CHAPTER 25: Current, Resistance and Electromotive Force

Current:

Motion of charge from one region to another.

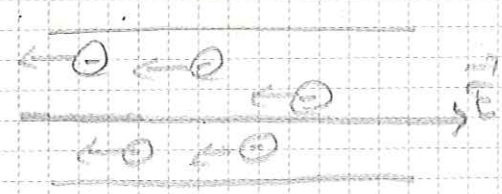
In a conductor, under a steady electric field $\vec{E}$, charges would be subject to constant force $\vec{F} = q\vec{E}$.

$\Rightarrow$ Motion with constant acceleration would be observed in vacuum, however in a conductor the charge will collide with ions in the material.

The resulting motion is then approximated by a motion with constant velocity. Constant velocity is called the drift velocity, $\vec{v}_D$.

This results in a constant electric current.

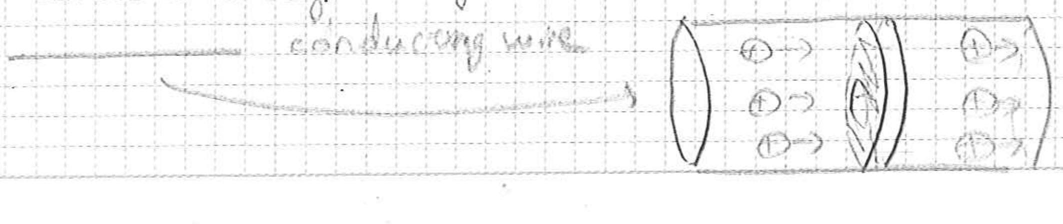
Current is conventionally accepted to flow in the direction of positive charge flow.



$\longrightarrow I$

Formal definition:

If we consider a segment of a conductor:



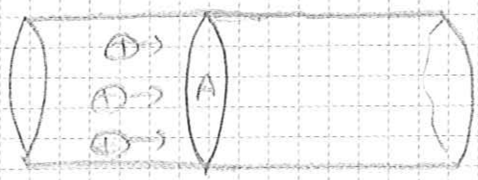
Current through a cross-sectional area A is defined as the net charge flowing through the area per unit time.

$$I = \frac{dQ}{dt}$$

SI unit of current is ampere.

$$(A) = \frac{(C)}{(s)}$$

Consider the cross section A:



Total charge flowing through A in a time Δt is:

$$\Delta Q = (A v_D \Delta t \underbrace{n}_{\text{concentration of particles}})\, q \longrightarrow \text{unit charge of the particles}$$

$$\Rightarrow \frac{\Delta Q}{\Delta t} = n q v_d A \Rightarrow \text{for } \Delta t \to 0;\ I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \boxed{n q v_D A}$$

We define the current per unit cross-sectional area;

$$\text{current density, } J = \frac{I}{A} = \boxed{n q v_D} \quad \overset{\text{units}}{\left(\frac{A}{m^2}\right)}$$

If the moving charges are negative, current density is in the direction opposite to drift velocity, $\vec{v}_D$

Therefore, we define $\vec{J} = n q \vec{v}_D$, vector current density.

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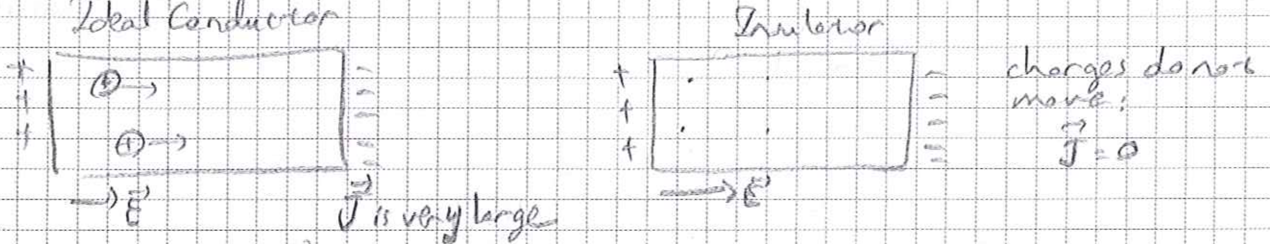
25.2. Resistivity:

The current density, $\vec{J}$, depends on the electric field, $\vec{E}$.



In general, there is a nearly linear relationship between $\vec{J}$ and $\vec{E}$. The proportionality constant

$$\boxed{\rho = \frac{E}{J}}$$ is called as the resistivity.

resistivity

In a material with large ρ given E results in less current. (more resistence!)

units: $(\Omega m) = \left(\frac{V/m}{A/m^2}\right) = \left(\frac{V\,m}{A}\right) = (\Omega m)$

Ohm

We define the conductivity as:

$$\sigma = \frac{1}{\rho} \longrightarrow J = \sigma E = \frac{1}{\rho} E$$

conductivity

Resistivity and Temperature:

At a constant T, resistivity is constant.

Resistivity of a metallic conductor changes with the dependence below;

$$\rho(T) = \rho_0 \left[1 + \alpha \left(T - T_0\right)\right]$$ reference temperature

temperature coefficient, $\alpha > 0$, for most metallic conductors.

ρ increases with T for most conductors.

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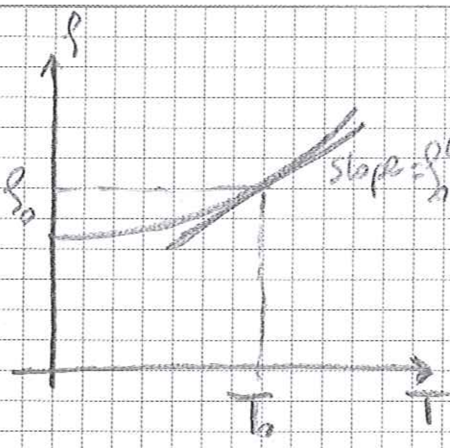
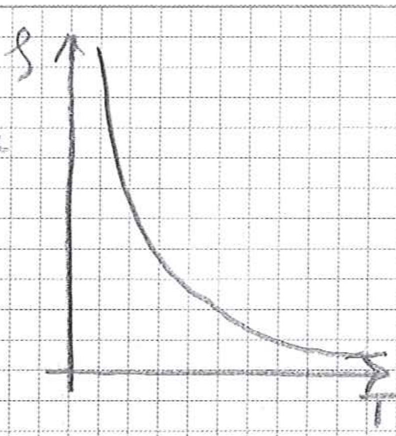
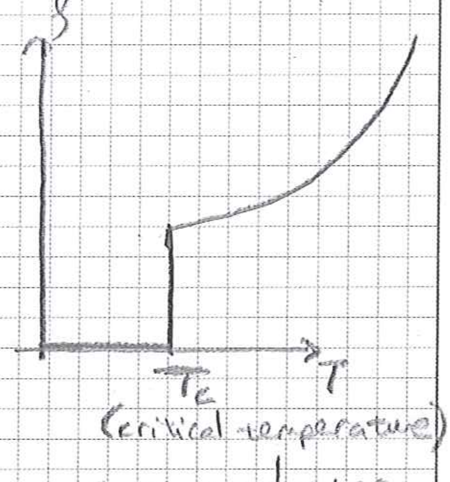






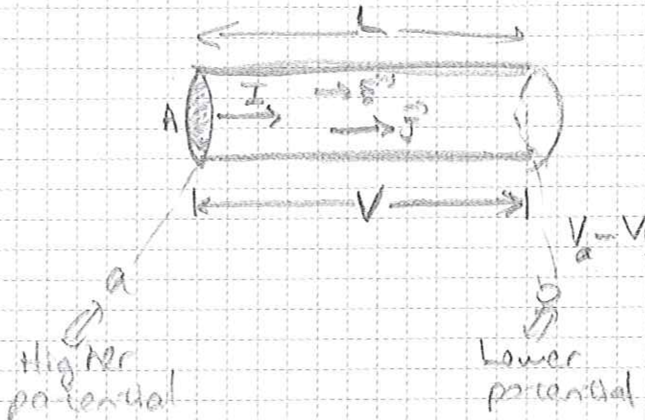
normal metal

semiconductor

superconductor

25.3 Resistance

Consider a conductor with uniform cross section A and length L.



Potential difference between a to b is:

$$V_a - V_b = V_{ab} = \int_a^b \vec{E} \cdot d\vec{l} = E \cdot L$$

$V = E L$, current $I = J A$ (current density)

$= \rho J L$

$V = \frac{\rho}{A} I L$

$\Rightarrow V = \frac{\rho L}{A} I \iff$ When ρ is constant V is proportional to I.

Ratio $\frac{V}{I} = R$ is called the resistance.

$\boxed{V = I R}$ Another way to write the Ohm's Law

$(V) = (A)(\Omega)$

Ex 25.2: Copper wire with a diameter of 1.02 mm, cross-sectional area $A = 8.2 \times 10^{-7} m^2$, carries a current $I = 1.67 A$. ($\rho = 1.72 \times 10^{-8} \Omega m$ for Copper)

(a) Electric field magnitude in the wire:

$$E = \rho J = \rho \frac{I}{A} = 1.72 \times 10^{-8} \Omega m \frac{1.67 A}{8.2 \times 10^{-7} m^2} = 0.035 V/m$$

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(b) Potential difference between 2 points 50m apart in the wire:

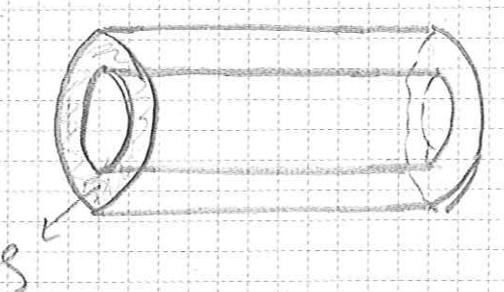
$$V = E\Delta L = 0.035\,\frac{V}{m} \times 50m = 1.75V$$

(c) Resistance of a 50m length of the wire:

$$R = \frac{\rho L}{A} = 1.72\times10^{-8}\,\Omega m \,\frac{50m}{8.2\times10^{-7}m^2} = 1.05\,\Omega$$

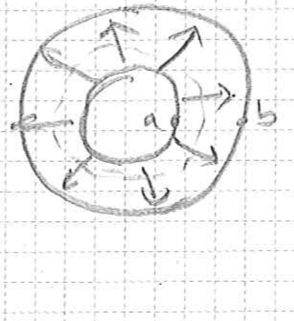
Ex 25.4: Resistance calculation

Consider a hollow cylinder of length L inner radius a, outer radius b.



Hollow cylinder made out of a material with resistivity ρ.

Potential difference is applied between the inner and outer surfaces, so that current flows radially.



What is the resistance of the radial current flow?

$V = IR$ $\qquad J = \frac{1}{\rho}E$

$$V_{ab} = \int_a^b \vec{E}\cdot d\vec{l} = \int \rho \vec{J}\cdot d\vec{l} = \rho \int_a^b \vec{J}\cdot d\vec{l} = \rho\int_a^b J(r)\,dr$$

continuity relationship

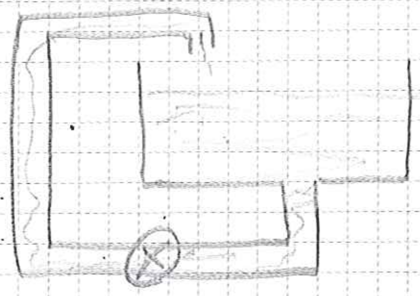
At a radial distance r: $J(r)\, 2\pi r L = I \Rightarrow J(r) = \frac{I}{2\pi r L}$

$$\Rightarrow V = \rho\int_a^b \frac{I}{2\pi r L}\,dr = \frac{\rho I}{2\pi L}\left(\ln(r_b) - \ln(r_a)\right)$$

$$\Rightarrow \boxed{R = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)}$$

25.4 Electromotive Force and Circuits

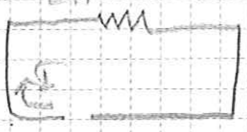
Consider a water circuit



Water would keep running if there is a water pump somewhere.

Otherwise the water flow will stop.

In an electric circuit there are such elements which direct the current from lower to higher potential.



"Batteries, electric generators, solar cells, thermocouples, fuel cells"

Inside a source of electromotive force:

$\vec{F}_e$: electric force applied to charge q
$\vec{F}_e = q\vec{E}$

$\vec{F}_n$: non-electrostatic force, in the direction "uphill" from − to +.

If a charge q is moved from b to a, $\vec{F}_n$ does a work:

$$W_n = q\mathcal{E}$$

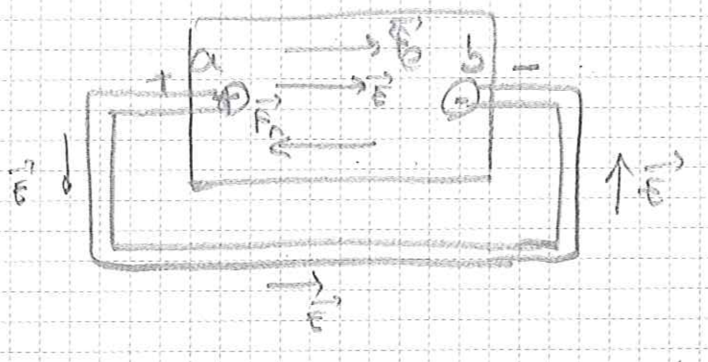
$$W_n - \underbrace{qV_{ab}}_{\text{work done by the electric force}} = \Delta K = 0 \quad \rightarrow \quad W_n = qV_{ab} = q\mathcal{E}$$

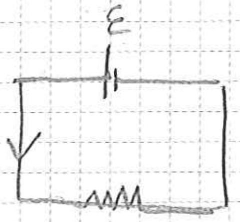
$$\rightarrow \boxed{V_{ab} = \mathcal{E}}$$

(emf)

electromotive force

If we now consider a complete circuit:





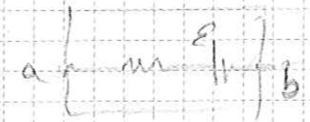
Inside the source of emf: $V_{ab} = \mathcal{E}$

In the circuit: $V_{ab} = IR$

$$\Rightarrow \boxed{\mathcal{E} = V_{ab} = IR}$$

Internal Resistance:

Any real source of emf has internal resistance such that potential difference is not equal to the emf.

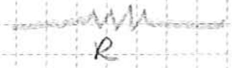


$$\boxed{V_{ab} = \mathcal{E} - Ir}$$

internal resistance

Symbols for Circuit Diagrams:

Conductor (negligible resistance)



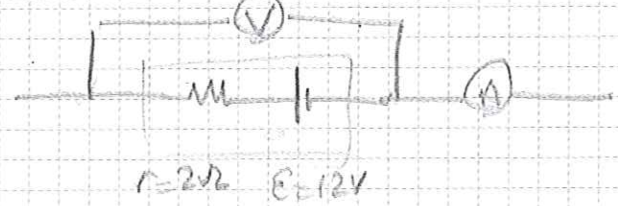
Resistor

Source of Emf (Longer vertical line represents the positive terminal)

Voltmeter (measures the pot. difference between its terminals)

Ammeter (measures current through it)

Ex 25.5: Open circuit

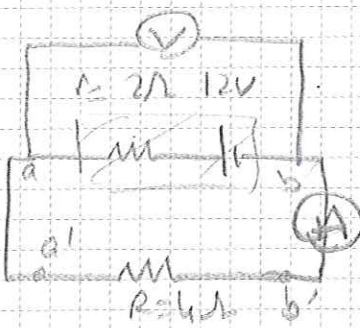


Readings of V and A

⇒ No current flow open circuit ⇒ A reads 0

V reads 12V

Ex 25.6:



Readings of V and A?

$V_{ab} = \mathcal{E} - Ir = IR$

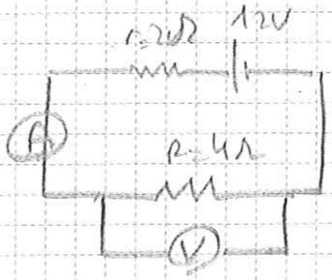
$\Rightarrow \mathcal{E} = I(r+R)$

$12V = I(6\Omega) \Rightarrow I = 2A$

A reads 2A

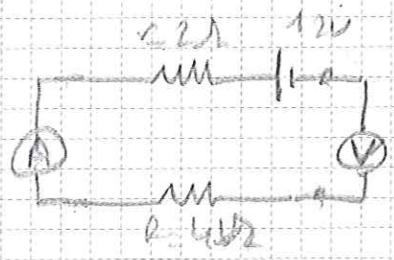
V reads $12 - 2A\,2\Omega = 8V$

Ex 25.7:



$12V = 6I \Rightarrow I = 2A$

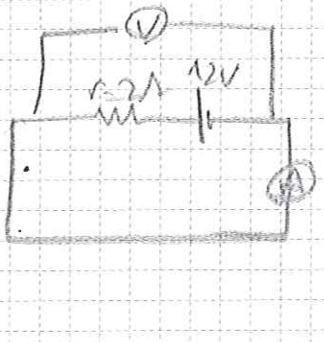
$V = 4I = 8V$



$I = 0$

$V_{ab} = 12V$

Ex 25.8:



V reads 0 V

$12V - I\,2\Omega = 0$

$\Rightarrow \boxed{I = 6A}$

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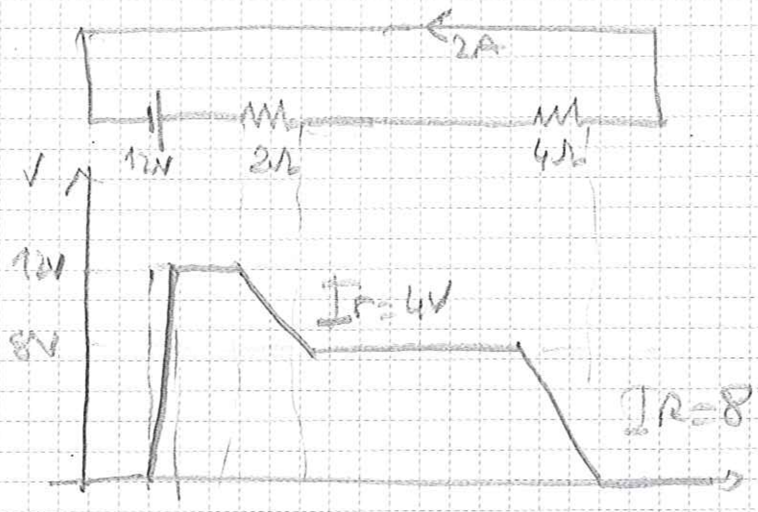
6b



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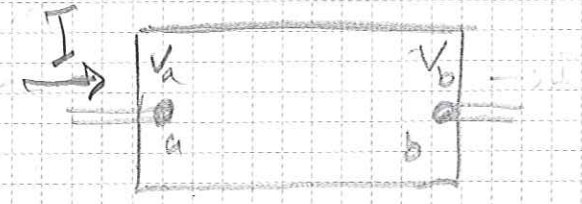
25.5 Energy and Power in Electric Circuits:

Consider a circuit element and a constant current flow.



A charge q is passing from terminal a to terminal b. Since the current flow is constant there is no change in the kinetic energy of the charge between the two terminals.

$$\Rightarrow \Delta K = 0 = \underbrace{W_e}_{\text{work done by the electric forces}} + W_n = q(V_a - V_b) + W_n$$

$$\Rightarrow W_n = -qV_{ab}$$

if $V_{ab} > 0 \Rightarrow$ work done by the electric forces is converted into heat, (internal energy)

if $V_{ab} < 0 \Rightarrow$ This is a source of electromotive force.

Chemical energy
solar energy
fuel energy
⋮
} converted to electrical energy.

$\Rightarrow qV$: is the amount of energy delivered or extracted

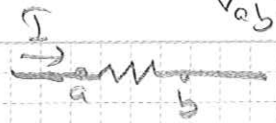
Time rate of change of the energy transfer:

$$P = \frac{d}{dt}(qV) = V\frac{dq}{dt} = \boxed{VI}$$

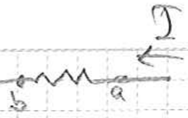
rate at which energy is delivered or extracted from the circuit element.

$$P = VI$$

$$\frac{J}{C} \cdot \frac{C}{sec} = \left(\frac{J}{sec}\right) = \text{Watt}$$



$V_{ab} > 0$



$V_{ab} > 0$

Pure Resistor:

For a pure resistor: $P = VI = I^2R = \frac{V^2}{R}$ is the power delivered to the resistor.

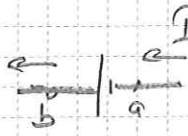
$V = IR$

electric energy is converted to heat.

Power Output of a Source:

Consider a battery with an emf of $\mathcal{E}$, and no internal resistance. The power that this source can deliver is:

$$P = VI = \mathcal{E} I$$



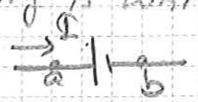
$V_{ab} < 0$

If the source has an internal resistance r:

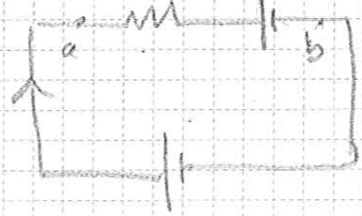
$$P = VI = (\mathcal{E} - Ir)I = \mathcal{E}I - I^2 r$$

Power Input to a Source:

Suppose that another source with a larger emf is connected to another source of emf.



$V_{ab} > 0$

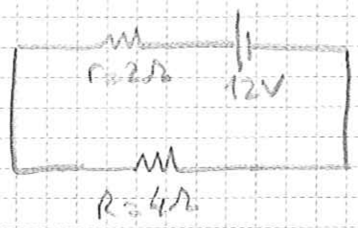


Such that current flows in the opposite direction.

$$\Rightarrow V_{ab} = Ir + \mathcal{E} \Rightarrow P = (Ir + \mathcal{E})I = \underbrace{\mathcal{E}I}_{\text{power input to the emf source}} + \overbrace{I^2 r}^{\text{power converted to heat}}$$

Example is charging of a battery.

Ex 25.9: Power input and output in a complete circuit.



Rate of energy conversion $\mathcal{E}I$

Rate of dissipation of energy I^2r

Net power output of the battery $\mathcal{E}I - I^2r$

$I = 2A \Rightarrow \mathcal{E}I = 24W$ $\quad \mathcal{E}I - I^2r = 16W$

$I^2r = 8W$

Ex 25.10:

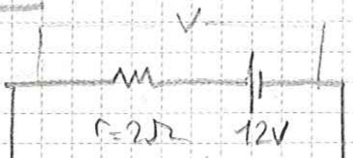
If 4Ω resistor is replaced by a 8Ω resistor. How is the dissipated power affected:

$P = VI$, for $R = 4\Omega$, $I = 2A$ $\quad P = 16W$

$= I^2R$ $\quad$ for $R = 8\Omega$ $\quad I = 1.2A$ $\quad P = (1.2A)^2 8\Omega = 11.52W$

$\Rightarrow$ Dissipated power decreased.

Ex 25.11: Power in a short circuit.



Rate of energy conversion $\mathcal{E}I$

— dissipation I^2r

Net power output of the battery

$I = \frac{12V}{2\Omega} = 6A \Rightarrow \mathcal{E}I = 72W$

$I^2r = 72W \Rightarrow$ No net power output from the battery.