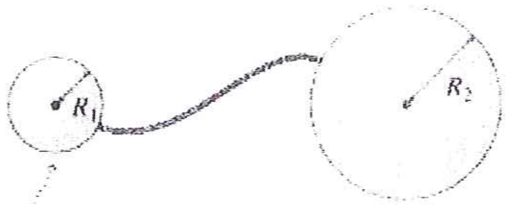
PHYS 102: General Physics - KOÇ UNIVERSITY
College of Sciences
Quiz 3 Oct 21, 2016

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes
Name: ID #: Signature:

Q. A spherical conductor with a radius R_1 carries a charge Q, while a second spherical conductor with a radius R_2 is neutral. The two conductors are later connected by a conducting wire as shown. Find the final total charge on each sphere.



initial charge Q   initially neutral

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1}, \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2}$$

$$V_1 = V_2^{(*)} \Rightarrow \frac{Q_1}{R_1} = \frac{Q_2}{R_2}^{(**)} \Rightarrow Q_1 = Q_2\frac{R_1}{R_2}$$

$$Q_1 + Q_2 = Q$$

$$Q_2\frac{R_1}{R_2} + Q_2 = Q$$

$$Q_2 = \frac{QR_2}{R_1+R_2}, \quad Q_1 = \frac{QR_1}{R_1+R_2}$$

(*) Since the two conductors are connected, we can think of them as a single conductor. A conductor has a constant potential inside. Therefore, potentials on the surface of both spheres should be equal.

(**) Total charge of each sphere is proportional to it's radius. However, this is not the case for surface charge density.

$$\frac{Q_1}{R_1} = \frac{Q_2}{R_2} \Rightarrow \frac{4\pi R_1^2\sigma_1}{R_1} = \frac{4\pi R_2^2\sigma_2}{R_2}$$

$R_1\sigma_1 = R_2\sigma_2$ Surface charge density is inversely proportional to the radius

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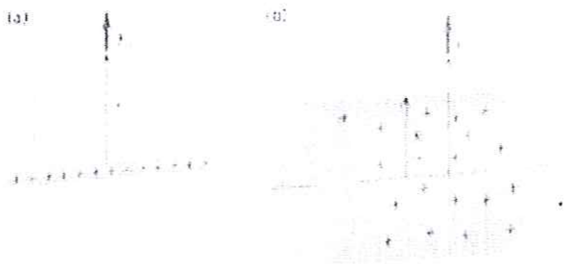
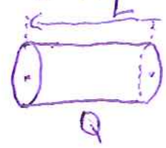
Q. Recall that the electric potential at a distance r from an very long, straight wire (Fig. a) with a *line* charge density λ is

$$V(r) = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$

where r_0 is the reference distance. Find the electric potential at a distance r from a very long, cylindrical conductor (Fig. b) with a radius R and a *surface* charge density σ. Use $V(R)$ as reference (that is, $V(R) = 0$). Express your answer separately for $r < R$ and $r > R$, in terms of σ, r, R and some constants.



$r \geq R$



$Q = 2\pi r L \cdot \sigma$

$\lambda = \frac{Q}{L} = 2\pi r \sigma$

$$V(r) \overset{(*)}{=} \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r} = \boxed{\frac{2\pi r\sigma}{2\pi\varepsilon_0}\ln\frac{R}{r}}$$

$r < R$

$$V(r<R) \overset{(**)}{=} \boxed{0}$$

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* Outside the cylinder ($r \geq R$), the cylinder has a potential identical to that of a wire.

** Inside a conductor, electric potential is constant and equal to surface potential. ($V(r=R) = 0$)

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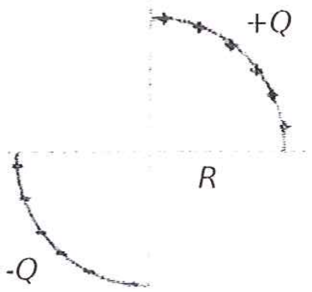
Q. The figure shows two arcs of a circle with radius R on which charges +Q and -Q have been spread uniformly.

(a) What is the value of the electric potential at the center of the circle?
(b) Show the direction of the electric field at the center with an arrow on the figure.
(c) Draw the equipotential line that passes through the center.

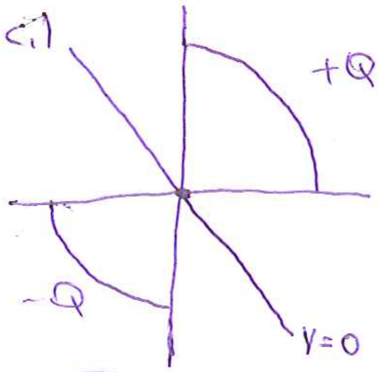


a.) $V_{+Q} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$

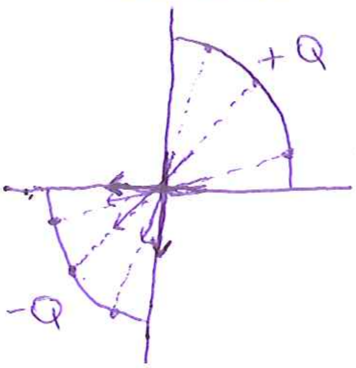
$V_{-Q} = \frac{1}{4\pi\varepsilon_0} \frac{-Q}{R}$

$V_{center} = V_{+Q} + V_{-Q} = 0$

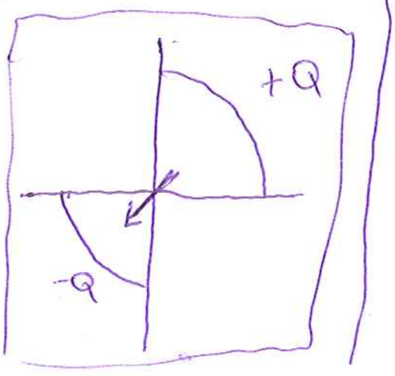
c.)



b.)



Summing all these vectors



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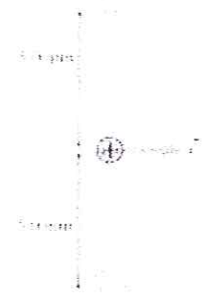
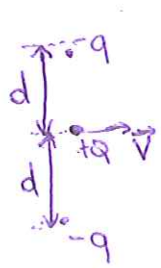
$$k \simeq 9\times10^{9}\ \mathrm{Nm^2/C^2}$$
$$\epsilon_0 \simeq 9\times10^{-12}\ \mathrm{C^2/Nm^2}$$
$$e\,(\text{proton charge}) \simeq 1.6\times10^{-19}\ \mathrm{C}$$
$$m_{proton} \simeq 1.6\times10^{-27}\ \mathrm{kg}$$
$$\pi \simeq 3$$

Q. The figure shows an arrangement of two -1.5 nC charges, each separated by 5.0 mm from a proton. If the two negative charges are held fixed at their locations and the proton is given an initial velocity v as shown in the figure, what is the minimum initial speed v that the proton needs to totally escape from the negative charges?





$$U_i = \frac{1}{4\pi\epsilon_0}\left(\frac{Q(-q)}{d} + \frac{Q(-q)}{d}\right) = -\frac{1}{2\pi\epsilon_0}\frac{Qq}{d}$$

$$K = \frac{1}{2}mv^2$$

$$U + K = 0$$

$$\frac{1}{2}mv^2 - \frac{1}{2\pi\epsilon_0}\frac{Qq}{d} = 0 \Rightarrow mv^2 = \frac{1}{\pi\epsilon_0}\frac{Qq}{d}$$

$$v = \sqrt{\frac{1}{m\pi\epsilon_0}\frac{Qq}{d}} = \sqrt{\frac{1}{(1.6\times10^{-27}\,\mathrm{kg}).3.(9\times10^{-12}\,\mathrm{C^2/Nm^2})}\;\frac{(1.6\times10^{-19}\,\mathrm{C})(1.5\times10^{-9}\,\mathrm{C})}{5\times10^{-3}\,\mathrm{m}}}$$

$$= 1.8\times10^{6}\ \mathrm{m/s}$$

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Q. A long, insulating cylinder carries a positive volume charge density ρ. From Gauss's law, we find that the electric field *inside* the cylinder is radially outward with magnitude

$$E(r) = \frac{\rho r}{2\epsilon_0}$$

where r is the distance from the cylinder's axis. Find the electric potential at r (for $r < R$) relative to the cylinder's surface (that is, $V(R) = 0$).

Electric potential at point b, with respect to point a can be calculated using:

$$V = -\int_a^b \vec{E} \cdot d\vec{l} \Rightarrow V = -\int_R^r \frac{\rho r'}{2\varepsilon_0} dr' = -\left. \frac{\rho r'^2}{4\varepsilon_0} \right|_R^r = \frac{-\rho r^2}{4\varepsilon_0} + \frac{\rho R^2}{4\varepsilon_0} = -\frac{\rho (r^2 - R^2)}{4\varepsilon_0}$$

$$= \frac{\rho (R^2 - r^2)}{4\varepsilon_0}$$