

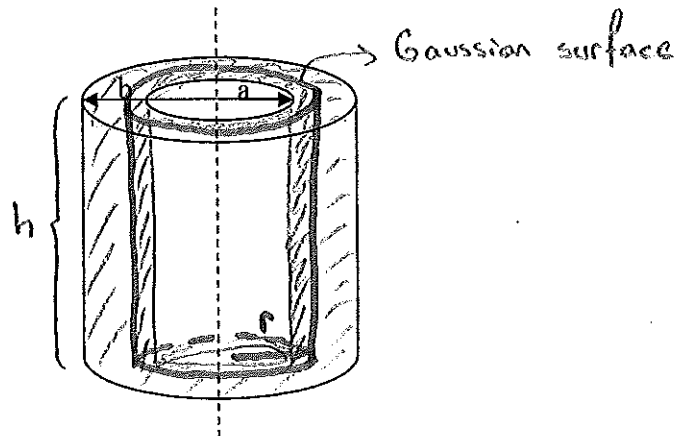
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

A very long hollow cylinder with inner radius a and outer radius b has positive charge uniformly distributed throughout it, with charge per unit volume ρ . Derive the expression for the electric field inside the volume at a distance r from the axis of the cylinder ($a < r < b$).



Gauss' Law:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

Φ_{total}

$$\Phi_{total} = \cancel{\Phi_{top}} + \cancel{\Phi_{bot}} + \Phi_{side}$$

\downarrow
 $E \cdot 2\pi r \cdot h$

$$\Phi_{total} = E 2\pi r h$$

$$Q_{enc} = \rho \cdot V_{enc}$$

\downarrow

(Volume inside the Gaussian surface - Volume of cavity of radius a)

$$V_{enc} = \pi r^2 h - \pi a^2 h = \pi h (r^2 - a^2)$$

$$Q_{enc} = \rho \pi h (r^2 - a^2)$$

$$E \cdot 2\pi r h = \frac{\rho \pi h (r^2 - a^2)}{\epsilon_0} \quad (\text{Gauss' Law})$$

$$\vec{E} = \frac{\rho (r^2 - a^2)}{2\epsilon_0 r} \hat{r}$$

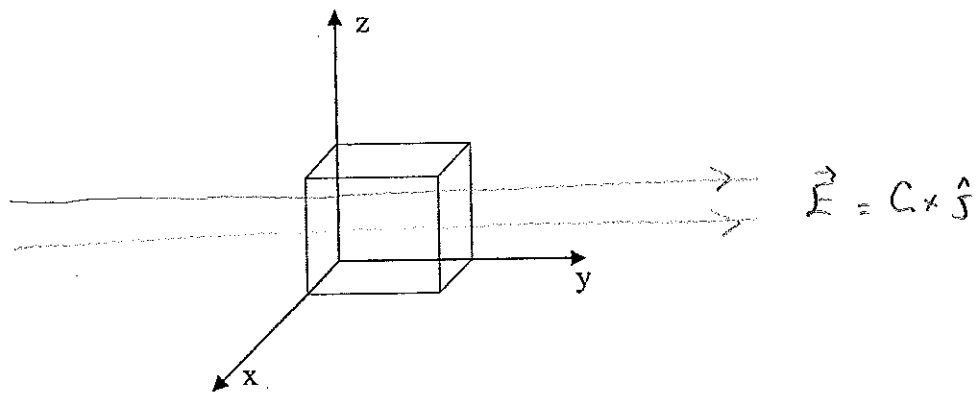
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

A cube has sides of length L . It is placed with one corner at the origin as shown below. The electric field is non-uniform and given by $\vec{E} = Cx\hat{j}$, where C is a positive constant, and x represents the x -coordinate. Find the total outward electric flux through the surface of the cube as a function of L , and C . Based on the Gauss' Law comment on whether total charge enclosed by the cube is zero or not.



$$\Phi_{\text{total}} = \Phi_{\text{up}} + \Phi_{\text{bottom}} + \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{left}} + \Phi_{\text{right}}$$

Φ_{up} : $(\hat{n} \text{ is in } +z \text{-direction})$
 $\Phi_{\text{up}} = E \cdot A \cdot \cos\theta = CxL^2 \cdot \cos 90^\circ = 0$

Φ_{bottom} : $(\hat{n} \text{ is in } -z \text{ direction})$
 $\Phi_{\text{bottom}} = E \cdot A \cdot \cos\theta = CxL^2 \cos 90^\circ = 0$

Φ_{front} : $(\hat{n} \text{ is in } +x \text{ direction})$
 $\Phi_{\text{front}} = E \cdot A \cdot \cos\theta = CxL^2 \cos 90^\circ = 0$

Φ_{back} : $(\hat{n} \text{ is in } -x \text{ direction})$
 $\Phi_{\text{back}} = E \cdot A \cdot \cos\theta = CxL^2 \cos 90^\circ = 0$

continues on the back

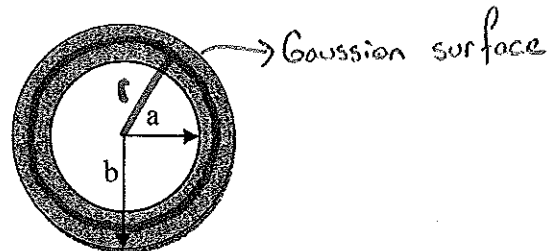
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

An insulating spherical shell with inner radius a and outer radius b has positive charge uniformly distributed throughout it, with charge per unit volume ρ . Derive the expression for the electric field inside the volume at a distance r from the center of the sphere ($a < r < b$).



Gauss' Law:

$$\underbrace{\oint \vec{E} \cdot d\vec{a}}_{\Phi_{\text{total}}} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\boxed{\Phi_{\text{total}} = E \cdot 4\pi r^2}$$

$$Q_{\text{enc}} = \rho \cdot V_{\text{enc}}$$

(Volume inside the Gaussian surface - volume of the cavity of radius a)

$$V_{\text{enc}} = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3$$

$$\boxed{Q_{\text{enc}} = \frac{4}{3}\pi \rho (r^3 - a^3)}$$

Gauss' Law:

$$E \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi \rho (r^3 - a^3)}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho (r^3 - a^3)}{3\epsilon_0 r^2} \hat{r}}$$

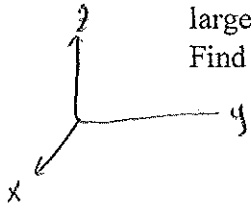
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

A small sphere with a mass m and carrying a charge q hangs from a thread near a very large, charged conducting sheet as shown below. The charge density on the sheet is $-\sigma$. Find the angle of the thread as a function of m , g , q , σ , and ϵ_0 .



Gauss' Law:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\Phi_{total} = \Phi_{left} + \Phi_{right} + \cancel{\Phi_{side}}$$

\downarrow \downarrow
 $E \cdot \pi r^2$ $E \cdot \pi r^2$

$$\Phi_{total} = 2E\pi r^2$$

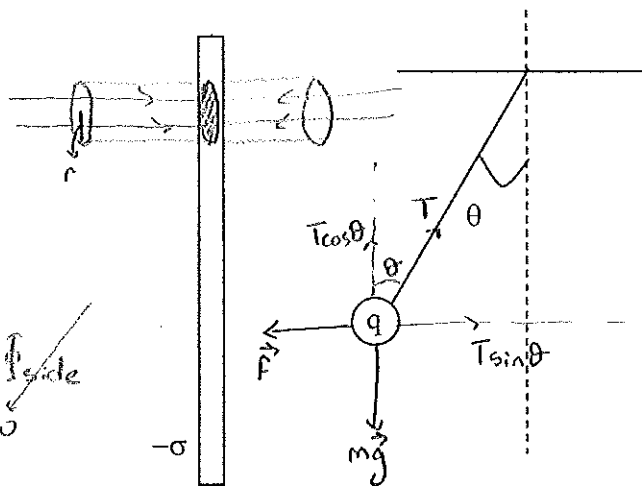
$$Q_{enc} = \sigma \pi r^2$$

$$2E\pi r^2 = \frac{\sigma \pi r^2}{\epsilon_0}$$

$$\vec{E} = \pm \frac{\sigma}{2\epsilon_0} \hat{y}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{F} = \vec{E} \cdot q = \left[-\frac{\sigma}{2\epsilon_0} \cdot q \hat{y} = \vec{F} \right]$$



$$\Sigma f_x = 0$$

$$\Sigma f_y = 0$$

$$F = \frac{\sigma}{2\epsilon_0} q = T \sin \theta$$

$$mg = T \cos \theta$$

$$\tan \theta = \frac{\sigma q}{2\epsilon_0 mg}$$

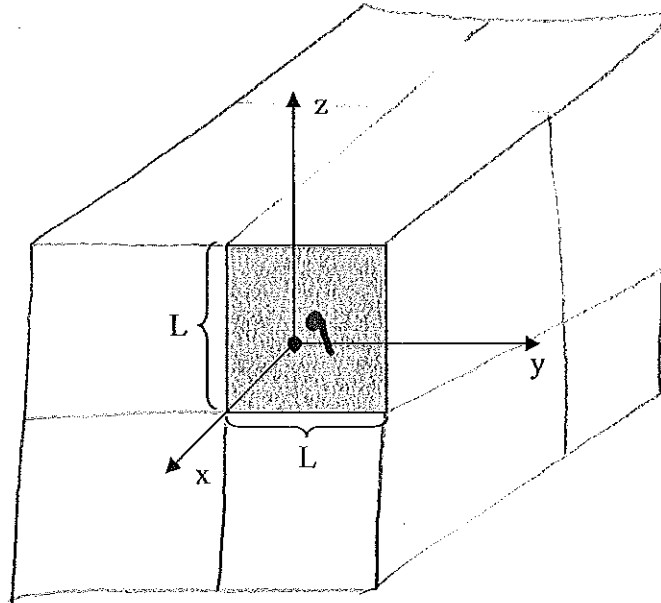
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

A flat, square surface with sides of length L (shown below) is described by the equations $x = L$, $0 \leq y \leq L$, and $0 \leq z \leq L$. Find the electric flux through the square due to a positive point charge q located at the origin as a function of q and ϵ_0 . (Hint: Think of the square as part of a cube centered on the origin)



Consider the cube itself as a Gaussian surface.

$$\text{Gauss' Law } \Phi_{\text{total}} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\Phi_{\text{total}} = \frac{q}{\epsilon_0}$$

The cube consist of 24 ($L \times L$)-square, since the charged particle is at the center, every little square has the same flux, because their distance from the particle are the same.

$$\boxed{\Phi_{\text{square}} = \frac{q}{24\epsilon_0}}$$