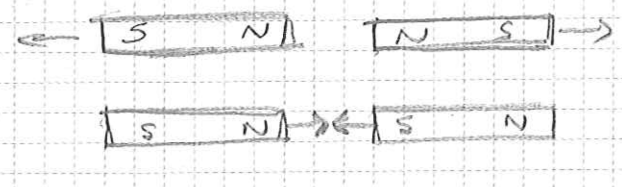
KOÇ ÜNİVERSİTESİ

Chapter 27: Magnetic Field and Magnetic Forces

Magnetism

There are materials called permanent magnets that attract and repel each other.

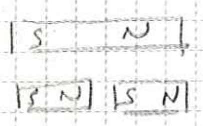


A magnet also attracts an unmagnetized object that contains iron. Both north and south poles would attract an unmagnetized object.

This is so far similar to electrically charged bodies.

Magnets produce magnetic fields, and a compass needle aligns itself with respect to the magnetic field lines.

⇒ One big difference is that magnetic poles in a magnet always come in pairs. If you divide a magnet into two, each part will have a N and S pole.



The real physics at the microscopic level is that;

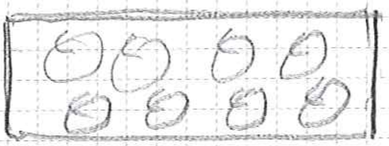
1)- A moving charge or a collection of moving charges produce a magnetic field.

If you place a compass next to a conducting wire, the needle's position will change with the current flow.

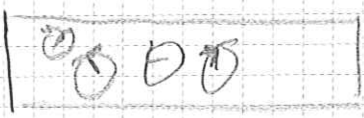
⇒ Stationary charges generate electric field
⇓ Coulomb's Law

Moving charges generate magnetic field
⇓ Biot-Savart Law

We can re-analyze the permanent magnets.



In a permanent magnet there is a coordinated motion of the atomic electrons.



In an unmagnetized body, motion of the atomic electrons are uncoordinated.

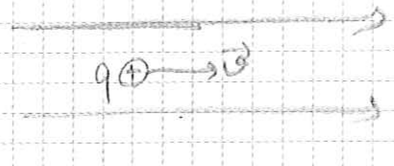
2) A moving charge (current) will respond to a magnetic field.
→ Magnetic force.

Magnetic Field; Magnetic Force

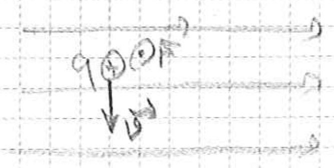
Consider a charged body with a net charge q. This body is moving with velocity $\vec{v}$ in a <u>constant</u> magnetic field $\vec{B}$.

The force applied to the charged body due to the magnetic field is:

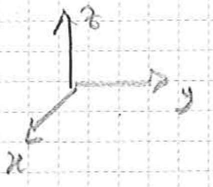
$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

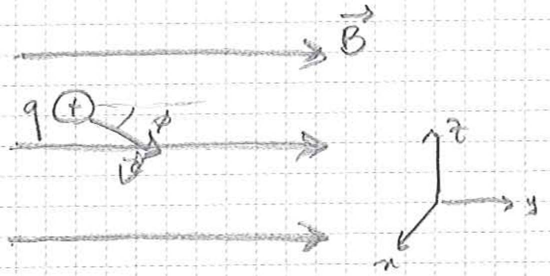


$\vec{F} = 0$



$\vec{F} = q\vec{v} \times \vec{B} = q\,v\,B\,\hat{i} \times \hat{j} = qvB\hat{k}$



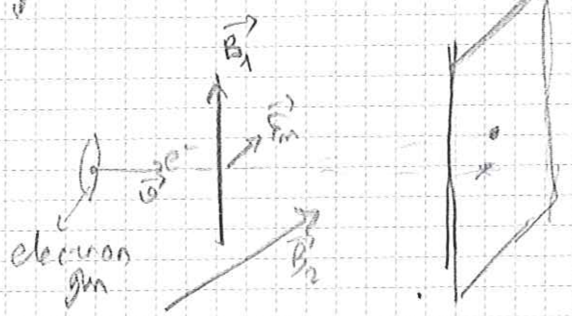


$\vec{F} = q\vec{v}\times\vec{B} = qvB\sin\phi\,\hat{k}$

Units: (B) : Tesla

$$1T = \frac{1N}{C\,m/s} = 1\,\frac{N}{Am}$$

Magnetic Force is used in tv screen, cathode ray tube.



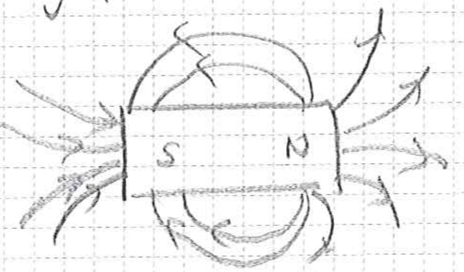
In a region where both electric and magnetic fields are present:

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)}$$

; Force is the vector sum of the electric and magnetic forces.

27.3 Magnetic Field Lines and Magnetic Flux

Magnetic field lines are the lines which are tangent to the magnetic field vector $\vec{B}$ at any point.

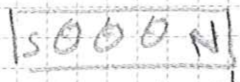


—Review—

Magnetism:

– A moving charge or a collection of moving charges produce a magnetic field.

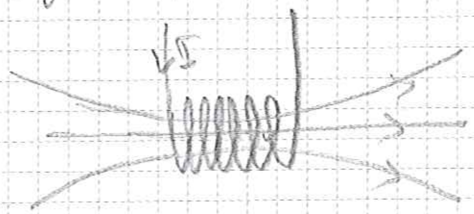
Permenent magnet



Atomic electrons move in a coordinated fashion.

$\vec{B}$: magnetic field is represented by $\vec{B}$

Magnetic field lines are drawn as the lines which are tangent to the magnetic field vector $\vec{B}$ at any point.



– A magnetic field will apply a magnetic force to a moving charge (current)

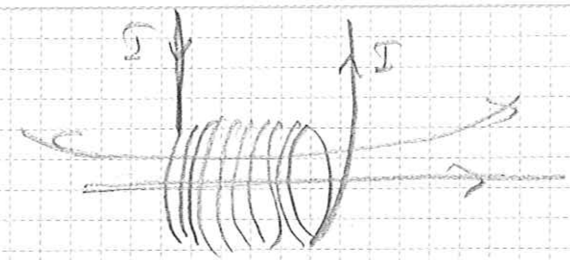
$$\vec{F}_m = q\,\vec{v} \times \vec{B}$$

In a region where both electric and magnetic fields are present:

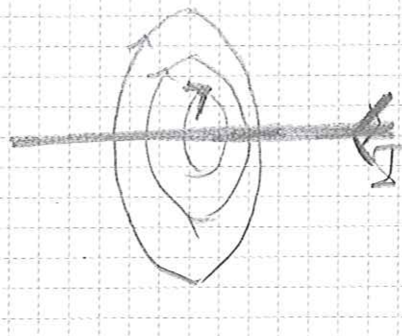
$$\vec{F} = \vec{F}_e + \vec{F}_m = q(\vec{E} + \vec{v} \times \vec{B}) \leftarrow \text{Lorentz Force}$$

Magnetic Flux

|
|
|



cylindrical current carrying solenoid.

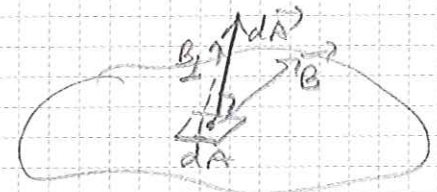


long, straight, current carrying wire

Magnetic Flux and Gauss' Law for Magnetism:

Magnetic flux through a surface;

$$\Phi_B = \int B_\perp dA = \int \vec{B}\cdot d\vec{A}$$



If $\vec{B}$ is uniform over a plane

$$\Phi_B = BA\cos\phi$$

SI unit of flux is Weber.

$$1\,Wb = 1\,T\,m^2$$

$\Rightarrow$ Gauss's Law: $\oint \vec{B}\cdot d\vec{A} = 0$, $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$ ← free charge

amount of net magnetic monopole is zero.

no magnetic monopole exist!

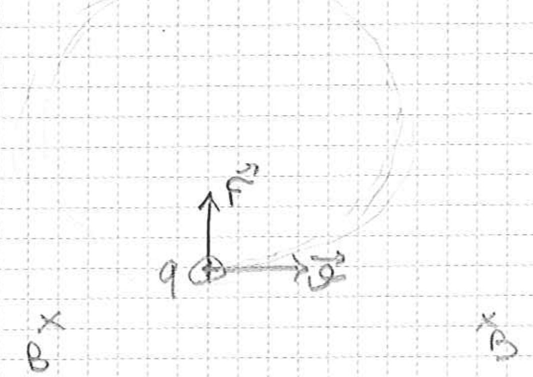
Magnetic field lines form closed loops

27.4 Motion of Charged Particles in a Magnetic Field

Consider a charged particle with charge q moving with velocity $\vec{v}$ under a magnetic field $\vec{B}$.

$$\vec{F} = q\vec{v} \times \vec{B}$$

$\vec{F}$ is always perpendicular to $\vec{v}$, direction of motion.
$\therefore$ $\vec{F}$ can never do work on the particle.
$\therefore$ Particle continues to its motion with constant speed
Only the direction of velocity changes.



Remember the uniform circular motion.
- The particle moves with constant speed.
- Constant radial acceleration
- The particle's velocity is perpendicular with B

$\Rightarrow$ Uniform circular motion.

Charged particle will undergo uniform circular motion.

$$F = |q| v B = m \underbrace{\frac{v^2}{R}}_{\text{constant acceleration}} \Rightarrow \boxed{R = \frac{mv}{|q|B}}$$ Radius of uniform circular motion.

Angular speed of the particle $\omega = \frac{v}{R} = \frac{v}{\frac{mv}{|q|B}} = \boxed{\frac{|q|B}{m}}$

$f = \frac{\omega}{2\pi} = \frac{|q|B}{2\pi m}$, is independent of Radius

f is called the cyclotron frequency.

If the particle has a velocity component parallel to $\vec{B} \Rightarrow$ Particle moves in a helix (84)

Ex 27.4: $q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg

uniform magnetic field directed along x-axis $B_x = 0.5$ T

At $t=0$ proton has $v_x = 1.5 \times 10^5$ m/s, $v_y = 0$, $v_z = 2 \times 10^5$ m/s

(a) At $t=0$, Force on proton and acceleration?

$$\vec{F} = q\vec{v} \times \vec{B} = q\vec{v} \times B_x \hat{i} = q(v_x \hat{i} + v_z \hat{k}) \times B_x \hat{i} = q v_z B_x \hat{j}$$

$$= 1.6 \times 10^{-14} \text{ N } \hat{j}$$

$$\vec{a} = \frac{\vec{F}}{m} = 9.58 \times 10^{12} \text{ m/s}^2 \ \hat{j}$$

(b) Radius of the helical path, the angular speed, distance traveled along the helix axis per revolution.

- circular motion in the yz-plane
- linear motion in the x direction

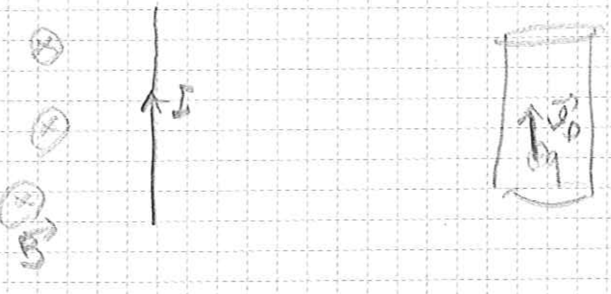
$$R = \frac{m v_z}{|q|B} = 4.18 \text{ mm}$$

$$w = \frac{v_z}{R} = \frac{|q|B}{m} = 4.79 \times 10^7 \text{ rad/s}$$

$$f = \frac{w}{2\pi} = \frac{1}{T} \Rightarrow T = \frac{2\pi}{w} = 1.31 \times 10^{-7} \text{ s} \Rightarrow \text{pitch} = v_x T = 19.7 \text{ mm}$$

27.6 Magnetic Force on a Current-Carrying Conductor

Consider a current carrying conductor in a constant magnetic field.



Current is made by many charge elements moving with an average velocity $\vec{v}_D$.

For a single charge element:

$$\vec{F} = q\,\vec{v}_D \times \vec{B}$$

Total force on all the charge elements

$$\vec{F} = \overbrace{nAl}^{N}\, q\, \vec{v}_D \times \vec{B}$$

Current in the wire is given as:

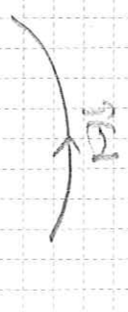
$$\vec{I} = \frac{\Delta q}{\Delta t} = \frac{q n A \vec{v}_D \Delta t}{\Delta t} = nA\vec{v}_D q$$

$$\Rightarrow \boxed{\vec{F} = l\,\vec{I} \times \vec{B}} = \boxed{I\,\vec{l} \times \vec{B}}$$

vector along the wire

If the conductor is not straight

$$\boxed{d\vec{F} = I\, d\vec{l} \times \vec{B}}$$



infinitesimal vector along the wire.

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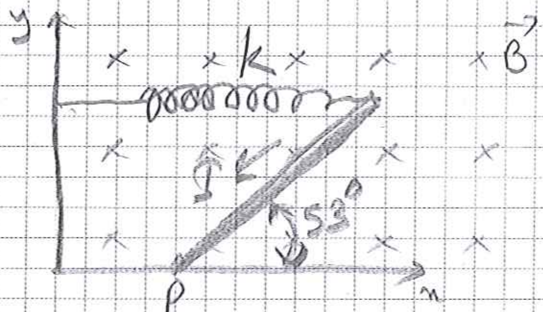
Problem 27.71:



A thin uniform rod with negligible mass and length 0.2 m.

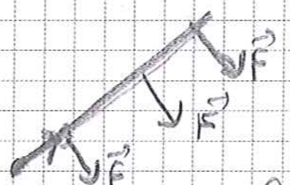
$k = 4.8\ N/m$

$B = 0.34\ T$

$I = 6.5\ A$

a) Torque due to the magnetic force on the rod, for an axis at P? perpendicular to the plane

$\vec{F} = I\, \vec{L} \times \vec{B}$

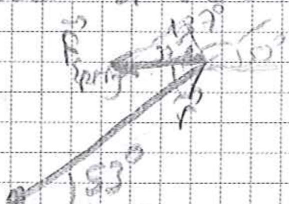


$$\vec{\tau} = -\hat{k} \int r\, dF = -\hat{k} \int r\, I B\, dr$$

$$= -IB\hat{k} \int_0^L r\,dr = -IB \frac{L^2}{2} \hat{k} = -\frac{6.5 \times 0.34 \times (0.2)^2}{2} \hat{k}$$

$$= -65 \times 34 \times 2 \times 10^{-2} \times 10^{-3} \hat{k} = -4420 \times 10^{-5} \hat{k} = -0.442\ Nm$$

b) When the rod is in equilibrium at an angle 53°, is the spring compressed or stretched?

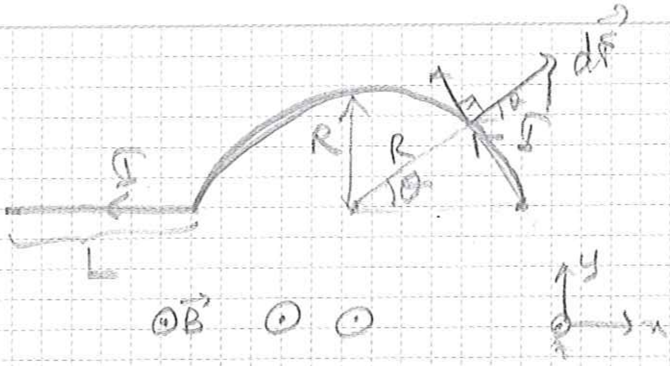


$\vec{\tau}_{spring} = \vec{r} \times \vec{F} = (0.2) \times F \times \sin 137°\, \hat{k}$

$\tau_{spring} = 0.2\, F \times \sin 53° = 0.2 \times \frac{4}{5} \times F = \frac{0.8}{5} F$

$\Rightarrow \frac{0.8}{5} F = 0.442 \Rightarrow \boxed{F \cong 2.5N}$ stretched

Ex 27.8:



What is the total magnetic force on the conductor?

On the straight portion:

$$\vec{F} = I\vec{L}\times\vec{B} = IL\,(-\hat{i})\times B\hat{k} = ILB\hat{j}$$

on the semicircular portion:

$d\vec{F} = I\,d\vec{l}\times\vec{B}$, the resulting net force will be in the y directiond

$$\Rightarrow F_y = \int_0^{\pi} IR\,d\theta\,B\sin\theta$$

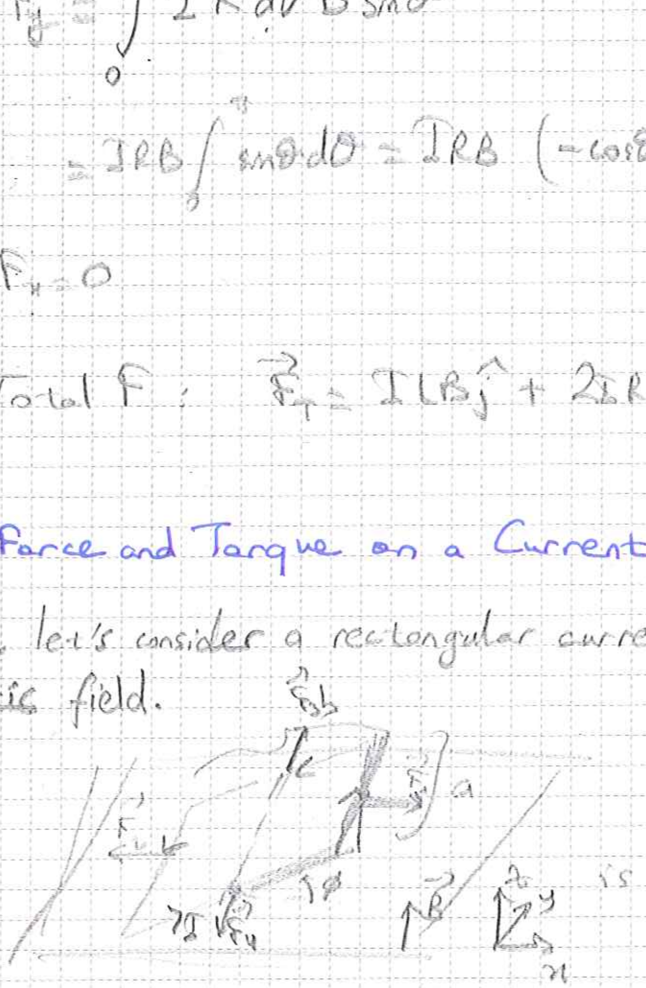
$$= IRB\int_0^{\pi}\sin\theta\,d\theta = IRB\left(-\cos\theta\Big|_0^{\pi}\right) = 2IRB$$

$F_x = 0$

$\Rightarrow$ Total F : $\vec{F}_T = ILB\hat{j} + 2IRB\hat{j} = IB(L+2R)\hat{j}$

27.7 Force and Torque on a Current Loop

First, let's consider a rectangular current loop in a uniform magnetic field.



is

Force on the righteren and lefteren sides of the loop

$\vec{F}_1 = IaB\hat{i}$

$\vec{F}_2 = -IaB\hat{i}$

Forces on the upper and lower sides

$$\vec{F_4} = \int I(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) \times B\hat{k}$$

$$= \int I\,dx\,B\,(-\hat{j}) = -IB\hat{j}\int_{-\frac{b}{2}\cos\phi} dx = -IB\hat{j}\,\Delta x = -IBb\cos\phi\,\hat{j}$$

$$\vec{F_3} = \int I\,dx\,B\,(-\hat{j}) = -IB\hat{j}\int_{\frac{b}{2}\cos\phi} dx = IBb\cos\phi\,\hat{j}$$

∴ The net force on the rectangular loop

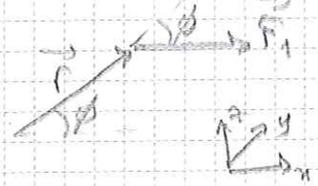
$$\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4} = 0$$

∴ The net torque on the rectangular loop around a point in the axis of rotation.

$$\vec{\tau} = \vec{r}\times\vec{F}$$

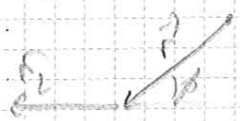
F_1, F_2 will have torque.

$$\vec{\tau_1} = \vec{r}\times\vec{F_1}$$



$$\Rightarrow \vec{\tau_1} = \hat{j}\,\frac{b}{2}F_1\sin\phi = \hat{j}\,\frac{b}{2}IaB\sin\phi$$

$$\vec{\tau_2} = \vec{r}\times\vec{F_2}$$



$$\vec{\tau_2} = \hat{j}\,\frac{b}{2}F_2\underbrace{\sin(\pi-\phi)}_{\sin\phi} = \hat{j}\,\frac{b}{2}IaB\sin\phi$$

⇒ Net torque $\vec{\tau} = \vec{\tau_1}+\vec{\tau_2} = \hat{j}\,b\,IaB\sin\phi$

$$\boxed{\tau = (IBa)(b\sin\phi)}$$

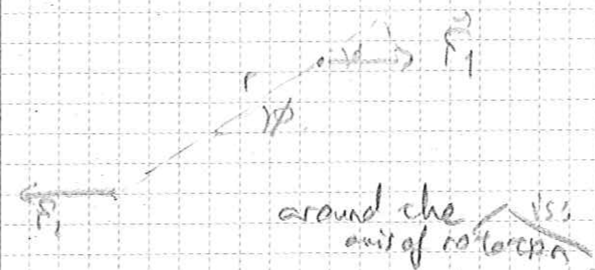
$\tau = IBA\sin\phi$ $A = ab$

We define the magnetic dipole moment vector $\vec{\mu}$

magnitude of $\vec{\mu}$ $\quad \mu = IA \quad \rightarrow \quad \tau = \mu B \sin\phi$

direction of $\vec{\mu}$ $\quad$ perpendicular to the loop, determined by the right-hand rule

$$\Rightarrow \boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$ in a compact form.



work done by the forces F_1, F_2 during an infinitesimal displacement $d\phi$ around the axis of rotation

$$dW_1 = -F_1 \sin\phi \frac{b}{2} d\phi$$
$$= -IaB \frac{b}{2} \sin\phi d\phi = -\frac{\mu B}{2} \sin\phi d\phi$$
$$dW_2 = -\frac{\mu B}{2} \sin\phi d\phi$$

$$\Rightarrow dW = dW_1 + dW_2 = -\mu B \sin\phi d\phi = -\tau d\phi$$

$$W = \int_{\phi_1}^{\phi_2} dW = U_1 - U_2 \Rightarrow -\mu B \int_{\phi_1}^{\phi_2} \sin\phi d\phi = \mu B(\cos\phi_2 - \cos\phi_1) = U_1 - U_2$$

$$\rightarrow U_1 = -\mu B \cos\phi_1$$

$$\therefore \boxed{U = -\vec{\mu} \cdot \vec{B}}$$ Potential energy for a magnetic dipole.

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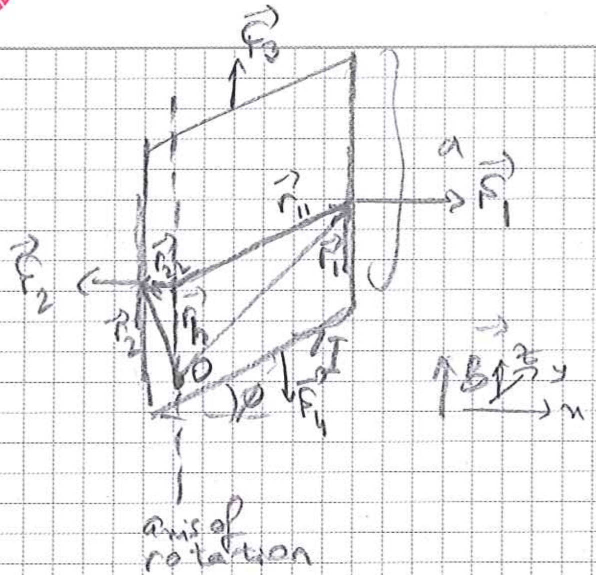

$\vec{r}_2 = \vec{r}_h + \vec{r}_{22}$

$\vec{r}_1 = \vec{r}_h + \vec{r}_{11}$

$\vec{\tau}_3 = \vec{\tau}_4 = 0$

$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \vec{r}_h \times \vec{F}_1 + \vec{r}_{11} \times \vec{F}_1$, $\vec{F}_1 = -\vec{F}_2 = IaB\hat{i}$

$\vec{\tau}_2 = \vec{r}_h \times \vec{F}_2 + \vec{r}_{22} \times \vec{F}_2$

$\Rightarrow \vec{\tau}_1 + \vec{\tau}_2 = \vec{r}_{11} \times \vec{F}_1 + \vec{r}_{22} \times \vec{F}_2 = (r_{11} F_1 + r_{22} F_1) \sin\phi$

$= \underbrace{(r_{11} + r_{22})}_{b} F_1 \sin\phi = IabB\sin\phi$

$= \vec{\mu} \times \vec{B}$

$\Rightarrow$ Around any axis of rotation on the plane of the loop

$\vec{\tau} = \vec{\mu} \times \vec{B}$ //

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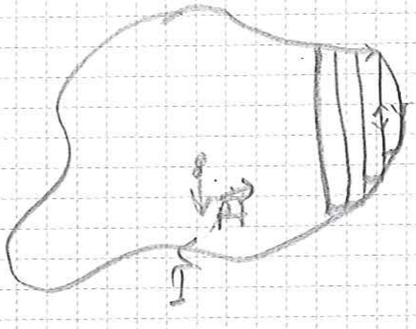
Consider a plane loop of any shape:



we can Approximate the plane loop by many rectangular loops.

$\Rightarrow \mu_1 = IA_1, \; \mu_2 = IA_2, \ldots$

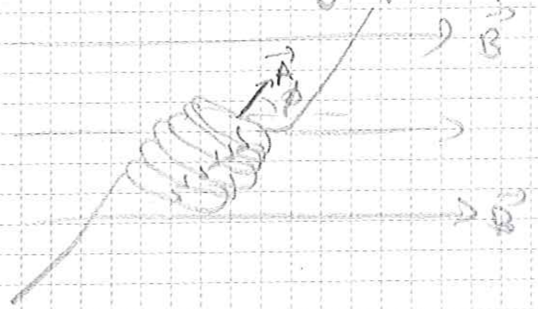
$\Rightarrow \vec{\tau}_1 = \vec{\mu}_1 \times \vec{B}, \quad \vec{\tau}_2 = \vec{\mu}_2 \times \vec{B}, \ldots$

$\Rightarrow \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \ldots = (\vec{\mu}_1 + \vec{\mu}_2 + \ldots) \times \vec{B}$

$$\boxed{\vec{\tau} = \underbrace{I\vec{A}}_{\vec{\mu}} \times \vec{B}}$$

Torque about any axis in the plane of the loop

Consider a solenoid,
a coil consisting of N planar loops,



Each loop has

$\vec{\tau}_1 = I\vec{A}_1 \times \vec{B} = I\vec{A} \times \vec{B}$

$\vec{\tau}_2 = I\vec{A}_2 \times \vec{B} = I\vec{A} \times \vec{B}$

$\vdots$

$\Rightarrow$ Total τ; $\vec{\tau} = NI\vec{A} \times \vec{B}$, $\boxed{\tau = NIAB\sin\phi}$

Ex 27.9: A circular coil, carries 5A current
radius = 0.05m, 30 turns
uniform $\vec{B}$ has magnitude 1.27

$\mu = ?$, $\tau = ?$

$\mu = IAN = (5A\pi(0.05m)^2) \times 30 = 1.18\,Am^2$

$\tau = \mu B\sin\phi = 1.18\,Am^2\, 1.27 = 1.41\,Nm$

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Ex 27.10: Coil in Ex 27.9 is rotated such that $\vec{\mu}$ is parallel to $\vec{B}$, what is the change in U:

$\Delta U = U_2 - U_1$, $U = -\vec{\mu} \cdot \vec{B}$

$$\left.\begin{array}{l} U_1 = -\vec{\mu} \cdot \vec{B} = 0 \\ U_2 = -\mu B = -1.41 J \end{array}\right\} \Rightarrow \Delta U = -1.41 J$$