$$

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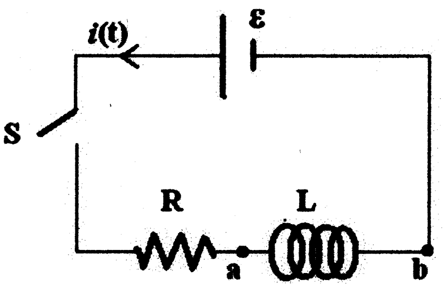
Section 4 **Quiz 11** **12 May 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

In the circuit shown below, the switch *S* is closed at time *t=0.* a) What is the current i just after *S* is closed? b) What is the current *i* long time after *S* is closed? c) Find an expression for V_{ab} as a function of time since *S* is closed.



a) Immediately after S is closed, $i = 0$, because current cannot develop instantaneously.

b) Long after S is closed, current reaches its steady state value $\left(\frac{di}{dt} = 0\right)$, and we have $i = \frac{\mathcal{E}}{R}$.

c) For the transient case we need to solve the following equation:

$\mathcal{E} - V_R - V_L = 0$ V_R : voltage drop due to resistor. $V_R = i \cdot R$
V_L : " " " " inductor. $V_L = L \cdot \frac{di}{dt}$

$$\mathcal{E} - i.R - L\frac{di}{dt} = 0 \quad (i \text{ is a function of time}, \ i = i(t))$$

$$\mathcal{E} - iR = L\frac{di}{dt} \qquad dt = L\frac{di}{\mathcal{E} - iR} \qquad \int_0^t dt = \int_0^{i(t)} L\frac{di}{\mathcal{E} - iR}$$

$$t = L\left[-\frac{1}{R}\ln(\mathcal{E} - iR)\right]_0^{i(t)} = -\frac{L}{R}\left[\ln(\mathcal{E} - i(t)R) - \ln\mathcal{E}\right]$$

$$-\frac{R}{L}t = \ln\frac{\mathcal{E} - i(t)R}{\mathcal{E}} \qquad 1 - i(t)\frac{R}{\mathcal{E}} = e^{-\frac{R}{L}t}$$

$$1 - e^{-\frac{R}{L}t} = i(t)\frac{R}{\mathcal{E}} \rightarrow i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right).$$

$\rightarrow$

$$V_{ab} = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R}e^{-\frac{R}{L}t}\right) = L\left(-\frac{\mathcal{E}}{R}\right)\left(-\frac{R}{L}\right)e^{-\frac{R}{L}t}$$

$$= \mathcal{E}e^{-\frac{R}{L}t}$$

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Section 5 **Quiz 11** **12 May 2011**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A long solenoid with length l and cross-sectional area A is closely wound with N_1 turns of wire. A coil with N_2 turns surrounds it at its center. Find the mutual inductance.

Mutual inductance, $M = \dfrac{N_2 \Phi_{B_2}}{i_1} = \dfrac{N_1 \Phi_{B_1}}{i_2}$

$\Phi_{B_2} = \Phi_{B_1} = B_1 \cdot A$ (because the outer solenoid surrounds the inner one)

B of a solenoid : $\mu_0 n I \rightarrow B_1 = \mu_0 \dfrac{N_1}{l} i_1$

$$\Rightarrow M = \frac{N_2 \left(\mu_0 \frac{N_1}{l} i_1 \right) A}{i_1} = \mu_0 \frac{N_1 N_2}{l} A$$