

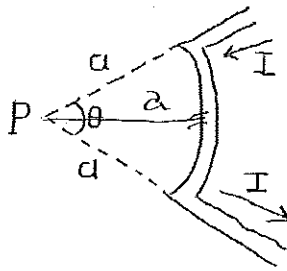
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

Calculate the magnetic field at point P created by the current-carrying wire segment shown in the figure. The wire consists of two straight portions and a circular arc of radius a , which subtends an angle θ .



Note that two straight segments will not produce any magnetic field, since their current moves toward point P.

Using Biot Savart we have:

$$dB = \frac{\mu_0 I dl}{4\pi a^2} \quad ; \quad dl = a d\theta$$

$$dB = \frac{\mu_0 I dl}{4\pi a^2} = \frac{\mu_0 I a d\theta}{4\pi a^2} = \frac{\mu_0 I d\theta}{4\pi a}$$

$$B = \int_0^\theta \frac{\mu_0 I}{4\pi a} d\theta = \boxed{\frac{\mu_0 I \theta}{4\pi a}} = \vec{B} \rightarrow \text{into the page}$$

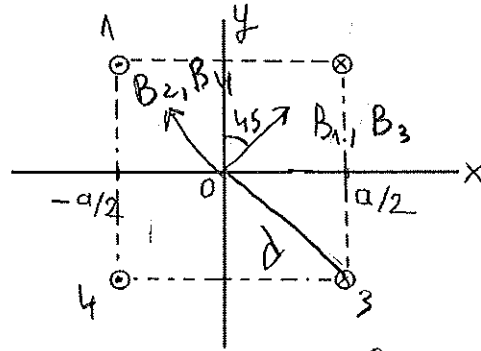
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Four wires each carrying a current of magnitude I are shown in the figure. The wires are located at the four corners of a square with side a . Two of the wires are carrying a current into the page, and the other two are carrying current out of the page. Calculate the y -component of the magnetic field at the center of the square.



Let's first draw the four forces and their y components:

$$B_y = B_{1y} + B_{2y} + B_{3y} + B_{4y} = 4 B_{1y} = 4 B_1 \cos 45 =$$

$$= 4 \cdot \frac{\mu_0 I}{2\pi d} \cdot \frac{\sqrt{2}}{2} = \frac{2\sqrt{2} \mu_0 I}{\sqrt{2} \pi d}$$

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{2} \left(\frac{a}{2}\right) = \frac{a}{\sqrt{2}}$$

$$B_y = \frac{\sqrt{2} \mu_0 I}{\pi \frac{a}{\sqrt{2}}} = \boxed{\frac{2 \mu_0 I}{\pi a} = B_y}$$

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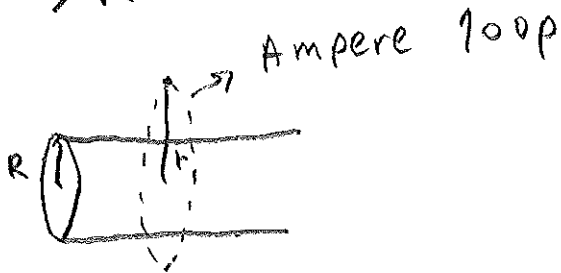
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A long straight wire of radius R carries a steady current I that is uniformly distributed through the circular cross section of the wire. Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Since we have cylindrical symmetry we will use the Ampere's law:

i) $r \geq R$



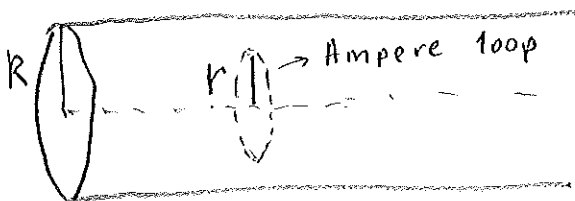
We choose circular Amperian loop passing through arbitrary point at distance r from the center:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$I_{enc} = I \Rightarrow B \oint dl = \mu_0 I_{enc} \Rightarrow$$

$$\Rightarrow B \cdot 2\pi r = \mu_0 I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

ii) $r < R$



Again we choose Ampere loop at distance r :

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$I_{enc} = \int \vec{j} \cdot \vec{dA} = j \pi r^2$$

where j is current per cross-section (area).

$$I = j \pi R^2 \Rightarrow j = \frac{I}{\pi R^2}$$

$$I_{enc} = \frac{I}{\pi R^2} \cdot \pi r^2 \Rightarrow I_{enc} = \frac{I r^2}{R^2}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} \Rightarrow B \cdot 2\pi r = \frac{\mu_0 I r^2}{R^2} \Rightarrow$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

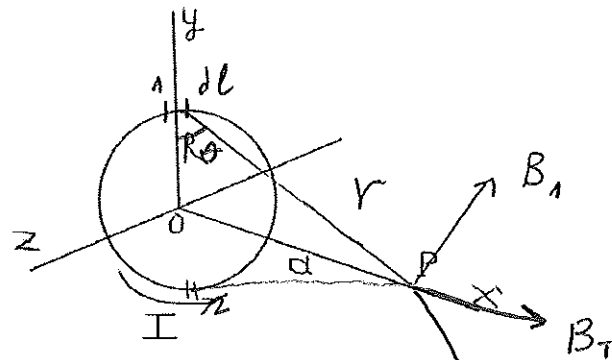
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Consider a circular wire loop of radius R located in the yz plane and carrying a steady current I as shown in the figure. Calculate the magnetic field at an axial point P a distance a from the center of the loop.



Due to the symmetry total field will be in the x direction. Thus it is enough to integrate dB_x . We will use the Biot-Savart law:

$$dB_x = dB \cos \theta = \frac{\mu_0 I}{4\pi} \frac{dl}{r^2} \cdot \frac{R}{r} \quad ; \quad dl = R d\theta$$

From Pitagora's theorem we have:

$$r = \sqrt{R^2 + a^2}$$

$$B_x = \int_0^{2\pi} dB_x = \int_0^{2\pi} \frac{\mu_0 I}{4\pi} \frac{R^2}{(R^2 + a^2)^{3/2}} d\theta = \frac{\mu_0 I}{4\pi} \frac{R^2}{(R^2 + a^2)^{3/2}} \cdot 2\pi$$

$$B = B_x = \frac{\mu_0 I R^2}{2(R^2 + a^2)^{3/2}}$$

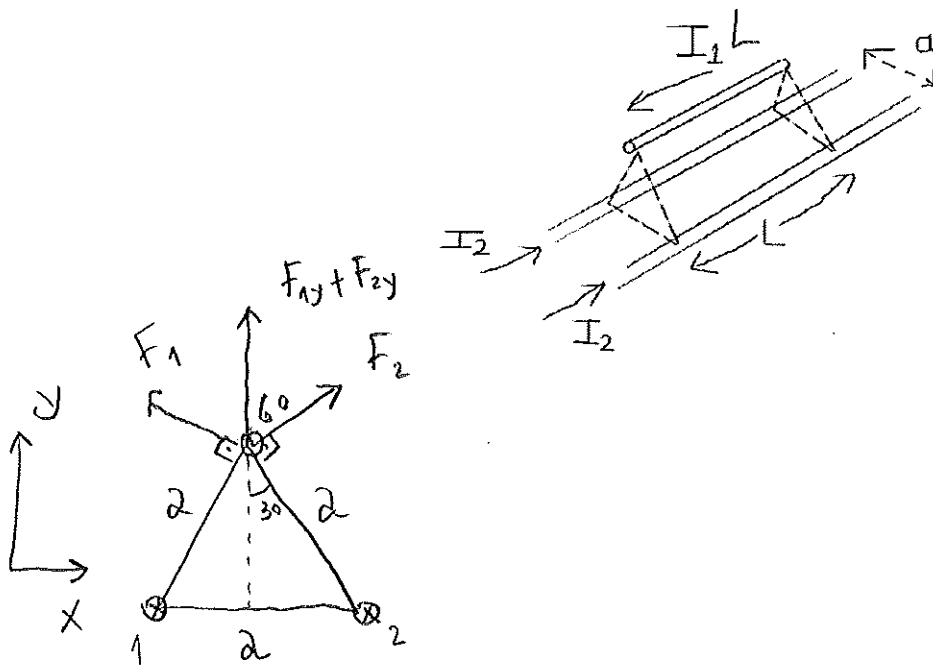
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Two infinitely long, parallel wires are lying on the ground a distance a apart as shown in the figure. A third wire, of length L and mass m , carries a current of I_1 and is levitated above the first two wires, at a horizontal position midway between them. The infinitely long wires carry equal currents I_2 in the same direction, but in the direction opposite that in the levitated wire. What current must the infinitely long wires carry so that the three wires form an equilateral triangle?



The total force on y -direction must be zero

$$F_y = 0 = F_{grav.} + F_{1y} + F_{2y}$$

$$F_{grav.} = -mg$$

$$F_{1y} = F_{2y} = B \cdot I_1 \cdot L \cdot \cos 60 = \frac{\mu_0 I_2}{2\pi a} I_1 L \cdot \frac{1}{2} = \frac{\mu_0 I_1 I_2 L}{4\pi a}$$

$$F_y = 0 = -mg + \gamma \cdot \frac{\mu_0 I_1 I_2 L}{2 \pi a} \Rightarrow$$

$$\Rightarrow \mu_0 I_1 I_2 L = mg \cdot 2\pi a \Rightarrow$$

$$\Rightarrow \boxed{I_2 = \frac{mg \cdot 2\pi a}{\mu_0 I_1 L}}$$