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Phys 102:

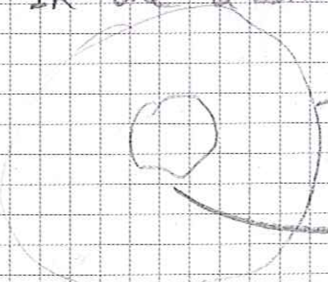
Chap 21: Electric Charge and Electric Field:

If you take two glass rods and rub them with silk, they will repel each other.

(+ + + +)   (+ + + +)

This is because each obtained a "positive electric charge", and like charges repel each other.

In the atomic structure:



→ electron cloud
e

In a neutral atom e=p

→ nucleus made of proton and neutron
(p, n)

Nucleus is very very dense, protons and neutrons are held together by an attractive interaction (strong nuclear force) which overcomes the electric repulsion of the protons.

⇒ Protons are held in the nucleus, they are normally not transported.

An atom is charged by addition or removal of electrons.

If one or more electrons are removed ⇒ positive ion is obtained
gained ⇒ negative

H^+ — 0e, 1p 1n

H^- — 2e, 1p, 1n

For a Macroscopic Body:

A macroscopic body can be positively or negatively charged by removing or adding electrons.

Two very important principles:

- Principle of conservation of charge: The algebraic sum of all the electric charges in any closed system is constant.

⇒ If you rub together a glass rod and a piece of silk both initially uncharged, the rod acquires + charge and silk − charge with the same magnitudes.

Total charge is conserved

- The magnitude of charge of the electron or proton is a natural unit of charge. Every observable amount of charge is always an integer multiple of the basic unit.

$Q = ne$, n: integer
e: natural unit of charge, $e = 1.602 \times 10^{-19}$ C

Conductors, Insulators and Induced Charges:

Conductor: Materials which permit electric charge to move easily from one region to another.
copper wire, most metals, ...

Insulator: Materials in which electrons do not move easily from one region to another. (Low mobility)
plastic, most nonmetals, ...

Semiconductors: Materials which are neither good conductors nor good insulators.
Si, GaA
⇑
chip scale electronic technology relies on Si.

Charging by Induction:

We can charge a metal ball by touching it with an electrically charged plastic rod.

It is also possible to charge a metal ball without touching ↙ induction.

. Bring a negatively charged rod into close vicinity of a metal sphere standing on an insulating stand.
(Positive charge will be induced on the left and negative charge on the right)

. Bring the − side of the sphere to contact with the earth. Earth will sink all the electrons because it is a good conductor.

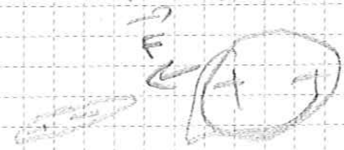
. Disconnect the wire from the ground and remove the rod.

→ A net positive charge is left on the sphere.

Now disconnect the wire to ground and then remove the rod. $\Rightarrow$ A net positive charge is left on the sphere.

A Charged body can exert forces on objects that are not charged $\rightarrow$ Induced charge effect.

When a $-$ charged rod is brought to the vicinity of a conducting sphere, $+$ charges will have a larger attractive force than the repelling force on the $-$ charges in the sphere.



Net effect $\Rightarrow$ Net attractive force.

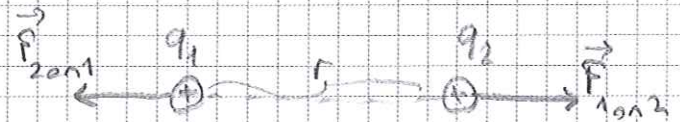
This is also true for an insulator. Charges in molecules will be shifted, polarization. Net effect of attraction is then felt.

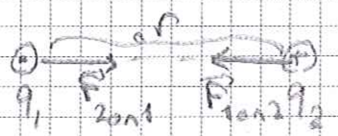
$\rightarrow$ The same is true if a $+$ charged body was brought to the vicinity of an insulator.

Net effect is once more an attractive force.

Coulomb's Law:

Interaction forces of charged particles is governed by Coulomb's Law.





Consider two point charges q_1, q_2 which are very small in comparison with distance r between them.

The magnitude of the electric force between q_1 and q_2 is:

$$|\vec{F}_{1on2}| = |\vec{F}_{2on1}| = k\frac{|q_1 q_2|}{r^2}$$

Direction of the electric force is along the line joining the two point charges. When q_1 and q_2 have the same sign the electric force is repulsive, when they have opposite sign the electric force is attractive.

$$\vec{F}_{1on2} = -\vec{F}_{2on1}$$

Comment: Electric force is very similar to the gravitational force, $F_G = G\frac{M_1 M_2}{r^2}$, Gravitational force is always attractive while the electric force can be repulsive or attractive.

In SI units: q_1, q_2 have units of Coulomb.

$$k = \frac{1}{4\pi\epsilon_0} \cong 9{,}0\times10^{9}\ Nm^2/C^2,\quad \epsilon_0 = 8{,}854\times10^{-12}\ C^2/Nm^2$$

we will often use $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$.

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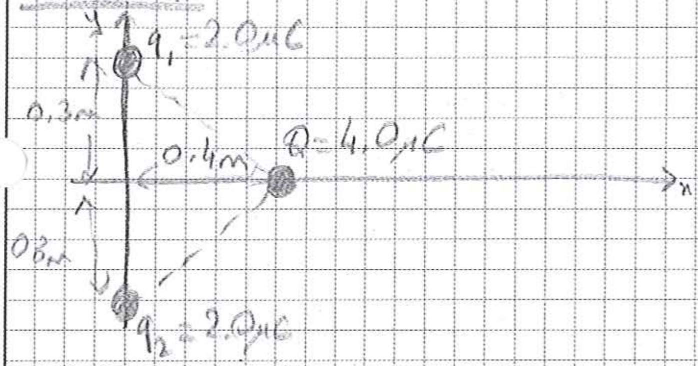
Superposition of Forces:

When two charges apply electric forces to a third charge, the total force acting on that charge is the vector sum of the forces that the two charges would apply individually.

Ex. 21.4:



Magnitude and direction of the net force on Q

net force:

$$\sum \vec{F} = 2F_1\cos\alpha = 2\times\frac{4}{5}\times 9\times10^{9}\,Nm^2/C^2\times\frac{2\times10^{-6}C\times4\times10^{-6}C}{(0.5m)^2}$$

$$= 0.46N\,\hat{i}$$

Electric Field and Electric Forces

Consider the interaction between two point charges A and B

We can express the electric force $\vec{F}_0$ applied from body A to body B as a result of the electric field caused by the body A.

Consider only the body A;

- "A" will cause an electric field at all points in the space.
- If a point charge q_0 is placed at the position of the body B, the point charge will experience the force $\vec{F}_0$ due to the electric field caused by the body A at point P.

Similarly the electric field generated by the body B will cause the electric force applied on A.

(Electric field of a body will not cause an electric force to the same body.)

Electric field vector $\vec{E}$ is defined as; electric force $\vec{F}_0$ experienced by a test charge q_0 divided by the charge q_0:

$$\boxed{\vec{E} = \frac{\vec{F}_0}{q_0}}$$, Electric force per unit charge

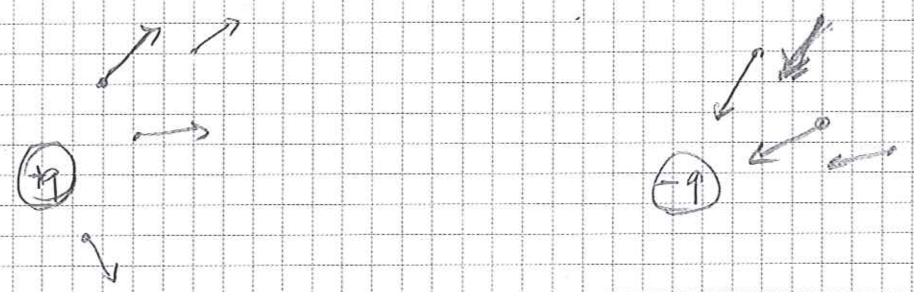
in SI units $(\vec{E}) = (N/C)$

Consider the source as being a point charge:

Let the point charge q be located at the source point S.

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\hat{r}$$, $\hat{r}$: unit vector that points along the line from source point to field point.

$$\Rightarrow \vec{E} = \frac{\vec{F}_0}{q_0} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$ Electric field due to a point charge q.



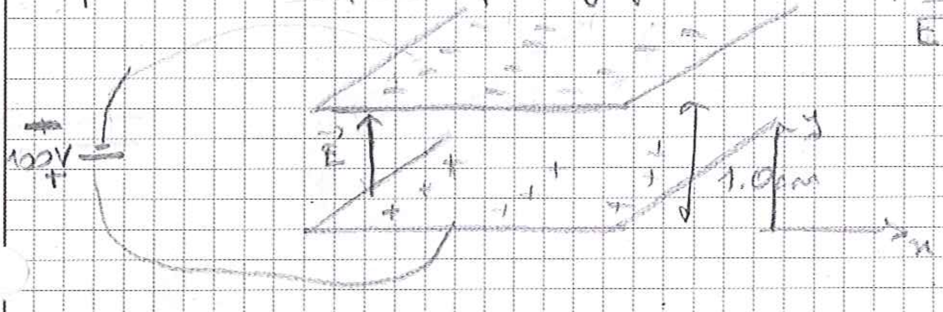
Electric field of a point charge points away from a positive charge ($\hat{r}$ direction) but toward a negative charge ($-\hat{r}$ direction)

Electric field from a point charge is a special case. In general a charge distribution will lead to a general electric field:

$$\vec{E}(x,y,z) = E_x(x,y,z)\,\hat{i} + E_y(x,y,z)\,\hat{j} + E_z(x,y,z)\,\hat{k}$$

Ex. 21.7: Electric field between two parallel conducting plates is uniform, pointing from the + plate to − plate.

$$\vec{E} = 10^4\,N/C\,\hat{j}$$



a) If an electron is released from rest at the upper plate what is its acceleration?

ignoring the gravitational force: $\vec{F}_y = -e\vec{E} = -e \times 10^4 N/C\,\hat{j}$

$\Rightarrow$ acceleration $\vec{a}_y = \dfrac{\vec{F}_y}{m_e} = -\dfrac{e\vec{E}}{m_e} = -\dfrac{e \times 10^4 N/C}{m_e}\,\hat{j}$

$= -1.76 \times 10^{15}\,m/s^2\,\hat{j}$

b) what speed and kinetic energy does it acquire while traveling 1.0 cm to the lower plate?

$\rightarrow$ Motion with constant acceleration:

$\vec{v}_y(t) = -a_y t\,\hat{j}$, $d = \frac{1}{2} a_y t_d^2 \Rightarrow t_d = \sqrt{\dfrac{2d}{a_y}}$

$$\Rightarrow \vec{v}_y(t_d) = -a_y\sqrt{\frac{2d}{a_y}}\,\hat{j} = -\sqrt{2da_y}\,\hat{j} = 5,9\times10^{6}\ m/s$$

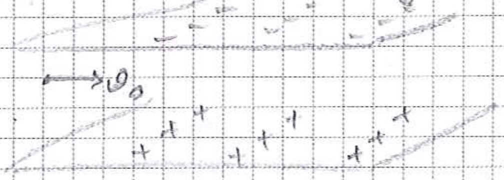
$$K = \frac{1}{2}m_e v_y^2 = 1,6\times10^{-17}\ J.$$

c) How much time is required for it to travel this distance?

$$t_d = \sqrt{\frac{2d}{a_y}} = 3,4\times10^{-9}\ s$$

Ex 21, 8:

What is the equation of the trajectory of an electron launched with an initial horizontal velocity v_0



$$\vec{F}_y = -e\times10^{4}\ N/C\ \hat{j} = -eE\hat{j}$$

$$\Rightarrow \vec{a}_y = -\frac{eE}{m_e}\hat{j}$$

$$\left.\begin{aligned} x(t) &= v_0 t \\ y(t) &= y_0 - \frac{1}{2}a_y t^2 \end{aligned}\right\}$$

Electric Field Calculations:

In order to calculate the electric field caused by a distribution of charges, we use the superposition of electric forces.

$\cdot\ \vec{E} = ?$

Imagine a charge distribution made up of point charges $q_1, q_2, q_3, \ldots$ Force applied by this charge distribution to a test charge q_0 placed at a point P is:

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = q_0\vec{E}_1 + q_0\vec{E}_2 + q_0\vec{E}_3 + \ldots$$

∴ Therefore, electric field at point P due to the distribution of charges is:

$$\vec{E}_0 = \frac{\vec{F}_0}{q_0} = \underbrace{\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots}$$

Sum of the electric fields due to each point charge in charge distribution.

This means integration.

If the charge distribution is along a line, we will use linear charge density $\lambda = \frac{Q}{L}$

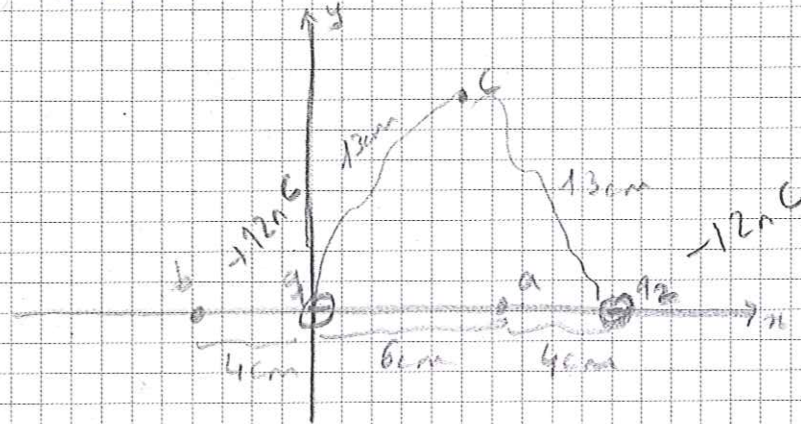
over a surface, surface charge density $\sigma = \frac{Q}{A}$

over a volume, volume charge density $\rho = \frac{Q}{V}$.

Examples

Ex 21.9: Electric field of an electric dipole

$q_1 = -q_2 = 12nC$



What is the electric field caused by q_1 and q_2 at a, b and c?

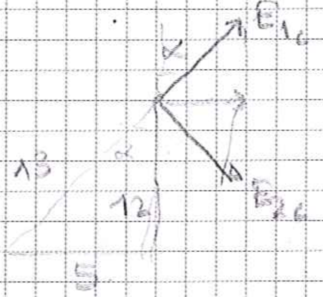
a) at point a: $E_x = E_{1x} + E_{2x} = \frac{1}{4\pi\epsilon_0} \frac{12\times10^{-9}C}{(6\times10^{-2}m)^2} + \frac{1}{4\pi\epsilon_0} \frac{12\times10^{-9}C}{(4\times10^{-2}m)^2}$

$= 9.8\times10^4 N/C$

b) At point b:

$$\vec{E}_{1b} = -\frac{1}{4\pi\epsilon_0}\frac{12\times10^{-9}C}{(4\times10^{-2}m)^2}\hat{i} + \frac{1}{4\pi\epsilon_0}\frac{12\times10^{-9}C}{(14\times10^{-2}m)^2}\hat{i} = -(6.2\times10^{4}N/C)\hat{i}$$

c) At point c:



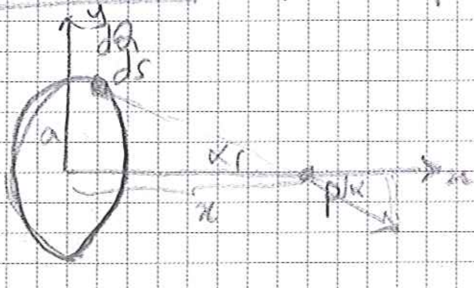
$$\vec{E}_{xc} = \frac{1}{4\pi\epsilon_0}\frac{12\times10^{-9}C}{(13\times10^{-2}m)^2}\cdot \sin\alpha\,\hat{i} + (\text{same})$$

$$= 2\,\frac{1}{4\pi\epsilon_0}\frac{12\times10^{-9}C}{(13\times10^{-2}m)^2}\times\frac{5}{13}\,\hat{i}$$

$$= 4.9\times10^{3}\,N/C\,\hat{i}$$

$$\vec{E}_{yc} = 0$$

Ex 21.10: Field of a ring of charge



A ring shaped conductor with radius a carries a total charge Q uniformly distributed around it. What is the electric field at a point P lying along x axis?

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2+a^2}\cdot\cos\alpha\,, \quad \cos\alpha = \frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{x^2+a^2}\cdot\frac{x}{\sqrt{x^2+a^2}}\,dQ\,, \quad dQ = \lambda\,ds\,, \quad \lambda = \frac{Q}{2\pi a}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}\,ds\,, \quad ds = a\,d\theta$$

$$\Rightarrow E_x = \int dE_x = \int_0^{2\pi}\frac{\lambda}{4\pi\epsilon_0}\frac{1}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}\,a\,d\theta = \frac{2\pi a\lambda}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}}$$

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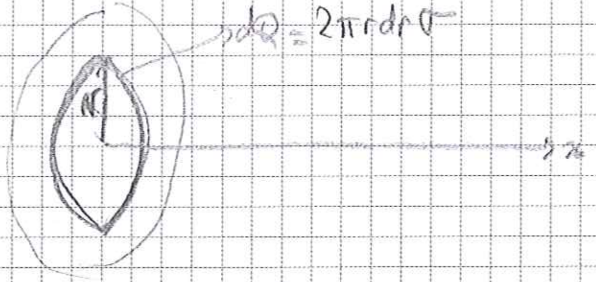
Ex 21.12 : Field of a uniformly charged disk.



Disk with a uniform charge density σ. What is the electric field at a point along the axis of the disk a distance x from its center.

For a ring of radius r

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q x}{(x^2+r^2)^{3/2}} \hat{i}$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dQ\, x}{(x^2+r^2)^{3/2}} \hat{i} \quad , \quad dQ = 2\pi r\, dr\, \sigma$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2\pi \sigma r\, dr}{(x^2+r^2)^{3/2}} x \hat{i}$$

⇒ The total Electric field:

$$E_x = \int dE_x = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{2\pi \sigma r\, dr}{(x^2+r^2)^{3/2}} x$$

$$= \frac{\sigma x}{2\epsilon_0} \underbrace{\int_0^R \frac{r}{(x^2+r^2)^{3/2}} dr}_{= -\frac{1}{(x^2+r^2)^{1/2}}}$$

$$= \frac{\sigma x}{2\epsilon_0} \left(\frac{-1}{(x^2+r^2)^{1/2}} \right) \Bigg|_0^R = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}} \right)$$

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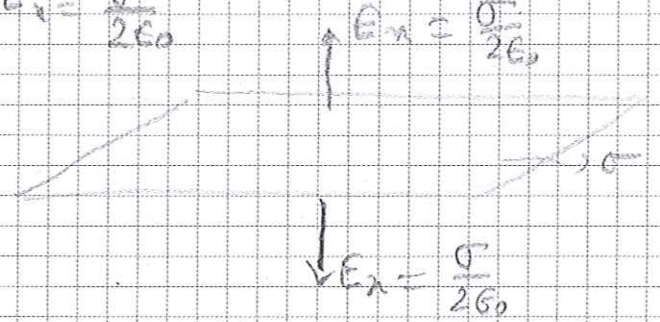
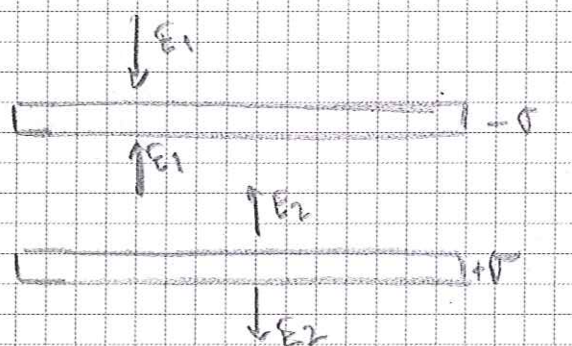
In the limit of very large R

$$E_x \simeq \frac{\sigma}{2\epsilon_0}$$, independent from the distance to the uniformly charged disk.

∴ The electric field produced by an infinite plane sheet is $E_x = \frac{\sigma}{2\epsilon_0}$



Ex. 21.13 Field of two oppositely charged infinite sheets



Two infinite parallel sheets are separated by a distance d. The lower sheet has a charge density $+\sigma$, the upper one $-\sigma$. What is the E-field bw the sheets, above the upper sheet, and below the lower sheet.

→ Find the E-fields due to individual sheets separately
- Then use the superposition principle

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \frac{\sigma}{\epsilon_0}\hat{j} & \text{between the sheets} \\ 0, & \text{below the lower sheet} \end{cases}$$

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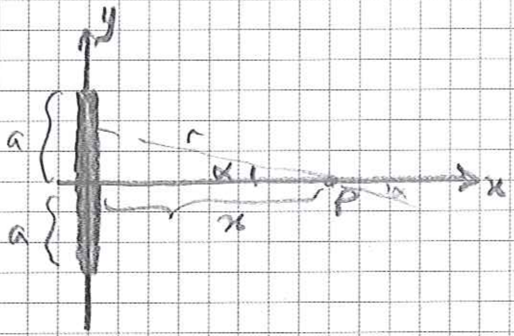
Ex 21.11: Field of a line of charge



A charge Q is distributed uniformly along a line with length $2a$. What is the electric field on the axis along the middle of the line?

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\cos\alpha \quad , \quad dQ = \lambda\,dy \quad , \quad r = \frac{x}{\cos\alpha}$$

$$\Rightarrow dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2}\cos^3\alpha \quad , \quad dQ = \lambda\,dy \; , \; y = x\tan\alpha$$

$$\Rightarrow dQ = \lambda x\, d(\tan\alpha) = \lambda x \frac{1}{\cos^2\alpha}\,d\alpha$$

$$\Rightarrow dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\cos^{\cancel{3}}\alpha}{x^{\cancel{2}}}\lambda x \frac{1}{\cancel{\cos^2\alpha}}\,d\alpha$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\cos\alpha}{x}\lambda\,d\alpha$$

$$\Rightarrow E_x = \int_{-\alpha_{max}}^{\alpha_{max}} \frac{\lambda}{4\pi\varepsilon_0 x}\cos\alpha\,d\alpha = \frac{\lambda}{4\pi\varepsilon_0 x}2\sin(\alpha_{max}) = \frac{2\lambda}{4\pi\varepsilon_0 x}\frac{a}{\sqrt{a^2+x^2}}$$

$$\lambda = \frac{Q}{2a} \Rightarrow \boxed{E_x = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{x\sqrt{x^2+a^2}}\right)}$$

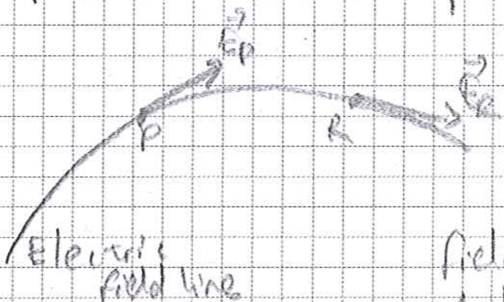
In the limit of very large a: $a \gg x$

$$\Rightarrow E_x \simeq \frac{Q}{4\pi\varepsilon_0 a}\frac{1}{x} = \frac{2\lambda}{4\pi\varepsilon_0}\frac{1}{x} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 x}}$$

Electric Field Lines

Field lines help visualizing electric fields.

An electric field line is an imaginery line drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point.

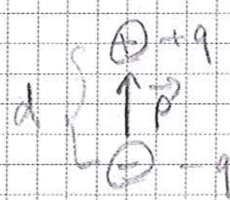


At any point, without a charge, the electric field has a unique direction, only one field line can pass through each point of the field. → Field lines never intersect.

→ Field lines are drawn close together at points where the field intensity is large.

Electric Dipoles

Electric dipole is a pair of point charges with equal magnitude and opposite sign separated by a distance d.

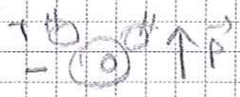


We define the electric dipole moment vector $\vec{p}$, as the vector with magnitude $p = qd$ and direction from the − to + charge.

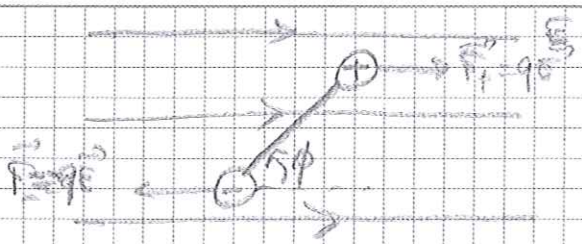
Electric dipoles are important because:

— many physical systems (molecules (water), TV antennas, ...) are explained by electric dipoles.

Water is such a good solvent because of its electric dipole moment.



What happens to an electric dipole under applied constant electric field.



Consider a uniform external electric field $\vec{E}$

The net force applied to the dipole is $\vec{F}_+ + \vec{F}_- = 0$
The net torque with the respect to the center of the dipole is:

$$\tau = 2(qE)\frac{d}{2}\sin\phi = qEd\sin\phi$$

$$\tau = pE\sin\phi \Rightarrow \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Work done by the electric force during an infinitesimal displacement $d\phi$:

$$dW = \tau\, d\phi \quad \text{, from chapter 10}$$

$$= -pE\sin\phi\, d\phi$$

Because the torque is in the direction of decreasing ϕ.

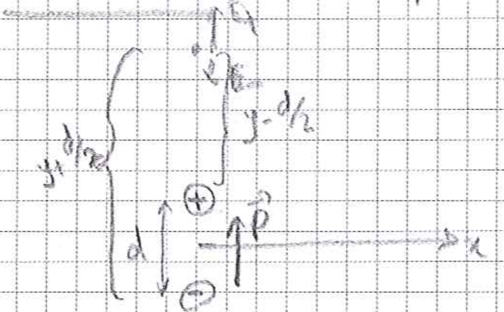
$$\Rightarrow W = \int_{\phi_1}^{\phi_2} -pE\sin\phi\, d\phi = pE\cos\phi_2 - pE\cos\phi_1$$

$$= -\Delta U = U_1 - U_2$$

$$\Rightarrow U = -pE\cos\phi = \boxed{-\vec{p}\cdot\vec{E}}$$ potential energy for a dipole

Potential energy is minimum for $\phi = 0$, therefore dipoles orient themselves in $\phi = 0$ position under applied constant E-field.

Ex 21.15: Field of an Electric Dipole



An electric dipole is centred at the origin, with $\vec{p}$ in the direction of the y-axis. What is the E-field at a point on the y-axis for $y \gg d$?

$$E_y = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2}\right\}$$

Use the binomial expansion:

$$(1+x)^n = 1 + nx + n(n-1)\frac{x^2}{2} + \dots, \quad \text{for } x \to 0 \quad (1+x)^n \cong 1 + nx$$

to a good approximation

$$\Rightarrow \frac{1}{(y-d/2)^2} = \left(y - \frac{d}{2}\right)^{-2} = y^{-2}\left(1 - \frac{d}{2y}\right)^{-2} \cong y^{-2}\left(1 + \frac{2d}{2y}\right) = \frac{1}{y^2}\left(1 + \frac{d}{y}\right)$$

$$\frac{1}{(y+d/2)^2} \cong \frac{1}{y^2}\left(1 - \frac{d}{y}\right)$$

$$\Rightarrow E_y \cong \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{y^2}\left(\cancel{1} + \frac{d}{y}\right) - \frac{1}{y^2}\left(\cancel{1} - \frac{d}{y}\right)\right\} = \frac{q}{4\pi\epsilon_0}\,\frac{2d}{y^3}$$

$$= \frac{p}{2\pi\epsilon_0 y^3}, \quad p = qd$$