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Chapter 26: Direct Current Circuits

Direct current (dc) circuits:

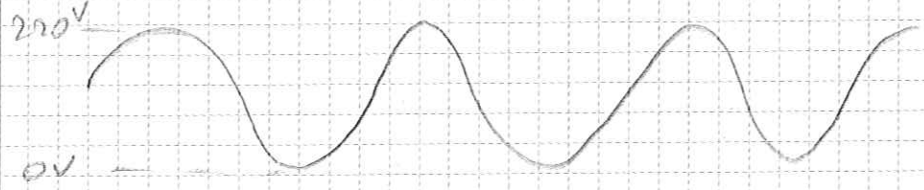
Current, electric potential in the circuit has no time dependence

5V DC Power Supply

9V DC Power Supply ...

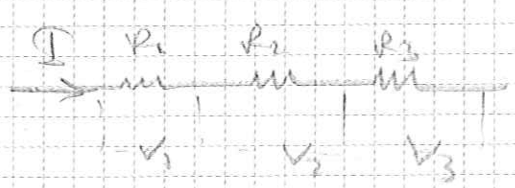
Alternating current (ac) circuits:

Current, electric potential in the circuit oscillates with time.



220V AC Voltage

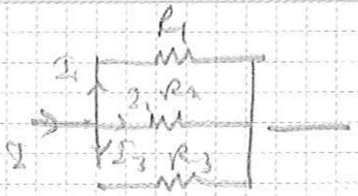
Resistors In Series and Parallel:



$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I\underbrace{(R_1 + R_2 + R_3)}_{R_{eq}}$$

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Equivalent resistance of resistors connected in series



$I = I_1 + I_2 + I_3$ ⟸ conservation of charge flow!

$V_1 = V_2 = V_3 = V$

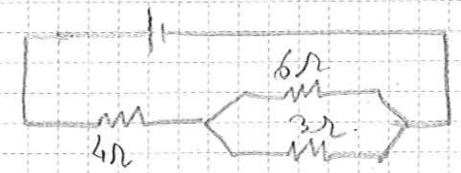
$$\Rightarrow I = I_1 + I_2 + I_3$$

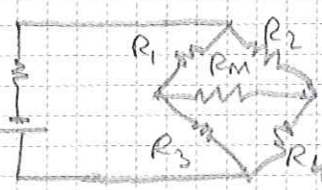
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_{eq}}$$

$$\Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

Equivalent resistance of resistors connected in parallel.

26.2 Kirchhoff's Rules:





Many practical resistor networks cannot be solved using the simple series-parallel resistor reduction techniques.

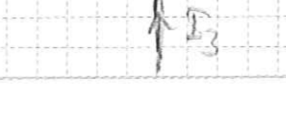
Kirchoff's Rules help analyzing complicated circuits.

Kirchhoff's Rules:

(i) The algebraic sum of the currents into any junction is equal to zero.

$\Sigma I = 0$, junction rule.

Junction: A point in the circuit where three or more conductors meet.



$I_1 + I_2 + I_3 = 0 \Rightarrow I_3 = \underbrace{-(I_1 + I_2)}$ flows in the opposite direction

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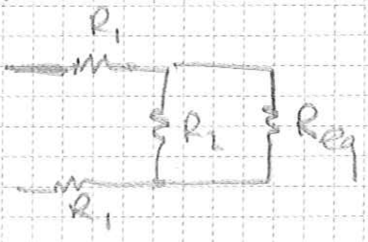
Prob 26.92

Equivalent resistance?



$$\frac{1}{R_2} + \frac{1}{R_{eq}} = \frac{R_{eq} + R_2}{R_{eq} R_2}$$

$$\Rightarrow R_{eq} = 2R_1 + \frac{R_2 R_{eq}}{R_{eq} + R_2}$$

$$\Rightarrow (R_{eq} - 2R_1)(R_{eq} + R_2) = R_{eq} R_2$$

$$\Rightarrow R_{eq}^2 + R_{eq}(\cancel{R_2} - 2R_1 - \cancel{R_2}) - 2R_1 R_2 = 0$$

$$R_{eq}^2 - 2R_1 R_{eq} - 2R_1 R_2 = 0$$

$$\Rightarrow R_{eq} = \frac{2R_1 \pm \sqrt{4R_1^2 + 8R_1 R_2}}{2} = R_1 \pm \underbrace{\sqrt{R_1^2 + 2R_1 R_2}}_{> R_1}$$

$$\Rightarrow \text{Solution is } \boxed{R_{eq} = R_1 + \sqrt{R_1^2 + 2R_1 R_2}}$$

$$R_{eq} \neq R_1 - \sqrt{R_1^2 + 2R_1 R_2} \Leftarrow$$ is not a physical solution

(ii) The algebraic sum of the potential differences in any loop (associated with emfs, and resistive elements) must equal zero.

$$\sum V = 0,$$ Kirchhoff's Loop Rule

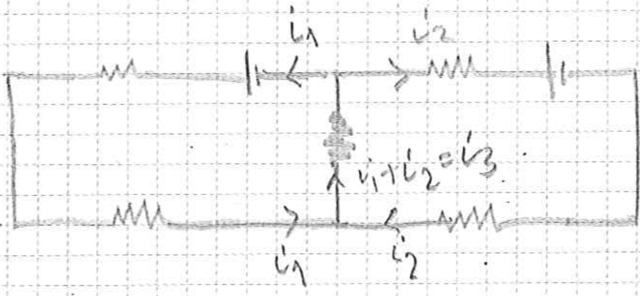
Loop: Any closed conducting path in the circuit.

This is clear because electrostatic force is conservative.

$$W_{a \to a} = -\Delta U = U_i - U_a = 0$$

How to solve circuits using Kirchoff's Law:

Given a circuit:

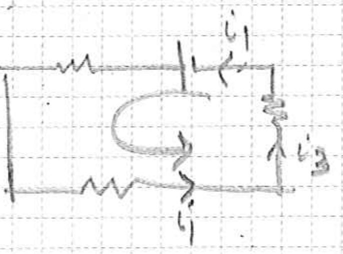


- Label currents with their directions. The directions need not be the correct directions

Write the Kirchoff's Junction Law in the junctions:

$$i_1 + i_2 = i_3$$

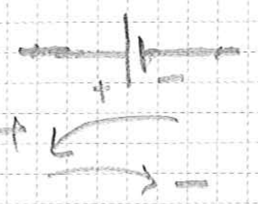
- Choose any closed loop in the circuit, designate a direction of travel.



Travel around the loop adding potential differences due to different elements.

Potential differences due to different elements:

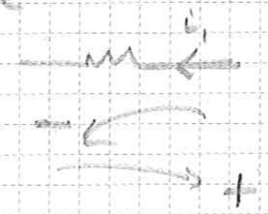
emf



Emf is counted + if you traverse from − to +.

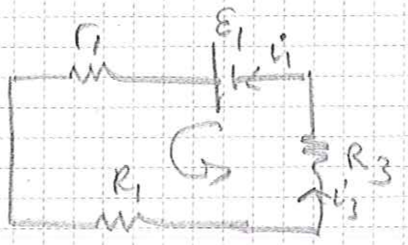
Emf is counted − if you traverse from + to −.

Resistance



Resistance is − if you traverse in the same direction as current flow.

Resistance is + if you traverse in the direction opposite to current.

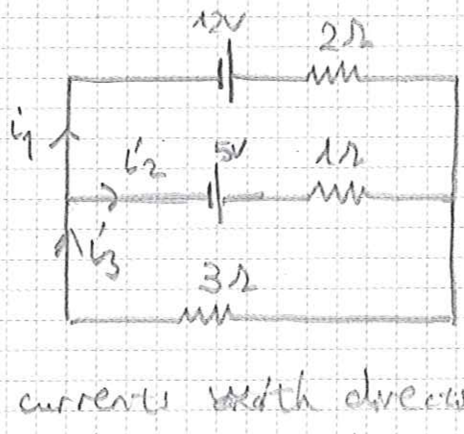


$$\cancel{V_a} + \mathcal{E}_1 - i_1 r_1 - i_1 R_1 - i_3 R_3 = \cancel{V_a}$$

$$\boxed{\mathcal{E}_1 - i_1 r_1 - i_1 R_1 - i_3 R_3 = 0}$$

!! Write as many linearly independent equations as the number of currents to solve for the circuit.

Ex 26.4: Consider the circuit:



What are the currents?

(i) Label currents with directions

(ii) Apply Kirchoff's Junction Law

$$i_1 + i_2 = i_3$$

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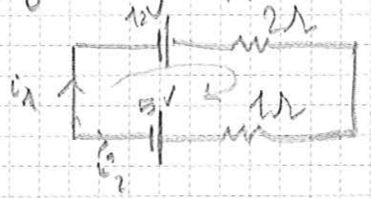
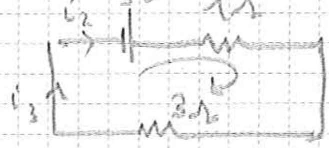
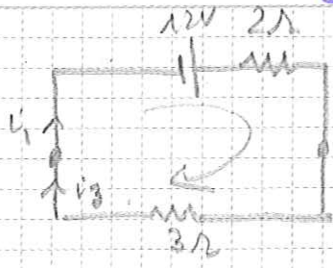
Apply Kirchoff's Loop Law:







$12 - 2i_1 + i_2 - 5 = 0$ $\quad$ $5 - i_2 - 3i_3 = 0$ $\quad$ $12 - 2i_1 - 3i_3 = 0$

$7 - 2i_1 + i_2 = 0$ $\quad$ $5 - i_2 - 3i_3 = 0$ $\quad$ $12 - 2i_1 - 3i_3 = 0$

2 linearly independent equations.

Add the first 2:

$$7 - 2i_1 + i_2 = 0$$
$$5 - i_2 - 3i_3 = 0$$
$$\overline{12 - 2i_1 - 3i_3 = 0}$$

∴ Use the equations;

$i_1 + i_2 = i_3$

$7 - 2i_1 + i_2 = 0$

$5 - i_2 - 3i_3 = 0 \Rightarrow 5 - i_2 - 3i_1 - 3i_2 = 5 - 3i_1 - 4i_2 = 0$

$\Rightarrow i_2 = \frac{5}{4} - \frac{3}{4} i_1$

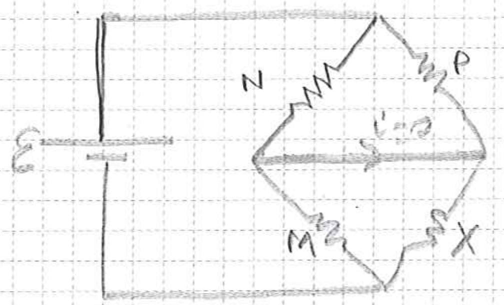
$\Rightarrow 7 - 2i_1 + \frac{5}{4} - \frac{3}{4} i_1 = \frac{33}{4} - \frac{11}{4} i_1 = 0 \Rightarrow \underline{i_1 = 3A}$

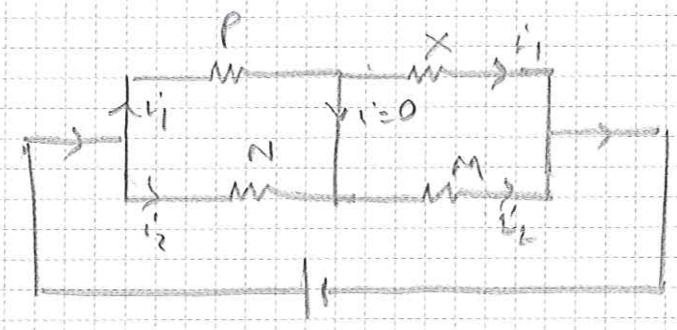
$\underline{i_2 = \frac{5}{4} - \frac{3}{4} i_1 = \frac{5}{4} - \frac{9}{4} = -1A}$

$\underline{i_3 = 2A}$

Prob 26.77

Show that $X = \frac{MP}{N}$





$i_1 P = i_2 N \Rightarrow \frac{i_1}{i_2} = \frac{N}{P}$

$i_1 X = i_2 M$

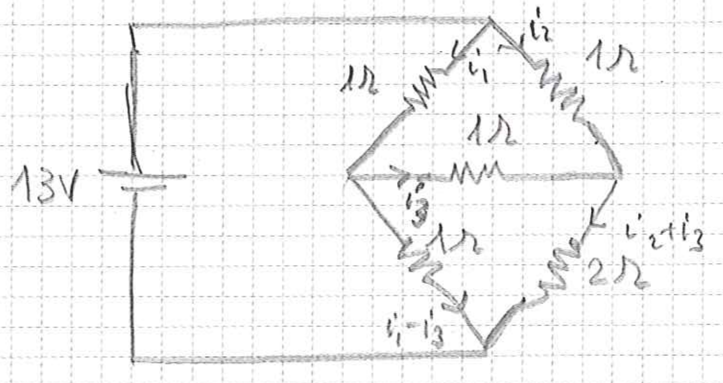
$\Rightarrow \frac{i_1}{i_2} = \frac{M}{X}$

$\Rightarrow \frac{M}{X} = \frac{N}{P} \Rightarrow \boxed{X = \frac{MP}{N}}$

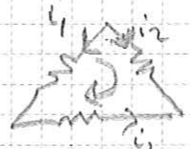
Wheatstone Bridge

Ex 26.6:

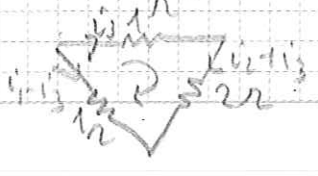
Find the current in each resistor.



3 Unknowns i_1, i_2, i_3 ⇒ Write 3 equations using Kirchoff's Loop Law:



$-i_2 + i_3 + i_1 = 0$



$-2(i_2+i_3) + i_1 - i_3 - i_3 = 0$

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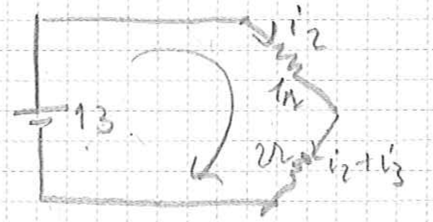

$$-i_2-2(i_2+i_3)+13=0$$

$$\Rightarrow \quad i_1-i_2+i_3=0$$

$$i_1-2i_2-4i_3=0$$

$$-3i_2-2i_3=-13$$

$$2i_1-i_3=13 \Rightarrow i_3=2i_1-13$$

$$\Rightarrow i_1-i_2+2i_1-13=0 \Rightarrow 3i_1-i_2=13$$

$$i_1-2i_2-4(2i_1-13)=0 \Rightarrow -7i_1-2i_2=-52$$

$$-13i_1=-52-26=-78 \Rightarrow i_1=6A$$

$$i_2=3i_1-13=18-13=5A$$

$$i_3=2i_1-13=12-13=-1A$$

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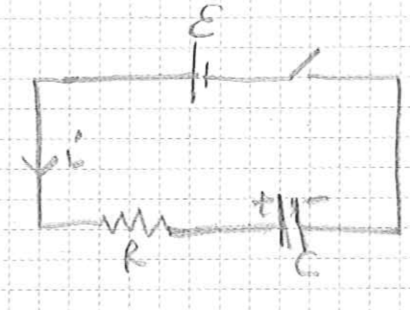
26.4 R-C Circuits

Circuits that involve charging or discharging of capacitors.



Charging a Capacitor:

Consider the capacitor being initially uncharged.

Then at time $t=0$ the switch is closed.

$\Rightarrow$ Capacitor then starts being charged.

$$C=\frac{Q}{V} \Rightarrow Q=CV \Rightarrow \frac{dQ}{dt}=C\frac{dV}{dt}=i$$

$$\mathcal{E}-iR-V=0 \Rightarrow \mathcal{E}-RC\frac{dV}{dt}-V=0$$

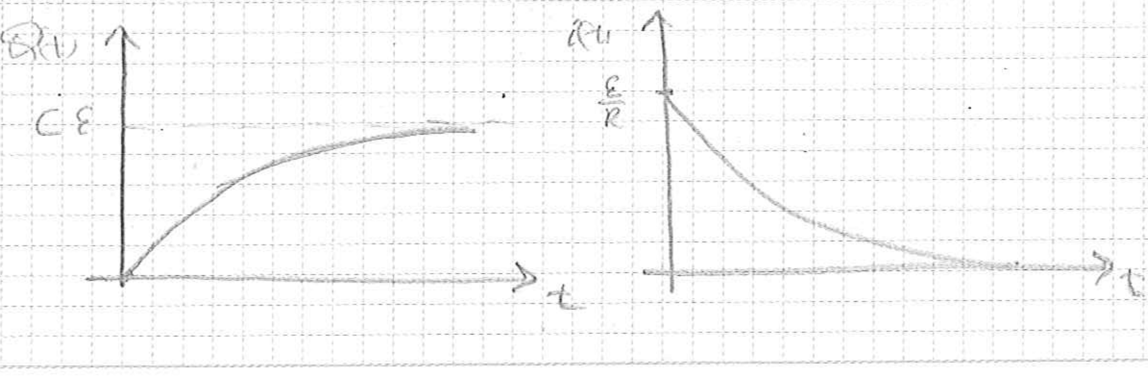
$$RC\frac{dV}{dt}+V-\mathcal{E}=0 \Rightarrow \int_0^V \frac{dV}{\mathcal{E}-V}=\int_0^t \frac{dt}{RC}$$

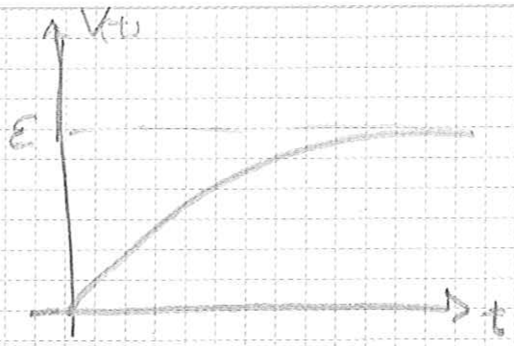
$$\Rightarrow -\ln(\mathcal{E}-V)\Big|_0^V=+\frac{t}{RC} \Rightarrow -\ln(\mathcal{E}-V)-\ln(\mathcal{E})=\frac{t}{RC}$$

$$\Rightarrow -\ln\left(\frac{\mathcal{E}-V}{\mathcal{E}}\right)=\frac{t}{RC} \Rightarrow \frac{\mathcal{E}-V}{\mathcal{E}}=e^{-t/RC} \Rightarrow \boxed{V=\mathcal{E}-\mathcal{E}e^{-t/RC}}$$

$$Q=CV \Rightarrow Q(t)=C\mathcal{E}\left(1-e^{-t/RC}\right) \quad //$$

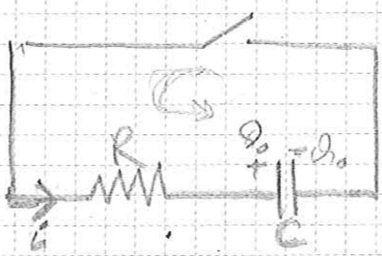
$$i=\frac{dQ}{dt}=\cancel{C}\mathcal{E}\left(\frac{1}{R\cancel{C}}e^{-t/RC}\right)=\frac{\mathcal{E}}{R}e^{-t/RC} \quad //$$





At time $t=RC$, current drops to $\frac{1}{e}$ of its initial value
$\tau = RC$ is called the time constant of the R-C circuit.

Discharging a Capacitor:



Consider an initially charged capacitor. At time $t=0$ the switch is closed.

$$\Rightarrow C = \frac{Q(t)}{V(t)}$$

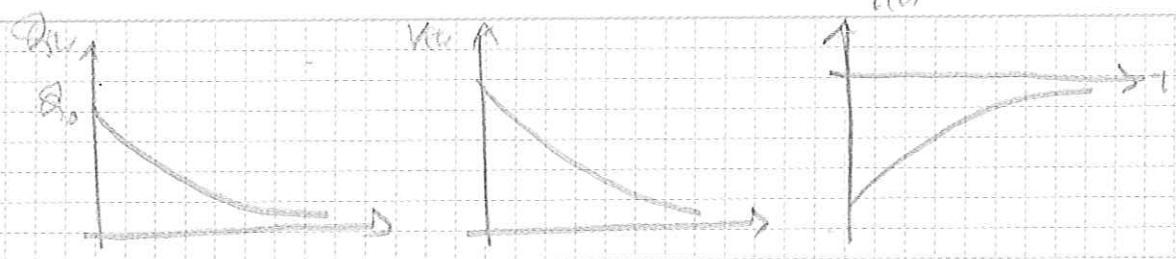
$$V(t) + i(t)R = 0 \Rightarrow \frac{Q(t)}{C} + \frac{dQ(t)}{dt}R = 0$$

$$\Rightarrow Q(t) = -RC\frac{dQ(t)}{dt} \Rightarrow \frac{dQ(t)}{Q(t)} = \frac{-dt}{RC} \Rightarrow \ln Q \Big|_{Q_0}^{Q} = \frac{-t}{RC}\Big|_0^t$$

$$\Rightarrow \ln\left(\frac{Q}{Q_0}\right) = \frac{-t}{RC} \Rightarrow \boxed{Q = Q_0 e^{-t/RC}}$$

$$V(t) = \frac{Q(t)}{C} = \boxed{\frac{Q_0}{C} e^{-t/RC}}$$

$$i(t) = +\frac{V(t)}{R} = \boxed{+\frac{Q_0}{RC} e^{-t/RC}}$$

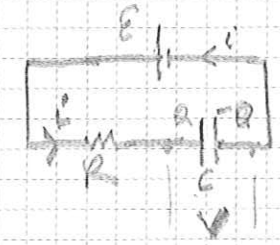


Time constant : $\boxed{\tau = RC}$

time at which the stored charge drops to $\frac{1}{e}$ of its initial value.

Energy Considerations

In charging of a capacitor:



$\mathcal{E}i$, rate at which the battery delivers energy to the circuit.

$$\mathcal{E}i = i^2R + Vi = i^2R + \frac{Q}{C}i$$

Total energy supplied by the battery during charging of the capacitor:

$$\int P\,dt = \int \mathcal{E}\frac{dQ}{dt}\,dt = \int_0^{Q_f} \mathcal{E}\,dQ = \mathcal{E}Q_f$$

↑ Final charge of the capacitor

Total energy stored in the capacitor

$$U = \frac{1}{2}Q_f V_f = \frac{1}{2} Q_f \mathcal{E}$$ ⇐ Half of the energy is stored in the capacitor. Half is dissipated in the resistor.

$$U = \int \frac{Q(t)}{C}\, i(t)\,dt = \int \frac{Q}{C}\frac{dQ}{dt}\,dt = \int \frac{Q}{C}\,dq = \frac{1}{2C}Q_f^2 = \boxed{\frac{1}{2} Q_f \mathcal{E}}$$

∴ This result does not depend on the values of R, E and C.

Discharging of a Capacitor

Power dissipated by the capacitor:

$$i\frac{Q}{C} = i^2R$$

$$\Downarrow$$

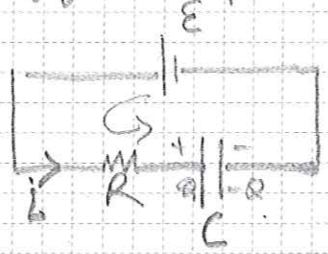
$$\int i\frac{Q}{C}\,dt = \int -\frac{dQ\,Q}{C} = -\frac{1}{2C}\,Q^2\Big|_{Q_0}^{0} = +\frac{1}{2C}Q_0^2 \Leftarrow$$ Total energy capacitor delivers to the circuit.

$$\int i^2R\,dt = \int \frac{Q_0^2}{R^2C^2}e^{-2t/RC}R\,dt = \frac{Q_0^2}{RC^2}\int e^{-2t/RC}dt$$

$$= \frac{Q_0^2}{RC^2}\;-\frac{RC}{2}\;e^{-2t/RC}\Big|_0^{\infty} = \frac{Q_0^2}{C}\left(\frac{-1}{2}\right)(0-1) = \frac{Q_0^2}{2C}$$ Energy dissipated in the resistor

RC - Circuits:

Charging a capacitor



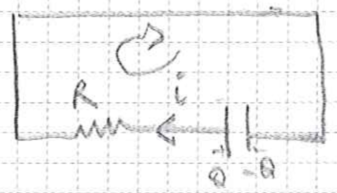
$\frac{dQ}{dt} = i$ ⟸ direction is important

$\mathcal{E} - iR - \frac{Q}{C} = 0 \Rightarrow \mathcal{E} = \frac{dQ}{dt}R + \frac{Q}{C}$

$\Rightarrow Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$

$i = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$

Discharging a capacitor:



$i = -\frac{dQ}{dt}$

$\Rightarrow \quad -iR + \frac{Q}{C} = 0$

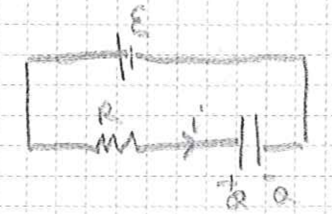
$\Rightarrow -\frac{dQ}{dt}R = \frac{Q}{C} \quad \Rightarrow \quad Q = Q_0 e^{-t/RC}$

$V = \frac{Q(t)}{C} = \frac{Q_0}{C} e^{-t/RC}$

$i(t) = \frac{V(t)}{R} = \frac{Q_0}{RC} e^{-t/RC}$

RC : time constant

Energy Considerations:



rate of energy dissipated is: $i^2R + Vi$

rate of energy delivered is: $\mathcal{E} i$

stored in the capacitor: $U = \int V \frac{dq}{dt} dt = \int \frac{Q}{C} dQ = \frac{1}{C}\frac{Q_f^2}{2}$

$= \frac{1}{2} Q_f V_f = \frac{1}{2} Q_f \mathcal{E}$

delivered by $\mathcal{E}$: $\int \mathcal{E} \frac{dq}{dt} dt = \int_0^{Q_f} \mathcal{E} dq = \mathcal{E} Q_f$