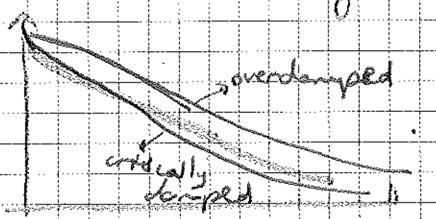
When $R^2 = \frac{4L}{C}$, the system no longer oscillates.

↳ critically damped case.



When $R^2 > \frac{4L}{C}$, → overdamped. Capacitor charge approaches zero even more slowly.

Ex: What resistance R is required to give an L-R-C circuit an oscillation frequency that is $\frac{1}{2}$ the undamped frequency.

undamped frequency: $\sqrt{\frac{1}{LC}}$

$$\frac{1}{2}\sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \Rightarrow \boxed{R = \frac{3L}{C}}$$

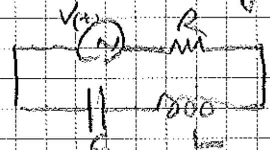
Chap 31 : Alternating Current

Circuits in which voltages and currents vary sinusoidally are called as alternating current circuits.

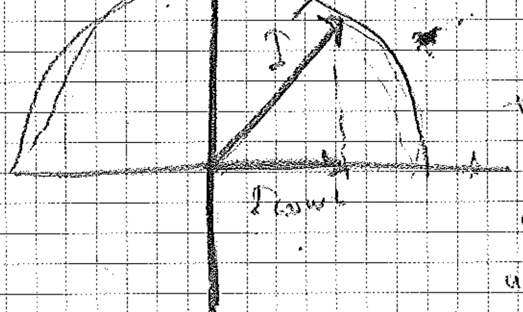
Phasors and Alternating Currents

"Most present day household and industrial power-distribution systems operate with alt. current." "In Turkey f = 50Hz"

A sinusoidal voltage is described as: $v = V\cos\omega t$



A sinusoidal current is described as $i = I\cos\omega t$



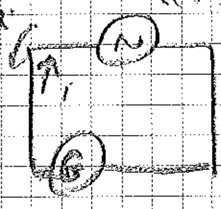
We use phasor diagrams to represent sinusoidally varying voltages and currents.

— The instantaneous value of a quantity that varies sinusoidally with time is represented by the projection onto a horizontal axis of a vector with a length I.

"Phasor is the vector in 2D whose projection to the horizontal axis gives the instantaneous value of the variable."

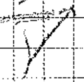
"The vector which rotates in the counter-clockwise direction with angular speed ω is called a phasor."

$i(t) = I\cos(\omega t)$



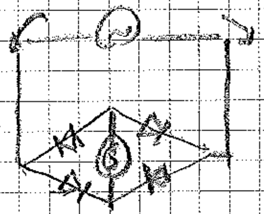
AC can be both positive and negative.

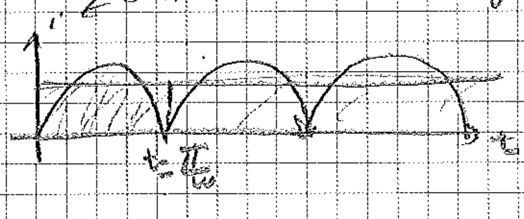
⇒ If we place a galvanometer (tool for current measurement) into an AC circuit it will not shown a value unless the oscillations are really slow.



⇒ So, we have to find other ways to describe and measure AC.

Full-wave rectifier circuit: ← Galvanometer reading





It is consisted of 4 diodes ⇒ these are elements which have 0 resistance in the forward and infinite resistance in the backward direction.

Galvanometer will display an average value:

$$\int_0^{\pi/\omega} I\sin\omega t\,dt = I\left(-\frac{\cos\omega t}{\omega}\right)\Big|_0^{\pi/\omega} = -\frac{I}{\omega}(-1-1) = \frac{2I}{\omega}$$

$$\frac{2I}{\omega} = I_{rav}\cdot\frac{\pi}{\omega} \Rightarrow \boxed{I_{rav} = \frac{2}{\pi}I}$$

rectified average value of a sinusoidal current.

AC is also described by the root mean square value:

$I_{rms} = \sqrt{\overline{i(t)^2}}$, square root of the time average of current squared.

$i(t) = I\cos\omega t$

$\Rightarrow i(t)^2 = I^2\cos^2\omega t = I^2\left(\frac{1+\cos 2\omega t}{2}\right) = \frac{I^2}{2} + \frac{I^2}{2}\cos 2\omega t$

$\overline{i^2(t)} = \overline{\left(\frac{I^2}{2}\right)} + \underbrace{\overline{\frac{I^2}{2}\cos 2\omega t}}_{=0} = \frac{I^2}{2} + 0 = \frac{I^2}{2}$

$\Rightarrow \boxed{I_{rms} = \frac{I}{\sqrt{2}}}$ root-mean-square value of a sinusoidal current

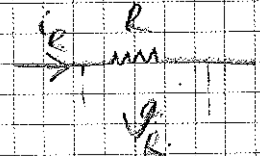
similarly $\boxed{V_{rms} = \frac{V}{\sqrt{2}}}$

31.2 Resistance and Reactance

. We will determine how to represent resistors, inductors and capacitors in ac circuits.

$\Rightarrow$ Find the relationships between phasors!

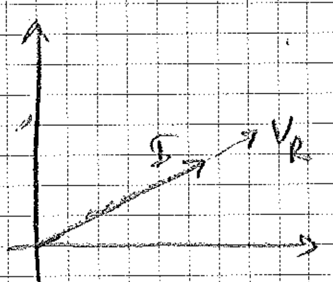
Resistor in an AC circuit



$i_R = I\cos\omega t$

$\Rightarrow v_R = IR\cos\omega t = V_R\cos\omega t$

$\Rightarrow \boxed{V_R = IR}$ phasor relationship.



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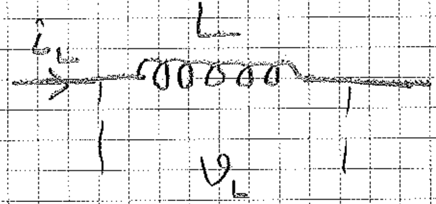
Inductor in an AC circuit



$i_L = I\cos\omega t$

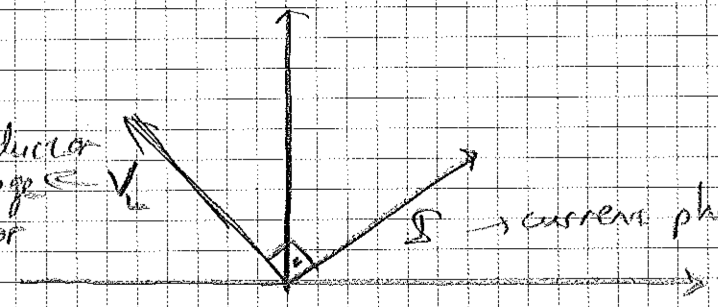
$$\mathcal{E} = v_L = L\frac{di_L}{dt} = -LI\omega\sin\omega t$$

$$v_L = -I\omega L\sin\omega t$$

$$v_L = \underbrace{I\omega L}_{V_L}\cos(\omega t + 90^\circ)$$

$$\Rightarrow V_L = I X_L \Rightarrow \boxed{X_L = \omega L}$$ inductive reactance

amplitude of voltage across an inductor



Ex 31.2: You want the current amplitude in a pure inductor to be 250μA, when voltage amplitude is 3,6V at f = 1.6 MHz

a) $X_L = ?$ $\quad X_L = \omega L = \frac{V_L}{I} = \frac{3.6\,V}{250\times10^{-6}A} = 14{,}4\,k\Omega$

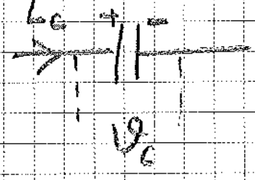
$L = ?$

$$L = \frac{14.4\times10^{3}\,\Omega}{2\pi\, 1.6\times10^{6}\,Hz} = 1{,}43\,mH$$

b) at $f = 16\,MHz$ $\quad I = ?$ if V_L is the same

$$X_L = 2\pi f L = \frac{V_L}{I} \Rightarrow I = \frac{V_L}{2\pi f L} \Rightarrow \boxed{I = 25\,\mu A}$$

Capacitor in an AC circuit



Choose the positive direction of current to be towards the + plate of the capacitor.
⇒ Capacitor is charging.

$$i_C = \frac{dq}{dt} > 0$$

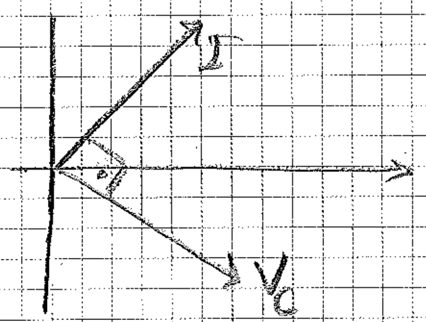
$$i_C = I\cos\omega t = \frac{dq}{dt}$$

$$\Rightarrow q(t) = \frac{I}{\omega}\sin\omega t \ , \ \text{for } q_0 = 0$$

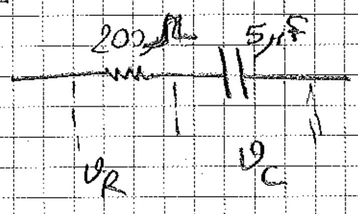
$$q(t) = \frac{I}{\omega}\cos(\omega t - 90^\circ)$$

$$\Rightarrow v_C = \frac{q}{C} = \left(\frac{I}{\omega C}\right)\cos(\omega t - 90^\circ)$$

$$V_C = \frac{I}{\omega C} = I X_C \Rightarrow \boxed{X_C = \frac{1}{\omega C}}$$ capacitive reactance



Ex 31.3: Resistor and capacitor in an AC circuit.



$$v_R = 1.2\,V\cos\left(2500\,rad/s\ t\right)$$

a) $i = ?$ $\quad i = \frac{v_R}{R} = \frac{1.2\,V}{200\,\Omega}\cos(2500\,rad/s\ t) = 6\,mA\cos(2500\,rad/s\ t)$

b) $X_C = ?$ $\quad X_C = \frac{1}{\omega C} = \frac{1}{2500\,rad/s \cdot 5\times10^{-6}F} = 80\,\Omega$

c) $v_C = ?$ $\quad v_C = X_C I\sin\omega t = 80\,\Omega \cdot 6\times10^{-3}A\ \sin(2500\,rad/s\ t) = 0.48\,V\sin(2500\,rad/s\ t)$

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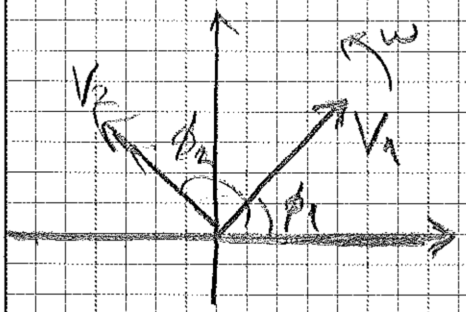
125.5

Phasor Analysis



$v(t) = V\cos(\omega t + \phi)$

$v_1(t) = V_1\cos(\omega t + \phi_1)$, $v_2(t) = V_2\cos(\omega t + \phi_2)$

Projection of V_1 to the horizontal axis

$$\vec{V}_1 = V_{1x}\hat{i} + V_{1y}\hat{j}$$
$$\vec{V}_2 = V_{2x}\hat{i} + V_{2y}\hat{j}$$

$\rightarrow v(t) = v_1(t) + v_2(t)$

$$\vec{V}_1 + \vec{V}_2 = \underbrace{(V_{1x} + V_{2x})}_{v(t) = v_1(t) + v_2(t)}\hat{i} + (V_{1y} + V_{2y})\hat{j}$$

Summation of the projections of V_1 and V_2 to the horizontal axis is the projection of the vectoral sum of V_1 and V_2 to the horizontal axis.

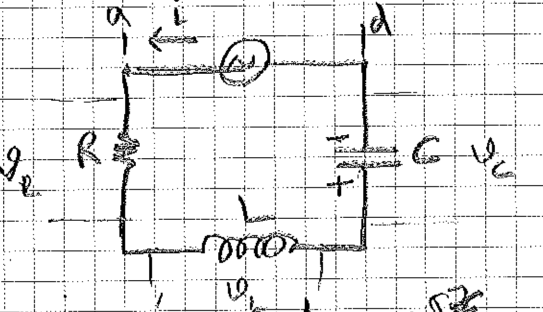
$\Rightarrow$ The phasor of $v_1(t) + v_2(t)$

is

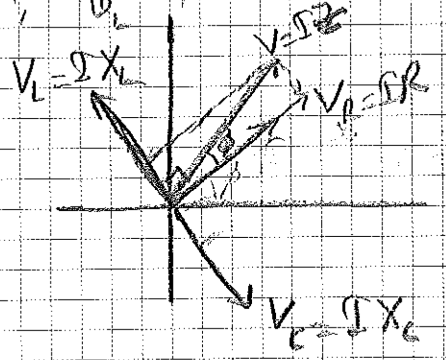
$$\vec{V}_1 + \vec{V}_2$$

31.3 L-R-C Series Circuit

Consider the circuit:



We will use a phasor diagram in order to solve for the circuit.



$$v_{ad} = V\cos(\omega t + \phi)$$

$$\Rightarrow \vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

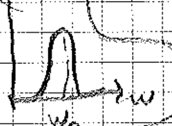
$$V = \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$

$$\boxed{V = I\sqrt{R^2 + (X_L - X_C)^2}}$$

We define the impedance Z of an AC circuit as:

$$V = IZ \Rightarrow \boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$ impedance of an L-R-C series circuit

Band pass filter:
$\omega \to \infty \quad Z \to \infty$
$\omega \to 0 \quad Z \to \infty$



$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Leftarrow \omega_R = \frac{1}{\sqrt{LC}}$$

∴ For a given amplitude V of the source voltage, the amplitude $I = \frac{V}{Z}$ will be different at different frequencies.

The angle ϕ between the voltage and current phasors is given by:

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \boxed{\frac{\omega L - \frac{1}{\omega C}}{R}}$$

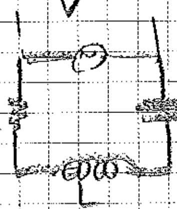
∴ If the current is $i = I\cos\omega t$, $\boxed{v = V\cos(\omega t + \phi) = IZ\cos(\omega t + \phi)}$

rms values are related as; $V_{rms} = \frac{V}{\sqrt{2}}$, $I_{rms} = \frac{I}{\sqrt{2}}$

$$= \frac{IZ}{\sqrt{2}}$$

$$\rightarrow \boxed{V_{rms} = I_{rms} Z}$$

Ex 31.4:



$R = 300\Omega$, $L = 60mH$, $C = 0.5\mu F$, $V = 50V$, $\omega = 10000 rad/s$

$\rightarrow X_L, X_C, Z, I, \phi, V_R, V_L, V_C = ?$

$X_L = \omega L = 10000 rad/s \times 60\times10^{-3} H = 600\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{10000 rad/s \; 0.5\times10^{-6} F} = 200\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\Omega)^2 + (400\Omega)^2} = 500\Omega$

$I = \frac{V}{Z} = \frac{50V}{500\Omega} = 0.1A$

$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{400}{300}\right) = 53°$

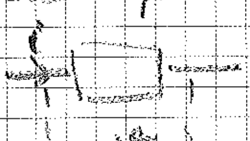
$V_R = IR = 0.1A \times 300\Omega = 30V$

$V_L = IX_L = 0.1A \times 600\Omega = 60V$

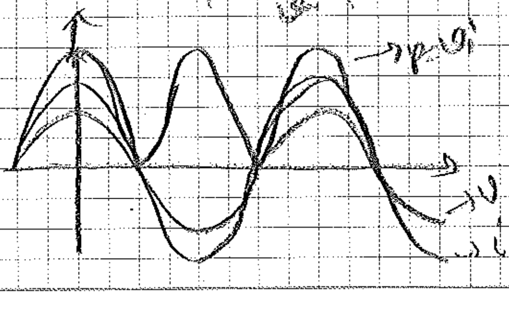
$V_C = IX_C = 0.1A \times 200\Omega = 20V$

31.4 Power in Alternating-Current Circuits

Instantaneous power delivered to a circuit element:

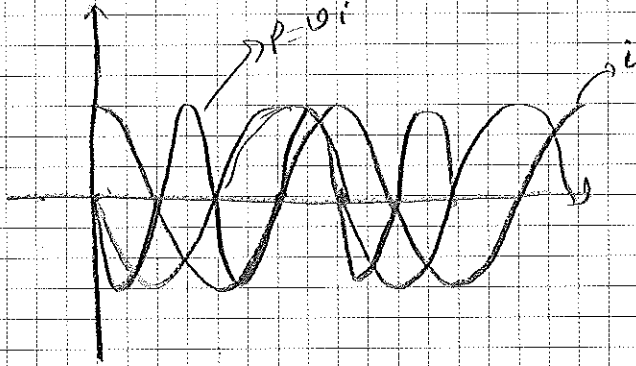


$p = vi$



$\leftarrow$ For a pure resistor

Pure inductor



For a pure resistor: $p = vi = VI\cos^2\omega t$

$$\Rightarrow P_{av} = VI\,\overline{\cos^2\omega t} = VI\left(\overline{\frac{1}{2}} + \overline{\frac{\cos 2\omega t}{2}}\right)$$

$$= \frac{VI}{2}$$

$$\Rightarrow \boxed{P_{av} = V_{rms} I_{rms}}$$

For an arbitrary circuit element:

$$p = vi = V\cos(\omega t + \phi) I\cos\omega t = VI\cos(\omega t + \phi)\cos(\omega t)$$

$$= VI\left\{\frac{1}{2}\left\{\cos(2\omega t + \phi) + \cos(\phi)\right\}\right\}$$

$$\Rightarrow P_{av} = \frac{VI}{2}\left\{\underbrace{\overline{\cos(2\omega t + \phi)}}_{0} + \overline{\cos(\phi)}\right\} = \frac{VI}{2}\cos\phi = \boxed{V_{rms} I_{rms}\cos\phi}$$

The factor $\cos\phi$ is called the power factor.

For a pure resistance $\phi = 0 \Rightarrow P_{av} = V_{rms} I_{rms}$

" " inductor $\phi = 90° \Rightarrow P_{av} = 0$

capacitor $\phi = -90° \Rightarrow P_{av} = 0$

no average power delivered to an inductor or a capacitor

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Ex 31.6: Power in a hair dryer

An electric hair dryer is rated $\overbrace{1500\,W}^{\text{average power}}$ at $\overbrace{120V}^{\text{rms voltage}}$

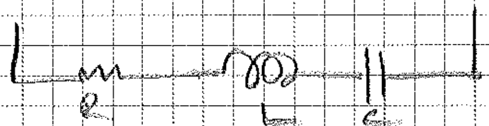
a) R=? $\quad P_{av} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} \Rightarrow R = \frac{(120V)^2}{1500W} = 9.6\Omega$

b) rms current ; $I_{rms} = \frac{V_{rms}}{R} = \frac{120V}{9.6\Omega} = 12.5A$

c) Maximum instant power

$$P_{max} = VI = 2 V_{rms} I_{rms} = 3000W$$

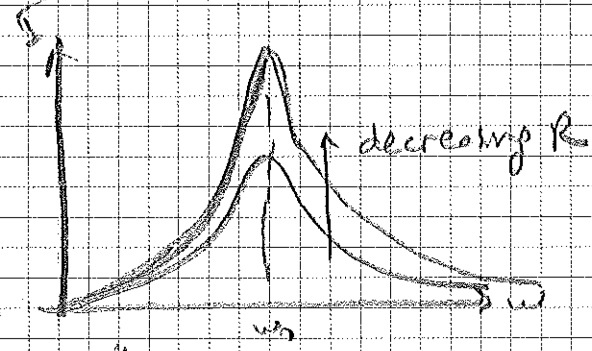
31.5 Resonance in Alternating Current Circuits

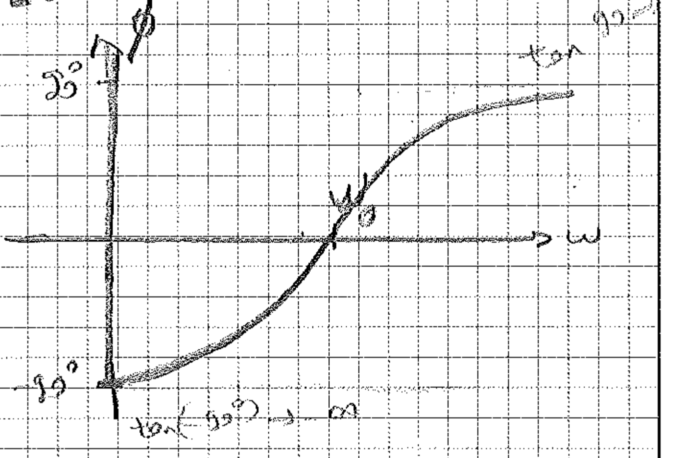


$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\tan\phi=\frac{X_L-X_C}{R}=\frac{\omega L-\frac{1}{\omega C}}{R},$$ angle ϕ between V and I

Z reaches to a minimum at $\omega_0=\frac{1}{\sqrt{LC}}$

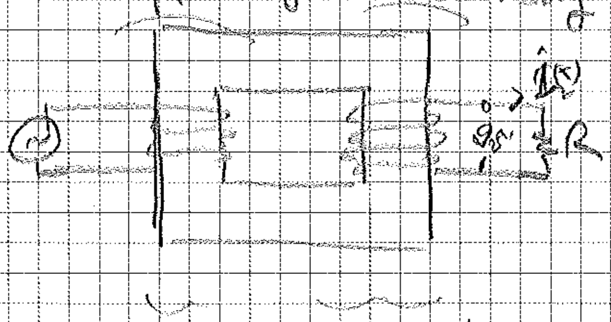




"Resonance"
Tuning is possible
by changing L or C.
"Analog Radio and Television receiving circuits"

31.6 Transformers

It is much easier to change voltage levels in AC as compared to DC.



Two coils of different windings.

Core made of a magnetizable material so that the magnetic field lines remain in the core.

When we assume that all the magnetic field lines are confined to the iron core, Φ_B is the same in both coils,

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_B}{dt}, \quad \mathcal{E}_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\Rightarrow \frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2} \Rightarrow \boxed{\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}}$$

Example: Power delivered from the primary:

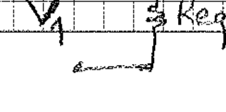
$$P_1 = \cancel{V_1 I_1} = \boxed{V_2 I_2}$$, there is no resistance in windings.

$$V_2 = V_1 \frac{N_2}{N_1} = I_2 R$$

$$\cancel{V_1 I_1} = \cancel{V_1} \frac{N_2}{N_1} V_1 \frac{N_2}{N_1} \frac{V_1}{R} \Rightarrow \boxed{\cancel{\frac{V_1}{I_1}} = \frac{R}{(N_2/N_1)^2}}$$

Transformer transforms the resistance

$$= \frac{(V_1)^2}{R} \left(\frac{N_2}{N_1}\right)^2$$



$$\Rightarrow P = V_1 \frac{V_1}{R_{eq}} = \frac{V_1^2}{R_{eq}} \Rightarrow \boxed{R_{eq} = \frac{R}{(N_2/N_1)^2}}$$