

Closed book. No calculators are to be used for this quiz.

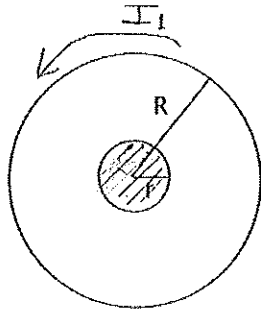
Quiz duration: 10 minutes

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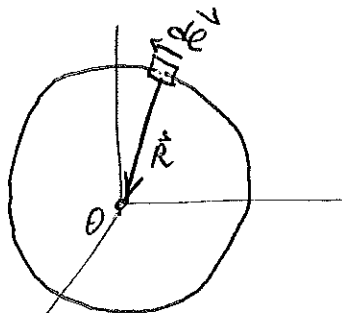
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Consider two single-turn co-planar, concentric coils of radii  $R$  and  $r$ , with  $R \gg r$ , as shown in Figure. What is the mutual inductance between the two loops?



Magnetic field ~~inside~~ at the ~~axis~~ center of the circular loop:



$$d\vec{B} = \frac{\mu_0}{4\pi} I_1 \frac{d\vec{l} \times \vec{R}}{R^2}$$

$$|\vec{B}| = \oint d\vec{B} = \frac{\mu_0}{4\pi} I_1 \int_{\theta=0}^{\theta=2\pi} \frac{R d\theta}{R^2} = \frac{\mu_0 I_1}{2R}$$

\* Since  $r \ll R$  field inside the inner circle can be taken as uniform and equal to its value at the center.

Thus magnetic flux inside the inner circle is,

$$\Phi_2 = B \cdot A = \frac{\mu_0 I_1}{2R} \pi r^2$$

The mutual inductance is then,

$$M = \frac{N_2 \Phi_2}{I_1} = 1 \cdot \frac{\frac{\mu_0 I_1}{2R} \pi r^2}{I_1} = \frac{\mu_0 \pi}{2R} \frac{r^2}{R}$$

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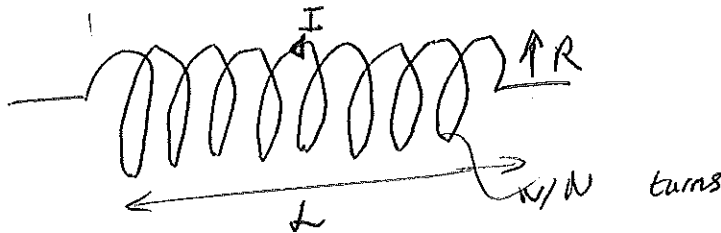
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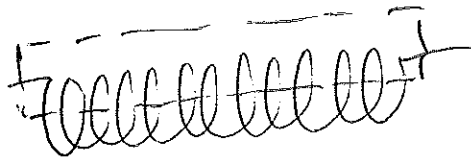
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Compute the self-inductance of a solenoid with  $N$  turns, length  $L$ , and radius  $R$  with a current  $I$  flowing through each turn.



field inside a solenoid



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B \cdot L = \mu_0 N I$$

$$B = \mu_0 \frac{N}{L} I$$

Assuming field inside the solenoid uniform, we can calculate flux through its cross-sectional area as

$$\Phi = B \cdot A = \mu_0 \frac{N}{L} I \pi R^2$$

So the inductance is,

$$L'_{sol.} = \frac{N \Phi}{I} = \frac{\mu_0 \pi N^2 R^2}{L}$$

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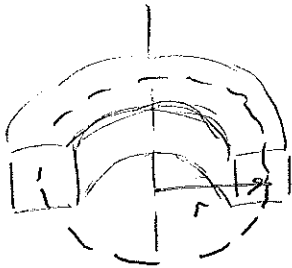
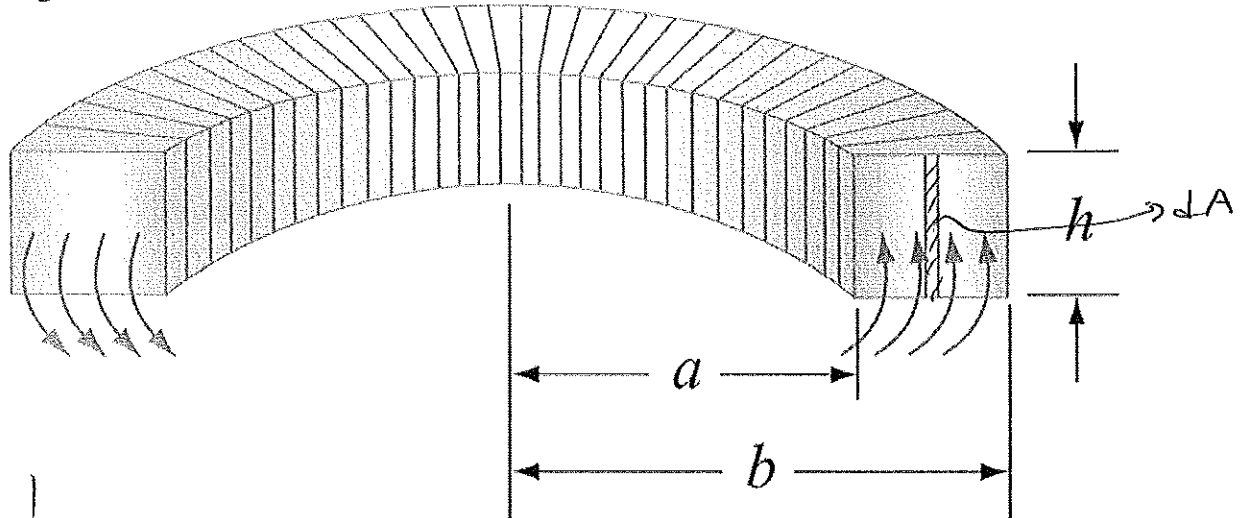
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Calculate the self-inductance of a toroid which consists of  $N$  turns and has a rectangular cross section, with inner radius  $a$ , outer radius  $b$  and height  $h$ , as shown in Figure.



field inside a toroid:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B \cdot 2\pi r = \mu_0 NI$$

$$B(r) = \frac{\mu_0 NI}{2\pi r}$$

flux through the strip:

$$d\Phi = \vec{B} \cdot d\vec{A} = \frac{\mu_0 NI}{2\pi r} h dr$$

total flux:

$$\Phi = \int d\Phi = \frac{\mu_0 NI h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 NI h}{2\pi} \ln\left(\frac{b}{a}\right)$$

Inductance

$$L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

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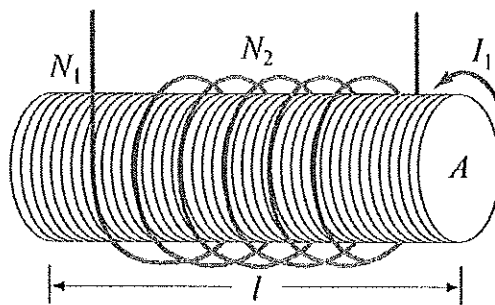
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A long solenoid with length  $l$  and a cross-sectional area  $A$  consists of  $N_1$  turns of wire. An insulated coil of  $N_2$  turns is wrapped around it, as shown in Figure. (i) Calculate the mutual inductance  $M$ , assuming that all the flux from the solenoid passes through the outer coil. (ii) Relate the mutual inductance  $M$  to the self-inductances and of the solenoid and the coil.



$$(i) M = \frac{N_2 \Phi_{B2}}{I_1}$$

$$\Phi_{B2} = BA \quad \text{where } B = \frac{\mu_0 N_1 I_1}{l}$$

$$= \frac{\mu_0 N_1 I_1 A}{l}$$

$$\Rightarrow M = \frac{\mu_0 N_1 N_2 I_1 A}{I_1 l} = \frac{\mu_0 N_1 N_2 A}{l}$$

$$(ii) \text{ Since } M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_1 \Phi_{B1}}{I_2} \Rightarrow I_2 = \frac{N_1 \Phi_{B1}}{M}$$

$$\text{and } L_1 = \frac{N_1 \Phi_{B1}}{I_1} \quad L_2 = \frac{N_2 \Phi_{B2}}{I_2}$$

$$\Rightarrow L_1 L_2 = M^2$$

$$\Rightarrow M = \sqrt{L_1 L_2}$$

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A long solenoid with length  $l$  and a radius  $R$  consists of  $N$  turns of wire. A current  $I$  passes through the coil. Find the energy stored in the system.

$$U = \frac{1}{2} L I^2$$

$$L = \frac{N \Phi_B}{I}, \quad \Phi_B = \vec{B} \cdot \vec{A} = BA = \left( \frac{\mu_0 N I}{l} \right) (\pi R^2)$$

magnitude of  
magnetic field of  
solenoid

$$\Rightarrow L = \frac{\mu_0 N^2 R^2 \pi l}{l}$$

$$\Rightarrow U = \frac{1}{2} \frac{\mu_0 N^2 I^2 R^2 \pi l}{l}$$