PHYS 102: General Physics 2 KOÇ UNIVERSITY Spring Semester 2012

College of Sciences

Section 1 Quiz 4 08 March 2012

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: Student ID: Signature:

A commercial coaxial (coax) cable has capacitance per unit length about 60 pF/m. If the outer radius of the coax cable is about 15mm, (i) derive the capacitance per unit length in relation to the dimensions of the cable; (ii) estimate the radius of the inner core. (You may assume there is only air between the outer and the inner conductors and take the permittivity constant as 10 pF/m. You may also take π=3, e = 3.)

(i) If the cable has inner radius r_a, outer radius r_b and length L, then by Gauss law the electric field inside the cable is

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0} \Rightarrow E\,2\pi r L = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{2\pi r L \epsilon_0} \text{ (radially outward)}$$

$$V_{ab} = \int_a^b \vec{E}\cdot d\vec{l} = \frac{Q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$\text{Capacitance per unit length} = \frac{C}{L} = \frac{Q}{V_{ab}L} = \frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)}$$

(ii) $\frac{C}{L}$ is given as 60 pF

$$\Rightarrow \frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)} = 60\,\text{pF} \Rightarrow \frac{2\cdot 3\cdot 10\,\text{pF/m}}{\ln\left(\frac{r_b}{r_a}\right)} = 60\,\text{pF}$$

$$\Rightarrow \ln\left(\frac{r_b}{r_a}\right) = 1$$

$$\Rightarrow \frac{r_b}{r_a} = e = 3$$

$$\Rightarrow \frac{15\,\text{mm}}{r_a} = 3 \Rightarrow r_a = 5\,\text{mm}$$

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Section 2 Quiz 4 08 March 2012

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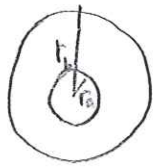
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A simple model of an atom consists of electron clouds located far away from positively charged nucleus. Ignore the nucleus and assume the electron cloud can be considered as a spherical shell of radius ~100 pm (picometer, pico = 0.000000000001). If some electrons jump to higher clouds, that you may assume at infinitely far away, argue that the atom can be described as an isolated spherical capacitor. (i) Derive the formula for the capacitance of an isolated spherical capacitor; (ii) using your result estimate the capacitance of an atom. (You may take the permittivity constant as 10 pF/m. You may also take π=3).

(i)



r_a: radius of inner sphere
r_b: radius of outer sphere

$$\oint_{\substack{\text{Sphere}\\ \text{with radius } r}} \vec{E}\cdot d\vec{A} = \int_{\text{Sphere}} E_r\, dA = E_r 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V_{ab} = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\,\frac{r_b - r_a}{r_a r_b}$$

$$\Rightarrow C = \frac{Q}{V_{ab}} = \frac{4\pi\varepsilon_0 r_a r_b}{(r_b - r_a)}$$

(ii) $r_a = 100$ pm, $r_b \to \infty$

$$\Rightarrow V_{ab} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a}\right) \Rightarrow C = 4\pi\varepsilon_0 r_a$$

$$= (4)(3)(10\,\text{pF/m})(100\,\text{pm})$$

$$= 12\times 10^{-9}\ \text{pF}$$

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Section 3 Quiz 4 08 March 2012

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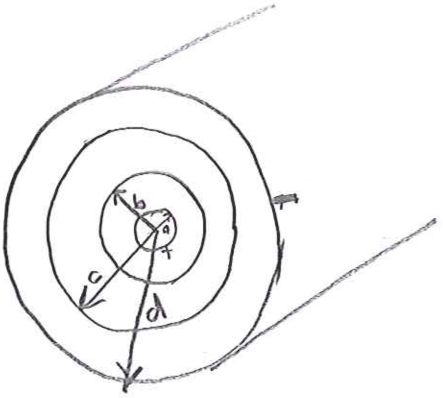
Quiz duration: 15 minutes

Name: Student ID: Signature:

Calculate the capacitance per unit length of a cable consisting of 4 coaxial cylinders of radii a<b<c<d.



We will first calculate capacitance of a single cylindrical capacitor with inner radius r_1 outer radius r_2 and length L.

Using Gauss law electric field can be found as:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \Rightarrow E\,2\pi r L = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{2\pi r L \varepsilon_0} \text{ (radially outward)}$$

and $V_{12} = \int_{r_1}^{r_2} \vec{E}\cdot d\vec{l} = \frac{Q}{2\pi\varepsilon_0 L}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{r_2}{r_1}\right)$

gives the potential difference between the cylindrical shells.

Assumption: Voltage is applied to the most inner and outer cylinders, having radius a and radius d respectively.

Capacitance per unit length $\rightarrow C = \frac{C}{L} = \frac{Q}{V_{12}L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_2}{r_1}\right)}$

The system ~~can~~ is equivalent to three cylindrical capacitors connected in series.

$$\frac{1}{C} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}}$$

$$= \frac{\ln\left(\frac{r_b}{r_a}\right)}{2\pi\varepsilon_0} + \frac{\ln\left(\frac{r_c}{r_b}\right)}{2\pi\varepsilon_0} + \frac{\ln\left(\frac{r_d}{r_c}\right)}{2\pi\varepsilon_0} = \frac{\ln\left(\frac{r_b}{r_a}\cdot\frac{r_c}{r_b}\cdot\frac{r_d}{r_c}\right)}{2\pi\varepsilon_0}$$

$$\Rightarrow C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_d}{r_a}\right)}$$

Remark: We see that the result is same as a cable consisting of two coaxial cylinders. The reason is that the cylinders with radius b and c are equipotential surfaces which will not give a contribution.

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Section 4 **Quiz 4** **08 March 2012**

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Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

Keys on a typical computer keyboard operate as parallel plate capacitors. Computer can detect the changes in their capacitances to check if a key is pressed or not. Assume each key is of area ~1 cm^2 and they can be pressed from 5mm down to ~ 0.1 mm. Dielectric constant of the material that fills the capacitors is given to be ~ 4. Calculate the change in the capacitance when a key is pressed. (You may take the permittivity constant as 10 pF/m and neglect 0.1 next to 5.)

C_1: capacitance before pressed
C_2: capacitance after pressed

$$C_1 = \frac{K\varepsilon_0 A_1}{d_1} = \frac{(4)(10\ pF/m)(1\times10^{-4}m^2)}{(5\times10^{-3}m)} = 0.8\ pF$$

$$C_2 = \frac{K\varepsilon_0 A_2}{d_2} = \frac{(4)(10\ pF/m)(1\times10^{-4}m^2)}{(1\times10^{-4}m)} = 40\ pF$$

$$\Delta C = C_2 - C_1 = 39.2\ pF$$

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Section 5 **Quiz 4** **08 March 2012**

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Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

A conducting layer of atmosphere, ionosphere, at about 60 km altitude surrounds Earth. (i) Assume the ground and the ionosphere are perfect spherical conductors, and calculate the capacitance. (ii) As the Earth has much larger radius than the altitude of the layer, show that your result reduces to that of a parallel plate system. (iii) If the potential difference between the ionosphere and the ground is about 6 MV, estimate the energy stored in the earth-ionosphere capacitor. (You may take the permittivity constant as 10 pF/m. You may also take π=3).

r_a: radius of inner sphere $\quad r_{Earth} \approx 6000$ km

r_b: radius of outer sphere

Electric field at a distance r such that $r_a < r < r_b$:

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Potential difference between the spheres:

$$V_{ab} = \int_a^b \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \Rightarrow C = \frac{4\pi\varepsilon_0 r_a r_b}{r_b - r_a}$$

(i) $$C = \frac{(4)(3)(10\,pF/m)(r_{Earth})(r_{Earth}+60\,km)}{60\,km} = 72{,}72\times10^9\,pF$$

(ii) If $r_a \gg r_b - r_a$ then $r_a \approx r_b$, $\quad r := r_a \approx r_b$

$d := r_b - r_a$

$$\Rightarrow C = \frac{4\pi\varepsilon_0 (r_a)(r_a)}{(r_b - r_a)} = \frac{4\pi\varepsilon_0 r^2}{d} = \frac{\varepsilon_0 A}{d}$$

($4\pi r^2$ → Area of sphere)

(iii) $$U = \frac{1}{2}CV^2 = \frac{1}{2}(72{,}72\times10^9\times10^{-12}F)(6\times10^6\,V)^2$$

$$= 1326{,}96\times10^8 \text{ joule}$$