

Section

Quiz 11

30 December 2016

Closed book. No calculators are to be used for this quiz.

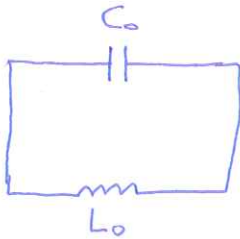
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

An LC circuit containing an inductor L_0 and a capacitor C_0 oscillates with a maximum current of I_0 . Assuming the capacitor has its maximum charge at time $t = 0$, calculate the energy stored in the inductor after t seconds.



$$I(t) = I_0 \cos(\omega_0 t + \phi)$$

$$\text{where } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$V(t) = L_0 \frac{dI}{dt} = -\omega_0 L_0 I_0 \sin(\omega_0 t + \phi)$$

$$V(t) = \frac{Q(t)}{C} \Rightarrow Q(t) = V(t) C_0$$

$$Q(t=0) = -\omega_0 L_0 I_0 \sin(\phi) \cdot C_0$$

for max charge at $t=0$,

$$\sin(\phi) = -1$$

$$\phi = \frac{3\pi}{2}$$

Energy stored in the inductor:

$$E_{\text{ind}} = \frac{1}{2} L I^2$$

At time t :

$$E_{\text{ind}}(t) = \frac{1}{2} L_0 I_0^2 \cos^2\left(\frac{t}{\sqrt{LC}} + \frac{3\pi}{2}\right)$$

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An ideal battery with voltage V_0 , a resistor with resistance R_0 , and an ideal inductor with inductance L_0 are all connected in series with an open switch. The switch is suddenly closed. How long after closing the switch will the current through the inductor reach one-half of its maximum value?



$$V_0 - iR_0 - L_0 \frac{di}{dt} = 0$$

$$V_0 - iR_0 = L \frac{di}{dt}$$

$$\frac{dt}{L_0} = \frac{di}{V_0 - iR_0}$$

$$\int_0^t \frac{dt}{L_0} = \int_0^i \frac{di}{V_0 - iR_0}$$

$$\frac{t}{L_0} = -\frac{1}{R_0} \ln(V_0 - iR_0) \Big|_0^i$$

$$i = \frac{V_0}{R_0} \left(1 - e^{-\frac{R_0 t}{L_0}}\right)$$

$$\text{For max } i, \left(1 - e^{-\frac{R_0 t}{L_0}}\right) = 0$$

$$i_{\max} = \frac{V_0}{R_0}$$

For half of max:

$$\frac{i_{\max}}{2} = i_{\max} \left(1 - e^{-\frac{R_0 t}{L_0}}\right)$$

$$1 - e^{-R_0 t/L_0} = \frac{1}{2}$$

$$e^{-\frac{R_0 t}{L_0}} = \frac{1}{2}$$

$$-\frac{R_0 t}{L_0} = \ln(0.5)$$

$$t = -\frac{L_0 \ln(0.5)}{R_0}$$