Section 1

Linear charge elasticity,$\lambda = \frac{Q}{\frac{2\pi R}{4}} = \frac{2Q}{\pi R}$

Infinitesimal charge,$dQ = \lambda dS$, $dS = rd\Theta$

field due to dQ :$dE = \frac{\lambda}{4\pi\epsilon_0}\frac{d\Theta}{r^2}$

Components of the field

$dE_x = dE\cos(\Theta) \rightarrow E_x = \int dE_x$

$dE_y = dE\sin(\Theta) \rightarrow E_y = \int dE_y$

The electric field at P:

$$dE_x = \frac{1}{2\pi^2\epsilon_0}\frac{Q}{r^2}\cos\Theta d\Theta$$

$$\begin{aligned} E_x &= \int dE_x \\ &= \frac{1}{2\pi^2\epsilon_0}\frac{Q}{R^2}\int_0^{\pi/2}\cos\Theta d\Theta \\ &= \frac{1}{2\pi^2\epsilon_0}\frac{Q}{R^2} \end{aligned}$$

$$\begin{aligned} E_y &= \int dE_y \\ &= \frac{1}{2\pi^2\epsilon_0}\frac{Q}{R^2}\int_0^{\pi/2}\sin\Theta d\Theta \\ &= \frac{1}{2\pi^2\epsilon_0}\frac{Q}{R^2} \end{aligned}$$

Section 2

Linear charge elasticity,$\lambda = \frac{Q}{\frac{2\pi R}{3}} = \frac{3Q}{2\pi R}$

Infinitesimal charge,$dQ = \lambda dS$, $dS = Rd\Theta$

field due to dQ :$dE = \frac{\lambda}{4\pi\epsilon_0}\frac{d\Theta}{R^2}$

Components of the field

$$dE_x = dE\cos(\Theta)$$

$$= \frac{-3}{8\pi^2\epsilon_0}\frac{Q}{R^2}\cos\Theta d\Theta$$

$$E_x \quad = \int dE_x = \frac{-3}{8\pi^2\epsilon_0}\frac{Q}{R^2}\int_{-\pi/3}^{\pi/3}\cos\Theta d\Theta \quad = \sin\Theta|_{-\pi/3}^{\pi/3}$$

$$E_x = \frac{3\sqrt{3}}{8\pi^2\epsilon_0}\frac{Q}{R^2}\hat{i}$$

$$dE_y = dE\sin\Theta \rightarrow E_y = \int dE_y = 0$$

all vertical components will cancel out due to the symmetry of the arc

Section 3

Linear charge elasticity,$\lambda = \frac{Q}{2\pi R/4} = \frac{2Q}{\pi R}$

Infinitesimal charge,$dQ = \lambda dS$, $dS = rd\Theta$

field due to dQ :$dE = \frac{\lambda}{4\pi\epsilon_0}\frac{d\Theta}{r^2}$

Let dE_+ be the field due to dQ_+ and let dE_-, be the field due to dQ_- then field at point P is superposition of these two $dE_+ + dE_-$

Due to symmetry of the problem, horizontal components cancel out. So,

$$E_x = 0$$

$$dE_y = dE_{+,y} + dE_{-,y} = 2dE_{+,y}$$

$$= 2\frac{1}{4\pi\epsilon_0}\frac{dQ_+}{r^2}\cos\Theta$$

$$E_y = \int dE_y = \frac{1}{\pi\epsilon_0}\frac{Q}{r^2}(\sin\Theta|_0^{\pi/2})\hat{j}$$

$$E_y = -\frac{1}{\pi^2\epsilon_0}\frac{Q}{r^2}$$

Section 4

Linear charge elasticity,$\lambda = \frac{Q}{\pi r}$

Infinitesimal charge,$dQ = \lambda dS$, $dS = rd\Theta$

field due to dQ :$dE = \frac{\lambda}{4\pi\epsilon_0}\frac{d\Theta}{r}$

Components of the field

$$dE_x = dE\sin(\Theta)$$

$$= \tfrac{1}{4\pi^2\epsilon_0}\tfrac{Q}{r^2}\sin\Theta d\Theta$$

$$E_x \quad = \textstyle\int_0^{pi} dE_x = -\tfrac{1}{4\pi^2\epsilon_0}\tfrac{Q}{r^2}\cos\Theta|_0^{pi}$$

$$E_x = \frac{1}{2\pi^2\epsilon_0}\frac{Q}{r^2}$$

$$dE_y = dE\sin\Theta \rightarrow E_y = \int dE_y = 0$$

all vertical components will cancel out due to the symmetry of the arc

Section 5

$E_X = \frac{1}{2\pi^2}$

Linear charge elasticity,$\lambda = \frac{Q}{\pi r}$

Infinitesimal charge,$dQ = \lambda dS$, $dS = rd\Theta$

field due to dQ :$dE = \frac{\lambda}{4\pi\epsilon_0}\frac{d\Theta}{r}$

Components of the field

$$\begin{aligned} dE_x &= dE\cos(\Theta) \\ &= \frac{1}{2\pi^2\epsilon_0}\frac{Q}{R^2}\cos\Theta d\theta \end{aligned}$$

$dE_y = dE\sin(\Theta) \rightarrow E_y = \int dE_y = 0$ The y component of the electric field is zero due to the symmetry of the problem

The electric field at P:

$$\begin{aligned} E &= \int_0^\pi [\frac{1}{4\pi^2\epsilon_0}\frac{Q}{r^2}\sin\Theta d\Theta]\hat{i} \\ &= -[\frac{1}{4\pi^2\epsilon_0}\frac{Q}{r^2}\cos\Theta]_0^\pi\hat{i} \\ &= \frac{1}{2\pi^2\epsilon_0}\frac{Q}{r^2}\hat{i} \end{aligned}$$