PHYS 102: General Physics 2 KOÇ UNIVERSITY Spring Semester 2013

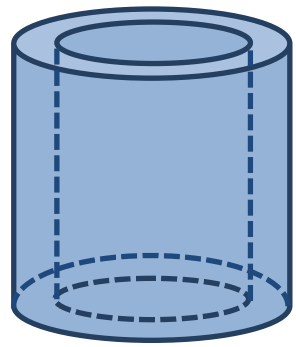
College of Arts and Sciences

Section Quiz 9 18 April 2013

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: Student ID: Signature:

A hollow cylindrical conductor along the z axis has inner radius R_1 and outer radius R_2. Total current I in the +z direction is distributed uniformly through the cross section of the conductor. Using Ampere's law, calculate the magnetic field $B(r)$ for $r < R_1$, $R_1 < r < R_2$, and $R_2 < r$.



Solution:

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$. Because of cylindrical symmetry, choose the integral path to be a circle of radius r.
From right-hand rule, the magnetic field is parallel to $d\vec{l}$ on the circle and the magnitude of B is constant on the circle (since all points on the circle are at the same distance from the center). These two properties are sufficient to evaluate the integral:

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B = \mu_0 I_{encl}$$

For $0 < r < R_1$, $I_{encl} = 0$ therefore $B = 0$.

For $R_1 < r < R_2$, $I_{encl} = I \frac{\pi(r^2 - R_1^2)}{\pi(R_2^2 - R_1^2)}$, therefore $B = \frac{I}{2\pi r} \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$

For $R_2 < r$, $I_{encl} = I$, therefore $B = \frac{I}{2\pi r}$

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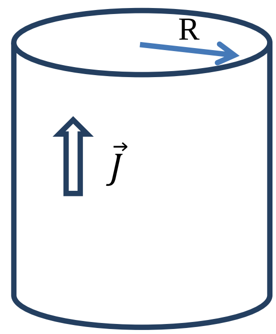
Name: Student ID: Signature:

A solid cylinder conductor on the z-axis has radius R. The cylinder has a current density $\vec{J} = \alpha r \hat{k}$ where α is a constant, r is the radial distance from the cylinder axis, and $\hat{k}$ is the unit vector in the z direction. Calculate the magnetic field <u>outside</u> the conductor using Ampere's law.



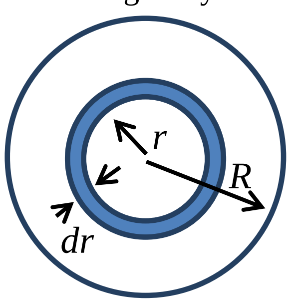
Solution:

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$. Because of cylindrical symmetry, choose the integral path to be a circle of radius r. From right-hand rule, the magnetic field is parallel to $d\vec{l}$ on the circle and the magnitude of B is constant on the circle (since all points on the circle are at the same distance from the center). These two properties are sufficient to evaluate the integral:

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B = \mu_0 I_{encl}$$

For $< r$, $I_{encl} = I_{total}$, therefore $B = \frac{I_{total}}{2\pi r}$

To find the total current, we integrate the current density from $r = 0$ to $r = R$. The current through a cylindrical shell of radius r and thickness dr is : $dI = J 2\pi r dr$.



$$I = \int_0^R J 2\pi r dr = 2\pi\alpha \int_0^R r^2 dr = \frac{2\pi\alpha}{3} R^3$$

Hence, $B = \frac{\alpha}{3} \frac{R^3}{r}$

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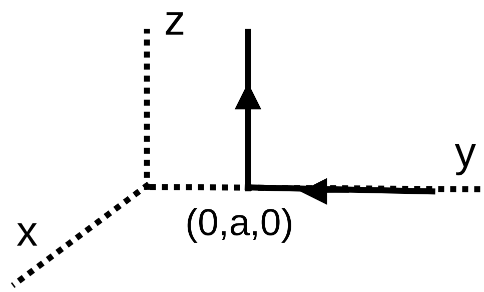
Name: Student ID: Signature:

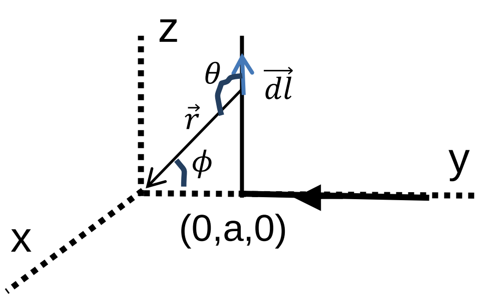
An infinite wire on the y axis is bent 90 degrees at y = a to extend in the +z direction. A current I flows through the wire in the shown direction. Find the total magnetic field at the origin (magnitude and direction)



Solution:

Apply Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l}\times\hat{r}}{r^2}$

For the wire segment on the y axis: $d\vec{l} = dy(-\hat{j})$ and $\hat{r} = -\hat{j}$. Thus, since $d\vec{l}\times\hat{r} = 0$ this wire segment will produce zero magnetic field at the origin.



For the wire segment parallel to z-direction: $d\vec{l} = dz\hat{k}$ and

$$d\vec{l}\times\hat{r} = +\hat{\imath}dz\sin\theta = -\hat{\imath}dz\cos\phi = -\hat{\imath}dz\frac{a}{r}.$$

$$dB = \frac{\mu_0}{4\pi}\frac{Iadz}{r^3} = \frac{\mu_0 I}{4\pi}\frac{adz}{(a^2+z^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_0^\infty \frac{adz}{(a^2+z^2)^{3/2}}$$

let $z = a\tan\alpha$ change of integration variable.This gives the value of the integral as 1/a. So the result is $\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{\imath})$

Note: One can also use the practical result that the wire segment in the z-direction is half of an infinite wire in the z-direction. So the magnetic field will be half of the infinite wire in the z-direction: $B = \frac{1}{2}B_{\infty-wire} = \frac{1}{2}\frac{\mu_0 I}{2\pi a}$.

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Section **Quiz 9** **18 April 2013**

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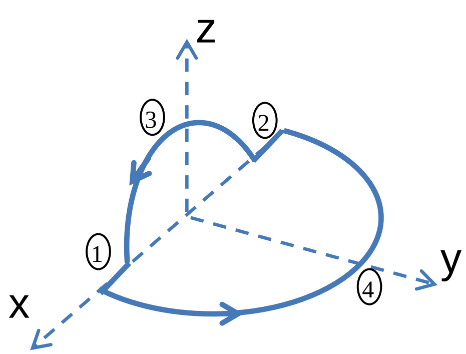
Name: **Student ID:** **Signature:**

Current I flows in a closed loop wire which is formed by a semicircle of radius r_1 in the x-y plane and a semicircle of radius r_2 in the x-z plane as shown in the figure $(r_1 > r_2)$. Find the magnitude and direction of the force acting on a charge q moving with velocity $\vec{v} = v_0\hat{\imath}$ at the origin. Express your answer using unit vectors.



$$\vec{F} = q\vec{v} \times \vec{B}$$
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$
$$\vec{F} = q\vec{v} \times (\vec{B}_1 + \vec{B}_2 + \vec{B}_3 + B_4)$$

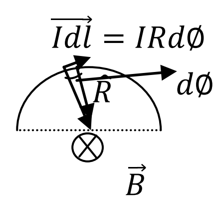
Here $\vec{B}_1$ refers to the magnetic field of the segment 1 and so on.

In order to find the force exerted on the charge we need first to calculate the magnetic field.

Biot-Savart law: $\vec{B} = \frac{\mu_0}{4\pi}\frac{\overrightarrow{Idl} \times \hat{r}}{r^2}$

For segment ① & ② this formula gives zero because $\overrightarrow{Idl} \times \hat{r} = 0$.

$\left|d\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{\left|\overrightarrow{Idl} \times \hat{R}\right|}{R^2} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{R^2}, \theta = \frac{\pi}{2}$,Which is shown in the below picture



$$B = \int_0^{\pi} \frac{\mu_0}{4\pi}\frac{IRd\emptyset}{R^2} = \frac{\mu_{0I}}{4R}$$

$$\vec{B}_3 = \frac{\mu_{0I}}{4r_1}\hat{\jmath}, \qquad \vec{B}_4 = \frac{\mu_0 I}{4r_2}\hat{k}$$

$$\vec{F} = \vec{F}_3 + \vec{F}_4$$
$$= q\vec{v} \times \left(\vec{B}_3 + \vec{B}_4\right)$$
$$= \frac{\mu_0 I q v_0}{4}\left(\frac{1}{r_1}\left(\hat{\imath} \times \hat{\jmath}\right) + \frac{1}{r_2}\left(\hat{\imath} \times \hat{k}\right)\right)$$
$$= \frac{\mu_0 I q v_0}{4}\left(\frac{\hat{k}}{r_1} - \frac{\hat{\jmath}}{r_2}\right)$$

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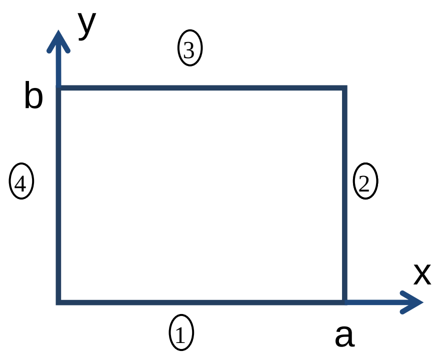
Name: Student ID: Signature:

A current density through the x-y plane generates a magnetic field $\vec{B} = Bx\hat{j}$. Find the magnitude and direction of the net current passing through the rectangular region shown in the figure, (Hint: Use Ampere's law on the rectangular path)



$\oint \vec{B}\,.\,\vec{dl} = \mu_0 I_{enc}$ Ampers law

We need to take the line integral therefore we divide the path in to 4 parts as it is shown in the figure above.

$$\cancelto{0}{\int_{x=0|_{y=0}}^{a} Bx\hat{j}\,.\,dx\hat{i}} + \int_{y=0|_{x=a}}^{a} Bx\hat{j}\,.\,dy\hat{j} + \cancelto{0}{\int_{x=a|_{y=a}}^{0} Bx\hat{j}\,.\,dx(\widehat{-i})} + \cancelto{0}{\int_{y=a|_{x=0}}^{0} Bx\hat{j}\,.\,dy(\widehat{-j})} = Ba$$

$$\oint \vec{B}\,.\,\vec{dl} = \mu_0 I_{enc} \Rightarrow Ba = \mu_0 I_{enc} \Rightarrow I = \frac{Ba}{\mu_0}$$

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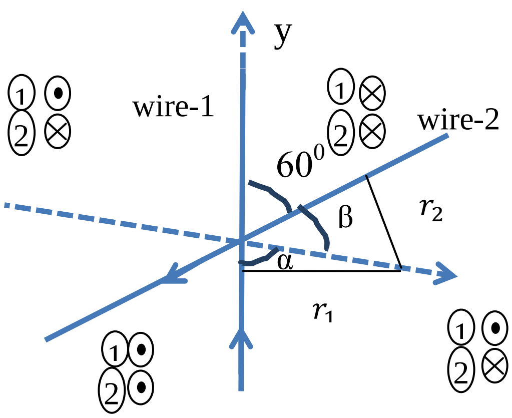
Name: Student ID: Signature:

Wire-1 on the y-axis carries a current I_0. Wire-2 carries a current $\frac{I_0}{2}$ and makes an angle of 60^0 with wire-1 on the x-y plane. Determine the location of points at which the net magnetic field is zero. (Assume that both wires are infinitely long)



it is clear from the figure the regions where the magnetic fields due to the wire-1 and wire-2 have opposite directions are the candidate for zero line of magnetic field. The next step is to find the line on which the magnetic fields' strength are equal:

$$\frac{B_1}{B_2} = 1 \rightarrow \frac{\frac{\mu I_1}{2\pi r_1}}{\frac{\mu I_2}{2\pi r_2}} = 1 \rightarrow r_2 = \frac{1}{2} r_1$$

$$\alpha + \beta = 120$$

$$\sin\alpha = 2\sin\beta$$

$$\sin(120 - \beta) = 2\sin\beta$$

$$\sin 120 \cos\beta - \cos 120 \sin\beta = 2\sin\beta$$

$$\frac{\sqrt{3}}{2}\cos\beta + \frac{1}{2}\sin\beta = 2\sin\beta$$

$$\cot\beta = \sqrt{3}$$

$$\beta = 30$$

Therefore the line of zero magnetic field is along x-axis