

Closed book. No calculators are to be used for this quiz.  
Quiz duration: 15 minutes

Name:

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Signature:

A particle with a charge of  $-1.24 \times 10^{-8} \text{ C}$  is moving with instantaneous velocity  $\vec{v} = (4.19 \times 10^4 \text{ m/s})\hat{i} + (-3.85 \times 10^4 \text{ m/s})\hat{j}$ . What is the force exerted on this particle by a magnetic field

a)  $\vec{B} = (1.40 \text{ T})\hat{i}$  and

b)  $\vec{B} = (1.40 \text{ T})\hat{k}$ ?



Magnetic Force on a Moving Charge:

$$\vec{F} = q \vec{v} \times \vec{B}$$

a) 
$$\vec{F} = (-1.24 \times 10^{-8} \text{ C}) [4.19 \times 10^4 \text{ m/s} \hat{i} - 3.85 \times 10^4 \text{ m/s} \hat{j}] \times (1.40 \text{ T} \hat{i})$$

since  $\hat{i} \times \hat{i} = 0$  &  $-\hat{j} \times \hat{i} = \hat{k}$

$$= -1.24 \times 3.85 \times 1.40 \times 10^4 \text{ N} \hat{k}$$

b) 
$$\vec{F} = (-1.24 \times 10^{-8} \text{ C}) [4.19 \times 10^4 \text{ m/s} \hat{i} - 3.85 \times 10^4 \text{ m/s} \hat{j}] \times (1.40 \text{ T} \hat{k})$$

$$\begin{aligned} \hat{i} \times \hat{k} &= -\hat{j} \\ -\hat{j} \times \hat{k} &= -\hat{i} \end{aligned}$$

$$= + [1.24 \times 4.19 \times 1.40 \times 10^4 \text{ N} \hat{j} + 1.24 \times 3.85 \times 1.40 \text{ N} \hat{i}]$$

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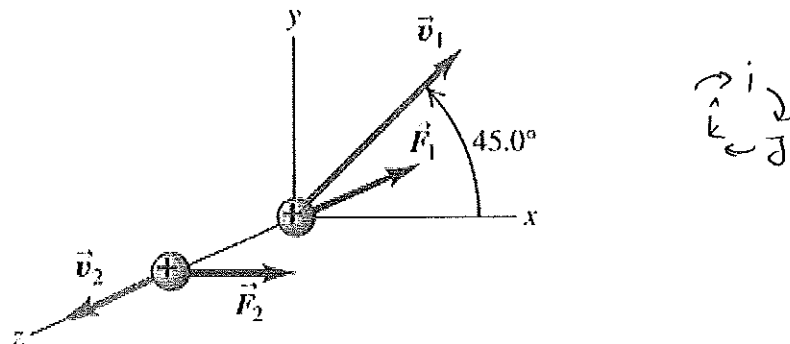
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When a particle of charge  $q > 0$  moves with a velocity of  $\vec{v}_1$  at  $45^\circ$  from the x axis in the xy-plane, a uniform magnetic field exerts a force  $\vec{F}_1$  along the  $-z$ -axis as shown in the figure. When the same particle moves with a velocity  $\vec{v}_2$  with the same magnitude as  $\vec{v}_1$  but along the  $+z$ -axis, a force  $\vec{F}_2$  of magnitude  $F_2$  is exerted along the  $+x$ -axis.

- What are the magnitude (in terms of  $q$ ,  $v$ , and  $F_2$ ) and direction of the magnetic field?
- What is the magnitude of  $\vec{F}_1$  in terms of  $F_2$ ?



let  $|\vec{v}_1| = v$  , so  $\vec{v}_1 = v \cos(45^\circ) \hat{i} + v \sin(45^\circ) \hat{j}$   
 $\vec{v}_1 = \frac{v\sqrt{2}}{2} (\hat{i} + \hat{j})$

$\vec{F}_1 = q \vec{v}_1 \times \vec{B}$  let  $\vec{B}$  be a generic field s.t  
 $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$\vec{F}_1 = q \frac{\sqrt{2}}{2} v (\hat{i} + \hat{j}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$   
 $= \frac{q\sqrt{2}}{2} v (B_y \hat{k} - B_z \hat{j} - B_x \hat{k} + B_z \hat{i})$  (1)

Since  $\vec{F}_1$  lies along the  $-z$ -axis:  $\vec{F}_1 = |\vec{F}_1| \hat{k}$   
 Thus, in the RHS of eqn.(1) only terms w/  $\hat{k}$  lives.

$B_z = 0 \Rightarrow (\vec{B} = B_x \hat{i} + B_y \hat{j})$   
 $\vec{F}_1 = \frac{q\sqrt{2}}{2} v (B_y - B_x) \hat{k}$  ( $B_y < B_x$ )

When the particle moves with

$$\vec{v}_2 = v \hat{k}$$

$$\begin{aligned}\vec{F}_2 &= q \vec{v}_2 \times \vec{B} \\ &= q v \hat{k} \times [B_x \hat{i} + B_y \hat{j}]\end{aligned}$$



$$= qv [B_x \hat{j} - B_y \hat{i}]$$

But  $\vec{F}_2 = |\vec{F}_2| \hat{i}$

$$\left\{ \Rightarrow B_x = 0 \Rightarrow \vec{B} = B_y \hat{j} \right.$$

And

$$|\vec{F}_2| = qv (-B_y) \Rightarrow B_y = \frac{-|\vec{F}_2|}{qv}$$

$$\vec{B} = -\frac{|\vec{F}_2|}{qv} \hat{j} \quad ; \text{ towards } -y \text{ axis}$$

with  $B_x = 0$

$$\begin{aligned}|\vec{F}_1| &= +qv \frac{\sqrt{2}}{2} B_y \\ \frac{|\vec{F}_1|}{|\vec{F}_2|} &= \frac{+qv \frac{\sqrt{2}}{2} B_y}{qv B_y}\end{aligned}$$

$$\left\{ \Rightarrow \frac{|\vec{F}_1|}{|\vec{F}_2|} = \frac{\sqrt{2}}{2} \right.$$

$$|\vec{F}_1| = \frac{\sqrt{2}}{2} |\vec{F}_2|$$

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A particle with charge  $q$  is moving with speed  $v$  in the  $-y$ -direction. It is moving in a uniform magnetic field  $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ .

- What are the components of the force  $\vec{F}$  exerted on the particle by the magnetic field?
- If  $q > 0$ , what must the signs of the components of  $\vec{B}$  if the components of  $\vec{F}$  are all nonnegative?
- If  $q < 0$ , and  $B_x = B_y = B_z > 0$ , find the direction of  $\vec{F}$  and find the magnitude of  $\vec{F}$  in terms of  $|q|$ ,  $v$ ,  $B_x$

$$\vec{v} = -v\hat{j}, \quad v > 0$$

$$a) \vec{F} = q\vec{v} \times \vec{B} = -qv\hat{j} \times [B_x\hat{i} + B_y\hat{j} + B_z\hat{k}]$$

$$= -qv(-B_x\hat{k} + B_z\hat{i})$$

$$= qvB_z\hat{i} + qvB_x\hat{k}$$

On the other hand,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

So  $F_x = qvB_z$ ,  $F_y = 0$  and  $F_z = qvB_x$

b) If  $q > 0$  and  $F_x, F_y, F_z \geq 0$

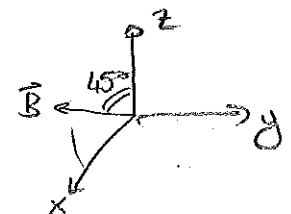
Then  $\begin{cases} -qvB_z \geq 0 \\ \downarrow \downarrow \\ B_z < 0 \end{cases} \Rightarrow B_z < 0$

$qvB_x \geq 0 \Rightarrow B_x \geq 0$ , sign of  $B_y$  is arbitrary.

c) If  $q < 0$  and  $B_x = B_y = B_z > 0$

$$\vec{F} = -qvB_x\hat{i} + qvB_x\hat{k} = -qvB_x(\hat{i} - \hat{k})$$

$$|\vec{F}| = qvB_x\sqrt{2}$$



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An open plastic bottle with an opening diameter of 2.5 cm is placed on a table. A uniform 1.75 T magnetic field directed upward parallel to vertical encompasses the bottle. What is the total magnetic flux through the plastic of the bottle.

diameter,  $R = 2.5 \times 10^{-2} \text{ m}$

radius,  $r = \frac{R}{2} = 1.25 \times 10^{-2} \text{ m}$

Gauss law for Magnetic fields

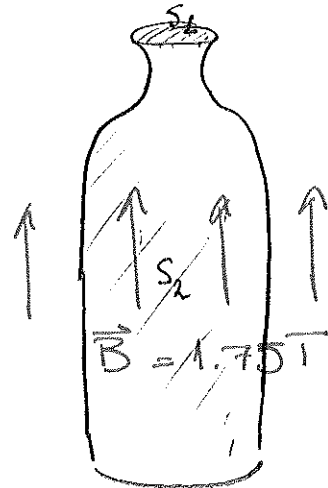
$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\int_{S_1} \vec{B} \cdot d\vec{A} + \int_{S_2} \vec{B} \cdot d\vec{A} = 0$$

$$\Rightarrow \int_{S_2} \vec{B} \cdot d\vec{A} = - \int_{S_1} \vec{B} \cdot d\vec{A}$$

$$= - B \int_{S_1} dA = \pi r^2 ?$$

$$= - 1.75 \text{ T} \times \pi (1.25 \times 10^{-2} \text{ m})^2$$



The total surface,  $S \equiv S_1 + S_2$

$S_1$ : opening surface

$S_2$ : plastic surface.

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A particle with charge  $q$  and initial velocity  $\vec{v}_0 = v_{x0}\hat{i} + v_{y0}\hat{j}$  enters a region of uniform electric and magnetic fields. The magnetic field in the region is  $\vec{B} = B_x\hat{i} + B_z\hat{k}$ . Calculate the magnitude and direction of the electric field in the region if the particle is to pass through undeflected.

Force due to magnetic field

$$\begin{aligned}\vec{F}_M &= q \vec{v}_0 \times \vec{B} = q (v_{x0}\hat{i} + v_{y0}\hat{j}) \times (B_x\hat{i} + B_z\hat{k}) \\ &= q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_{x0} & v_{y0} & 0 \\ B_x & 0 & B_z \end{vmatrix} = q (v_{y0}B_z)\hat{i} - q (v_{x0}B_z)\hat{j} \\ &= q (B_x v_{y0})\hat{k}\end{aligned}$$

Force due to electric field,

$$\vec{F}_E = q\vec{E} = q(E_x\hat{i} + E_y\hat{j} + E_z\hat{k})$$

If the particle passes through the region undeflected the net force on the particle is zero!

$$\text{So } \vec{F}_E = -\vec{F}_M$$

$$q(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) = -q[-v_{y0}B_z\hat{i} + v_{x0}B_z\hat{j} + B_x v_{y0}\hat{k}]$$

$$\Rightarrow E_x = -v_{y0}B_z, \quad E_y = v_{x0}B_z, \quad E_z = B_x v_{y0}$$

Thus,

$$\vec{E} = -v_{y0}B_z\hat{i} + v_{x0}B_z\hat{j} + B_x v_{y0}\hat{k}$$