Chapter 26: Direct Current Circuits

Direct current (dc) circuits:

Current, electric potential in the circuit has no time dependence

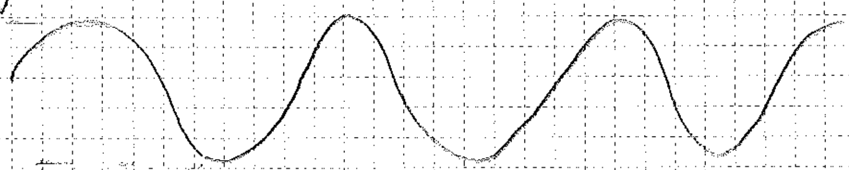
5V DC Power Supply

9V DC Power Supply ...

Alternating current (ac) circuits:

Current, electric potential in the circuit oscillates with time.

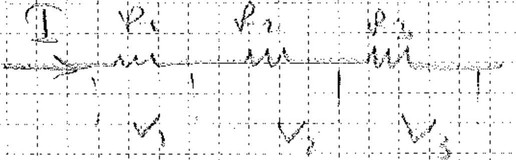
220V



0V

220V AC Voltage

Resistors In Series and Parallel:



$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I\,(\underbrace{R_1 + R_2 + R_3}_{R_{eq}})$$

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Equivalent resistance of resistors connected in series

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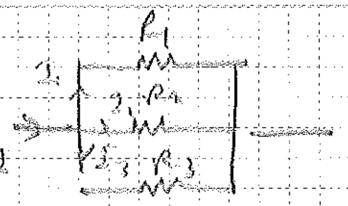


$I = I_1 + I_2 + I_3$ ← conservation of charge flow!

$V_1 = V_2 = V_3 = V$

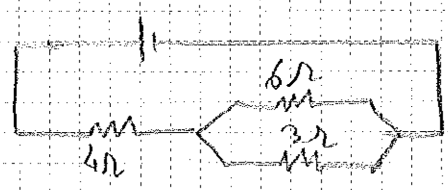
$\Rightarrow I = I_1 + I_2 + I_3$

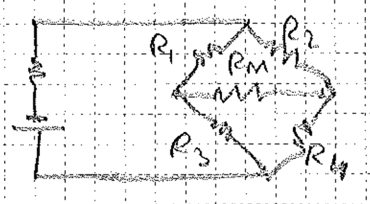
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_{eq}}$$

$$\Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Equivalent resistance of resistors connected in parallel.

26.2 Kirchhoff's Rules:





Many practical resistor networks cannot be solved using the simple series + parallel resistor reduction techniques.

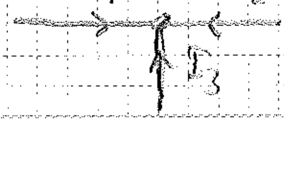
Kirchoff's Rules help analyzing complicated circuits.

Kirchhoff's Rules:

(i) The algebraic sum of the currents into any junction is equal to zero.

$\Sigma I = 0$, junction rule.

Junction: A point in the circuit where three or more conductors meet.



$I_1 + I_2 + I_3 = 0 \Rightarrow I_3 = -(I_1 + I_2)$ flows in the opposite direction

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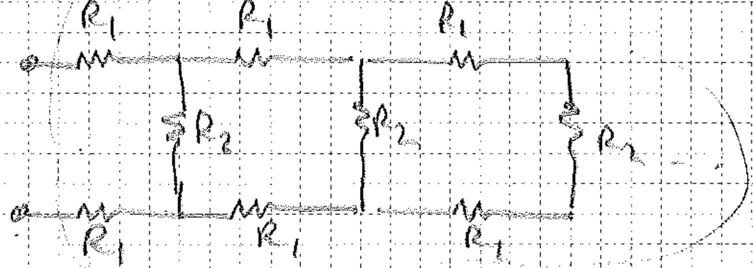
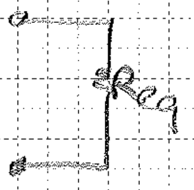
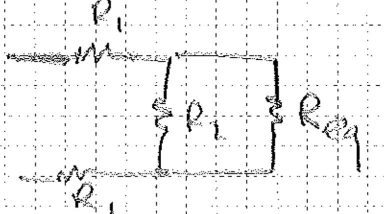
Prob 26.92





Equivalent resistance?



$$\frac{1}{R_2} + \frac{1}{R_{eq}} = \frac{R_{eq} + R_2}{R_{eq} R_2}$$

$$\Rightarrow R_{eq} = 2R_1 + \frac{R_2 R_{eq}}{R_{eq} + R_2}$$

$$\Rightarrow (R_{eq} - 2R_1)(R_{eq} + R_2) = R_{eq} R_2$$

$$\Rightarrow R_{eq}^2 + R_{eq}(\cancel{R_2} - 2R_1 - \cancel{R_2}) - 2R_1 R_2 = 0$$

$$R_{eq}^2 - 2R_1 R_{eq} - 2R_1 R_2 = 0$$

$$\Rightarrow R_{eq} = \frac{2R_1 \pm \sqrt{4R_1^2 + 8R_1R_2}}{2} = R_1 \pm \underbrace{\sqrt{R_1^2 + 2R_1R_2}}_{> R_1}$$

$$\Rightarrow \text{Solution is } \boxed{R_{eq} = R_1 + \sqrt{R_1^2 + 2R_1R_2}}$$

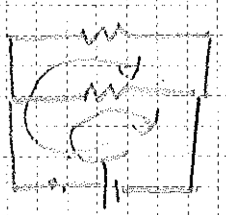
$R_{eq} \neq R_1 - \sqrt{R_1^2 + 2R_1R_2}$ ⟺ is not a physical solution

(ii) The algebraic sum of the potential differences in any loop (associated with emfs, and resistive elements) must equal zero.

$\sum V = 0$, Kirchhoff's Loop Rule

Loop: Any closed conducting path in the circuit.

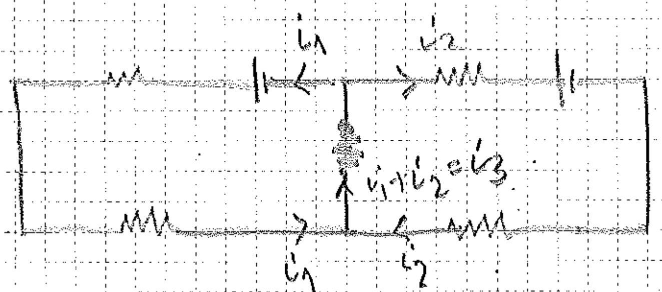
This is clear because electrostatic force is conservative.



$W_{a \to a} = -\Delta U = U_f - U_i = 0$

How to solve circuits using Kirchoff's Laws:

Given a circuit:

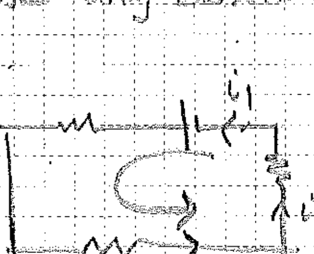


- Label currents with their directions. The directions need not be the correct directions.

Write the Kirchoff's Junction Law in the junctions:

$i_1 + i_2 = i_3$

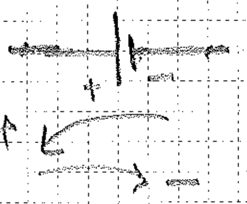
- Choose any closed loop in the circuit, designate a direction of travel.



Travel around the loop adding potential differences due to different elements.

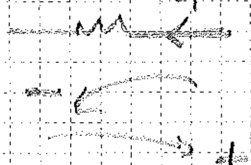
Potential differences due to different elements:

emf

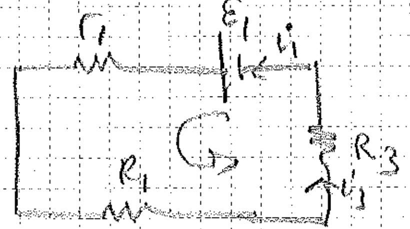


Emf is counted + if you traverse from − to +.
emf is counted − if you traverse from + to −.

Resistance



Resistance is − if you traverse in the same direction as current flow.
Resistance is + if you traverse in the direction opposite to current.



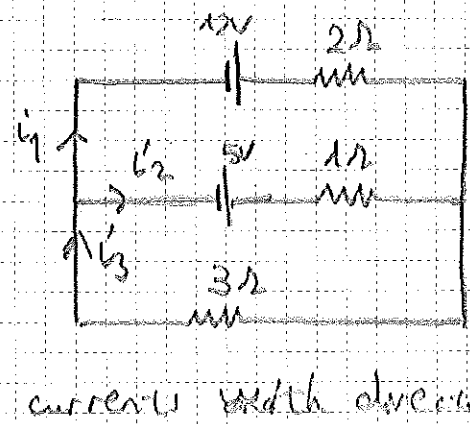
$$V_a + \mathcal{E}_1 - i_1 r_1 - i_1 R_1 - i_3 R_3 = V_a$$

$$\boxed{\mathcal{E}_1 - i_1 r_1 - i_1 R_1 - i_3 R_3 = 0}$$

4 - Write as many linearly independent equations as the number of currents to solve for the circuit.

Ex 26.4: Consider the circuit:

What are the currents?



(i) Label currents with directions

(ii) Apply Kirchoff's Junction Law

$$i_1 + i_2 = i_3$$

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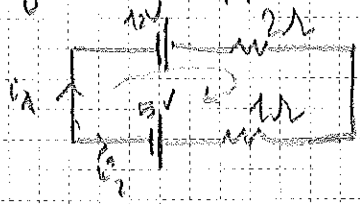
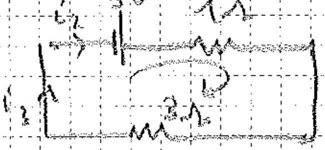
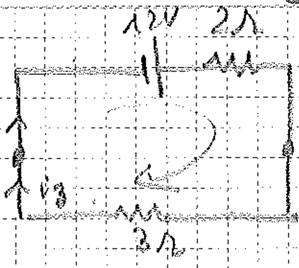
Apply Kirchoff's Loop Law:







$12 - 2i_1 + i_2 - 5 = 0$ $\qquad$ $5 - i_2 - 3i_3 = 0$ $\qquad$ $12 - 2i_1 - 3i_3 = 0$

$7 - 2i_1 + i_2 = 0$ $\qquad$ $5 - i_2 - 3i_3 = 0$ $\qquad$ $12 - 2i_1 - 3i_3 = 0$

2 linearly independent equations.

Add the first 2:
$$\begin{aligned} 7 - 2i_1 + i_2 &= 0 \\ 5 - i_2 - 3i_3 &= 0 \\ \hline 12 - 2i_1 - 3i_3 &= 0 \end{aligned}$$

∴ Use the equations:

$i_1 + i_2 = i_3$

$7 - 2i_1 + i_2 = 0$

$5 - i_2 - 3i_3 = 0 \Rightarrow 5 - i_2 - 3i_1 - 3i_2 = 5 - 3i_1 - 4i_2 = 0$

$\Rightarrow i_2 = \frac{5}{4} - \frac{3}{4} i_1$

$\Rightarrow 7 - 2i_1 + \frac{5}{4} - \frac{3}{4} i_1 = \frac{33}{4} - \frac{11}{4} i_1 = 0 \Rightarrow \underline{i_1 = 3A}$

$i_2 = \frac{5}{4} - \frac{3}{4} i_1 = \frac{5}{4} - \frac{9}{4} = \underline{-1A}$

$\underline{i_3 = 2A}$

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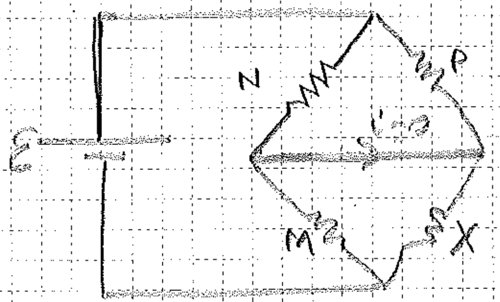
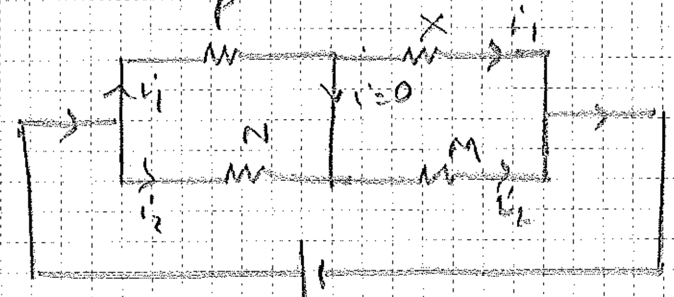
Prob 26.77



Show that $X = \frac{MP}{N}$



$i_1 P = i_2 N \Rightarrow \frac{i_1}{i_2} = \frac{N}{P}$

$i_1 X = i_2 M$

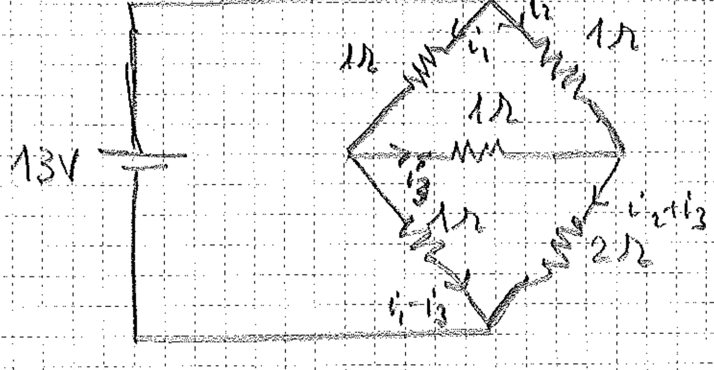
$\Rightarrow \frac{i_1}{i_2} = \frac{M}{X}$

$\Rightarrow \frac{M}{X} = \frac{N}{P} \Rightarrow \boxed{X = \frac{MP}{N}}$

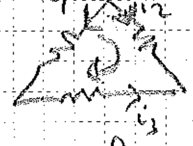
Wheatstone Bridge

Ex 26.6:

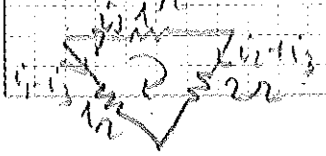
Find the current in each resistor.



3 Unknowns $i_1, i_2, i_3 \Rightarrow$ Write 3 equations using Kirchoff's Loop Law:



$-i_2 + i_3 + i_1 = 0$



$-2(i_2 + i_3) + i_1 - i_3 - i_3 = 0$

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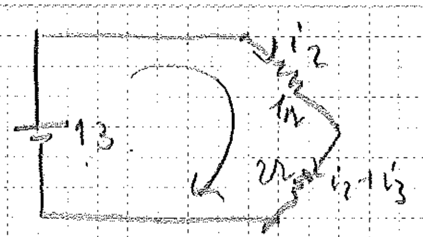

$-i_2 - 2(i_2+i_3) + 13 = 0$

$\Rightarrow \quad i_1 - i_2 + i_3 = 0$

$i_1 - 2i_2 - 4i_3 = 0$

$-3i_2 - 2i_3 = -13$

$2i_1 - i_3 = 13 \Rightarrow i_3 = 2i_1 - 13$

$\Rightarrow i_1 - i_2 + 2i_1 - 13 = 0 \Rightarrow 3i_1 - i_2 = 13$

$i_1 - 2i_2 - 4(2i_1 - 13) = 0 \Rightarrow -7i_1 - 2i_2 = -52$

$-13 i_1 = -52 - 26 = -78 \Rightarrow i_1 = 6A$ //

$i_2 = 3i_1 - 13 = 18 - 13 = 5A$ //

$i_3 = 2i_1 - 13 = 12 - 13 = -1A$ //

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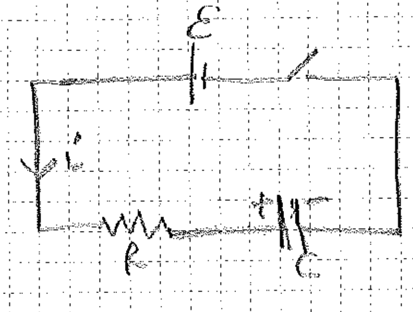
26.4 R-C Circuits

Circuits that involve charging or discharging of capacitors.



Charging a Capacitor:

Consider the capacitor being initially uncharged.

Then at time t=0 the switch is closed.

→ Capacitor then starts being charged.

$$C = \frac{Q}{V} \Rightarrow Q = CV \Rightarrow \frac{dQ}{dt} = C\frac{dV}{dt} = i$$

$$\mathcal{E} - iR - V = 0 \Rightarrow \mathcal{E} - RC\frac{dV}{dt} - V = 0$$

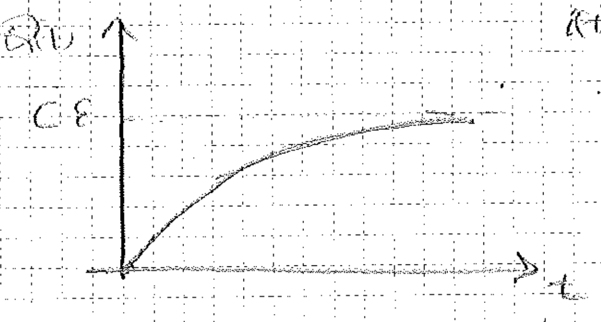
$$RC\frac{dV}{dt} + V - \mathcal{E} = 0 \Rightarrow \int_0^V \frac{dV}{\mathcal{E}-V} = \int_0^t \frac{dt}{RC}$$

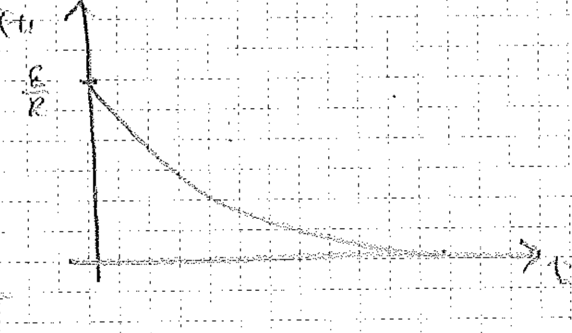
$$\Rightarrow -\ln(\mathcal{E}-V)\Big|_0^V = +\frac{t}{RC} \Rightarrow -\ln(\mathcal{E}-V) + \ln(\mathcal{E}) = \frac{t}{RC}$$

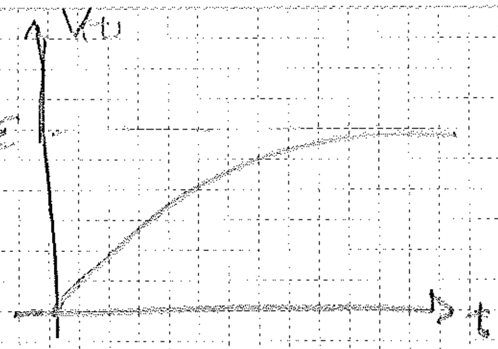
$$\Rightarrow -\ln\left(\frac{\mathcal{E}-V}{\mathcal{E}}\right) = \frac{t}{RC} \Rightarrow \frac{\mathcal{E}-V}{\mathcal{E}} = e^{-t/RC} \Rightarrow \boxed{V = \mathcal{E} - \mathcal{E}e^{-t/RC}}$$

$$Q = CV \Rightarrow Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) \text{ //}$$

$$i = \frac{dQ}{dt} = \cancel{C}\mathcal{E}\left(\frac{1}{R\cancel{C}}e^{-t/RC}\right) = \frac{\mathcal{E}}{R}e^{-t/RC} \text{ //}$$

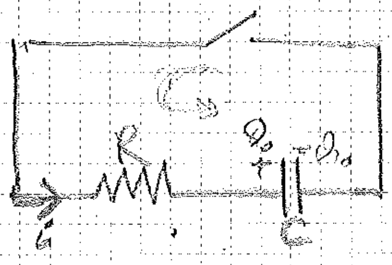






At time $t = RC$, current drops to $\frac{1}{e}$ of its initial value. $\tau = RC$ is called the time constant of the R-C circuit.

Discharging a Capacitor:



Consider an initially charged capacitor. At time $t=0$ the switch is closed.

$$\Rightarrow C = \frac{Q(t)}{V(t)}$$

$$V(t) + i(t)R = 0 \Rightarrow \frac{Q(t)}{C} + \frac{dQ(t)}{dt}R = 0$$

$$\Rightarrow Q(t) = -RC\frac{dQ(t)}{dt} \Rightarrow \frac{dQ(t)}{Q(t)} = \frac{-dt}{RC} \Rightarrow \ln Q\Big|_{Q_0}^{Q} = \frac{-t}{RC}\Big|_0^t$$

$$\Rightarrow \ln\left(\frac{Q}{Q_0}\right) = \frac{-t}{RC} \Rightarrow \boxed{Q = Q_0 e^{-t/RC}}$$

$$V(t) = \frac{Q(t)}{C} = \boxed{\frac{Q_0}{C} e^{-t/RC}}$$

$$i(t) = +\frac{V(t)}{R} = \boxed{+\frac{Q_0}{RC} e^{-t/RC}}$$

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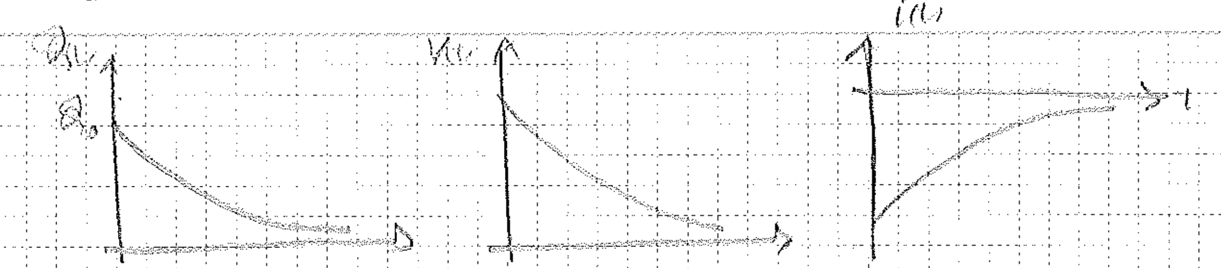
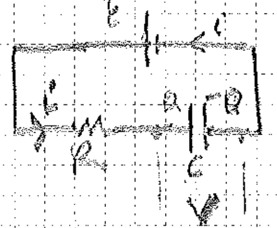


Time constant: $\boxed{\tau = RC}$

time at which the stored charge drops to $\frac{1}{e}$ of its initial value.

Energy Considerations

In charging of a capacitor:



$\mathcal{E}i$, rate at which the battery delivers energy to the circuit.

$$\mathcal{E}i = i^2R + VI = i^2R + \frac{Q}{C}i$$

Total energy supplied by the battery during charging of the capacitor:

$$\int P\,dt = \int \mathcal{E}\frac{dq}{dt}\,dt = \int_0^{Q_f} \mathcal{E}\,dQ = \mathcal{E}Q_f$$

↑ Final charge of the capacitor

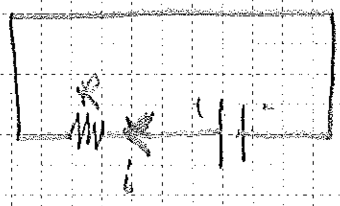
Total energy stored in the capacitor

$$U = \frac{1}{2}Q_f V = \frac{1}{2}Q_f\mathcal{E}$$ ⇐ Half of the energy is stored in the capacitor. Half is dissipated in the resistor.

$$U = \int \frac{Q(t)}{C}\, i(t)\,dt = \int \frac{Q}{C}\frac{dQ}{dt}\,dt = \int \frac{Q}{C}\,dq = \frac{1}{2C}Q_f^2 = \boxed{\frac{1}{2}Q_f\mathcal{E}}$$

∴ This result does not depend on the values of R, $\mathcal{E}$ and C.

Discharging of a Capacitor



Power dissipated by the capacitor:

$$i\frac{Q}{C} = i^2R$$

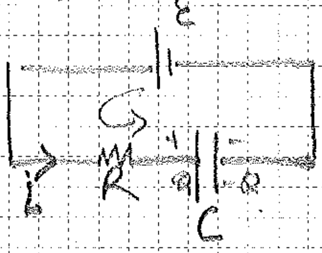
$$\int i\frac{Q}{C}\,dt = \int -\frac{dQ\,Q}{C} = -\frac{1}{2C}Q^2\Big|_{Q_0}^{0} = +\frac{1}{2C}Q_0^2$$ ⇐ Total energy capacitor delivers to the circuit.

$$\int i^2R\,dt = \int \frac{Q_0^2}{R^2C^2}e^{-2t/RC}R\,dt = \frac{Q_0^2}{RC^2}\int e^{-2t/RC}dt$$

$$= \frac{Q_0^2}{RC^2}\cdot -\frac{RC}{2}e^{-2t/RC}\Big|_0^{\infty} = \frac{Q_0^2}{C}\left(-\frac{1}{2}\right)(0-1) = \frac{Q_0^2}{2C}$$ Energy dissipated in the resistor

RC - Circuits:

Charging a capacitor



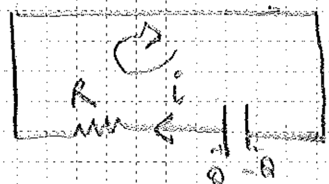
$\frac{dQ}{dt} = i$ ← direction is important

$\mathcal{E} - iR - \frac{Q}{C} = 0 \Rightarrow \mathcal{E} = \frac{dQ}{dt}R + \frac{Q}{C}$

$\Rightarrow Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$

$i = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$

Discharging a capacitor:



$i = -\frac{dQ}{dt}$

$\Rightarrow -iR + \frac{Q}{C} = 0$

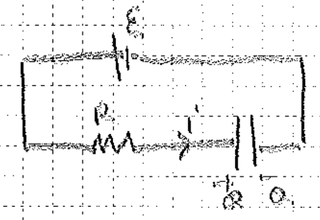
$\Rightarrow -\frac{dQ}{dt}R = \frac{Q}{C} \Rightarrow Q = Q_0 e^{-t/RC}$

$V = \frac{Q(t)}{C} = \frac{Q_0}{C} e^{-t/RC}$

$i(t) = \frac{V(t)}{R} = \frac{Q_0}{RC} e^{-t/RC}$

RC : time constant

Energy Considerations:



rate of energy dissipated is: $i^2R + Vi$

rate of energy delivered is: $\mathcal{E}i$

stored in the capacitor: $U = \int V \frac{dq}{dt}\,dt = \int \frac{Q}{C}\,dQ = \frac{1}{C}\frac{Q_f^2}{2}$

$= \frac{1}{2} Q_f V_f = \frac{1}{2} Q_f \mathcal{E}$

delivered by $\mathcal{E}$: $\int \mathcal{E}\frac{dq}{dt}\,dt = \int_0^{Q_f} \mathcal{E}\,dq = \mathcal{E}Q_f$