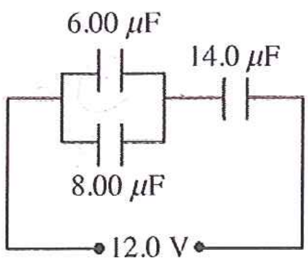
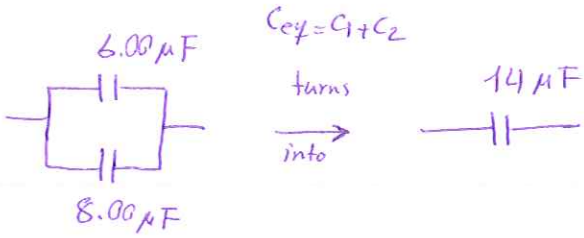
PHYS 102: General Physics - KOÇ UNIVERSITY
College of Sciences
Quiz 4 Nov 4, 2016

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes
Name: ID #: Signature:

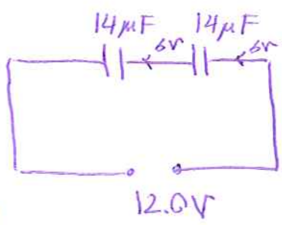
Q. Two capacitors of capacitance 6.00 μF and 8.00 μF are connected in parallel. The combination is then connected in series with a 12.0-V voltage source and a 14.0 μF capacitor, as shown in the figure. Find the charge on the 6.00 μF capacitor.



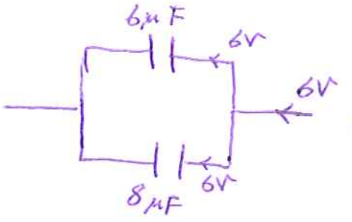
For parallel capacitors:



So the circuit will be like



They are series so every capacitor gets 6V!



: For parallel capacitors, each will get same 6V voltage.

Finally $\Longrightarrow$ $Q = CV = (6.00\,\mu F)(6\,V) = 36\,\mu C$

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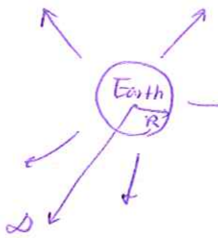
Q. Capacitance can be calculated also for a *single* conductor, by assuming the second conductor as being located at infinity. Using $u = \epsilon_0 E^2/2$ for electric field energy density and $U = Q^2/2C$ for the stored energy in a capacitor, calculate the capacitance of a conducting sphere with radius R.

Hint: Evaluate U by integrating the field energy density over spherical shells of volume $dV = 4\pi R^2 dr$.

$$\begin{cases} u = \varepsilon_0 \frac{E^2}{2} \\ U = \frac{Q^2}{C} \end{cases} \Rightarrow U = \int u\, dV$$



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2} \longrightarrow E^2 = \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{Q^2}{R^4}$$

Using hint: $U = \int u\, dV = \int \overbrace{\frac{1}{2}\varepsilon_0 E^2}^{u} \overbrace{(4\pi R^2 dr)}^{dV} = \frac{1}{2}\varepsilon_0 \int \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{Q^2}{R^4} (4\pi R^2 dr)$

$$U = \frac{1}{2} \cdot \frac{Q^2}{4\pi\varepsilon_0} \int_R^\infty \frac{dr}{R^2} = \frac{1}{2} \cdot \frac{Q^2}{4\pi\varepsilon_0} \left[\frac{-1}{r} \right]_R^\infty = \frac{1}{2} \cdot \frac{Q^2}{4\pi\varepsilon_0} \left(\frac{-1}{\infty} + \frac{1}{R} \right)$$

(∞ → infinity; R → Earth surface; $\frac{-1}{\infty}$ → Zero)

$$U = \frac{Q^2}{2} \cdot \frac{1}{4\pi\varepsilon_0 R} \quad (\mathrm{I}) \xrightarrow[\text{that}]{\text{we knew}} U = \frac{Q^2}{2C} \quad (\mathrm{II})$$

$$(\mathrm{I}), (\mathrm{II}) \Rightarrow \boxed{C = 4\pi\varepsilon_0 R}$$

Capacitance of Earth

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Q. Capacitance can be calculated also for a *single* conductor, by assuming the second conductor as being located at infinity. Use this idea and the definition $C = Q/V$ to calculate the capacitance of a conducting sphere with radius R. Substitute $\epsilon_0 \simeq 9\times10^{-12}$ F/m and $R = 6400$ km to estimate Earth's (a good conductor) capacitance.

$C = \frac{Q}{V}$ → for Earth $V = k\frac{Q}{R}$

$\Rightarrow C = \frac{Q}{k\left(\frac{Q}{R}\right)} = \frac{R}{k} \Rightarrow \boxed{C = \frac{R}{k}}$ Capacitance of sphere with radius R

$k = \frac{1}{4\pi\epsilon_0} \longrightarrow \boxed{C = 4\pi\epsilon_0 R}$ ✓

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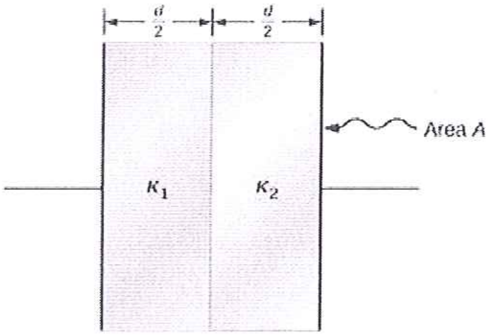
Q. A parallel-plate capacitor with plate area A has the space between the plates filled with two slabs of dielectric, one with dielectric constant κ_1 and one with κ_2 as shown in the figure. Each slab has thickness $d/2$, where d is the plate separation. Find the capacitance in terms of ϵ_0 and given parameters.

Hint: Every point on the interface between the two dielectrics has the same electric potential. Then, one can assume them to be separated by a conductor surface, and think of the system as composed of two capacitors.



We can assume two series capacitors:

$$C_1 = K_1 \frac{\varepsilon_0 A}{(\frac{d}{2})} = \frac{2K_1\varepsilon_0 A}{d}$$

$$C_2 = K_2 \frac{\varepsilon_0 A}{(\frac{d}{2})} = \frac{2K_2\varepsilon_0 A}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{d}{2\varepsilon_0 A K_1} + \frac{d}{2\varepsilon_0 A K_2} = \frac{d}{2\varepsilon_0 A}\left(\frac{K_1+K_2}{K_1K_2}\right) \Rightarrow C_{eq} = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$$

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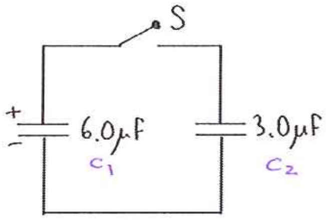
Q. In the circuit shown below, with the switch S open, the 6.0 μF capacitor has an initial charge of 6.0 μC while the 3.0 μF capacitor is uncharged. The switch is then closed and left closed for a long time. Calculate the initial and final values of the total electric energy stored in the two capacitors.

Initial charge

$Q_0 = 6.0\,\mu C$



$U = \frac{Q^2}{2C}$

$$\frac{C_1}{C_2} = \frac{Q_1}{Q_2} = \frac{6.0\,\mu F}{3.0\,\mu F} = 2 \Rightarrow \frac{Q_1}{Q_2} = 2$$

$\Rightarrow$ $Q_0 = Q_1 + Q_2$ (switch closed, Capacitors share charge)

$$\Rightarrow Q_0 = Q_1 + \frac{Q_1}{2} \longrightarrow Q_0 = \frac{3}{2} Q_1 \Rightarrow \begin{cases} Q_1 = 4.0\,\mu C \\ Q_2 = 2.0\,\mu C \end{cases}$$

$$Q_2 = \frac{1}{3} Q_0 \Rightarrow$$

Energy stored: 1) When switch is open:

$$U_0 = \frac{Q_0^2}{2C_1} = \frac{(6.0\,\mu C)^2}{2 \times 6.0\,\mu F} = 3 \times 10^{-6}\,J$$

2) When swith is closed:

$$U_{tot} = U_1 + U_2 = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \overbrace{\frac{(4.0\,\mu C)^2}{2 \times (6.0\,\mu F)}}^{1.33 \times 10^{-6}\,J} + \overbrace{\frac{(2.0\,\mu C)^2}{2 \times (4.0\,\mu F)}}^{5 \times 10^{-7}\,J} = 1.83 \times 10^{-6}\,J$$

Energy in (1) and (2) are not the same, so we don't have energy conservation!