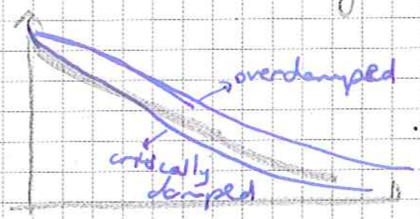
KOÇ ÜNİVERSİTESİ

122

When $R^2 = \frac{4L}{C}$, the system no longer oscillates.

↳ critically damped case.



When $R^2 > \frac{4L}{C}$, → overdamped. Capacitor charge approaches zero even more slowly.

Ex: What resistance R is required to give an L-R-C circuit an oscillation frequency that is $\frac{1}{2}$ the undamped frequency.

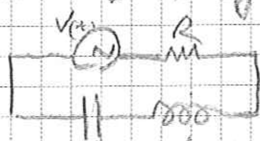
undamped frequency: $\sqrt{\frac{1}{LC}}$

$$\frac{1}{2}\sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \Rightarrow \boxed{R = \frac{3L}{C}}$$

Chap 31 : Alternating Current

Circuits in which voltages and currents vary sinusoidally are called as alternating current circuits.

"Most present day household and industrial power-distribution systems operate with alt. current." "In Turkey f=50Hz"



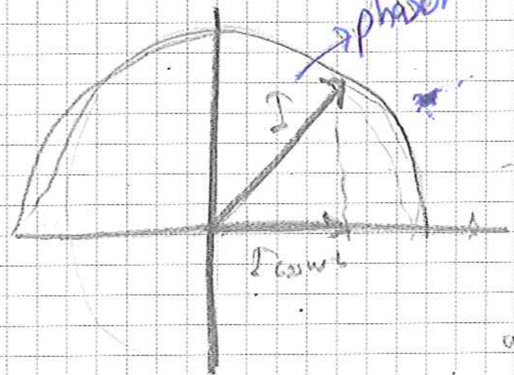
Phasors and Alternating Currents

A sinusoidal voltage is described as:

$$v = V\cos\omega t$$

A sinusoidal current is described as

$$i = I\cos\omega t$$



We use phasor diagrams to represent sinusoidally varying voltages and currents.

→ The instantaneous value of a quantity that varies sinusoidally with time is represented by the projection onto a horizontal axis of a vector with a length I.

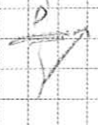
"Phasor is the vector in 2D whose projection to the horizontal axis gives the instantaneous value of the variable."

"The vector which rotates in the counterclockwise direction with angular speed ω is called a phasor."

$i(t) = I\cos(\omega t)$

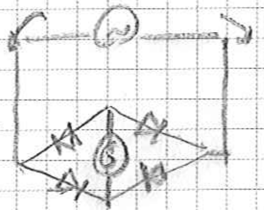
AC can be both positive and negative.

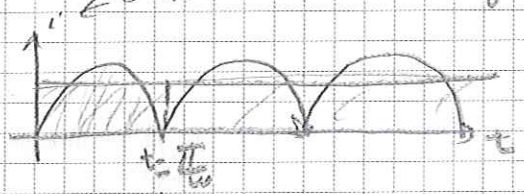
⇒ If we place a galvanometer (for current measurement) into an AC circuit it will not shown a value, unless the oscillations are really slow.



⇒ So, we have to find other ways to describe and measure AC.

Full-wave rectifier circuit:





It is consisted of 4 diodes ⇒ these are elements which have 0 resistance in the forward and infinite resistance in the backward direction.

Galvanometer will display an average value:

$$\int_0^{\pi/\omega} I\sin\omega t\, dt = I\left.\frac{-\cos\omega t}{\omega}\right|_0^{\pi/\omega} = -\frac{I}{\omega}(-1-1) = \frac{2I}{\omega}$$

$$\frac{2I}{\cancel{\omega}} = I_{rav}\cdot\frac{\pi}{\cancel{\omega}} \Rightarrow \boxed{I_{rav} = \frac{2}{\pi}I}$$

rectified average value of a sinusoidal current.

KOÇ ÜNİVERSİTESİ

123.5

AC is also described by the root mean square value:

$I_{rms} = \sqrt{\overline{i(t)^2}}$, square-root of the time average of current squared.

$i(t) = I\cos\omega t$

$\Rightarrow i^2(t) = I^2\cos^2\omega t = I^2\left(\frac{1+\cos 2\omega t}{2}\right) = \frac{I^2}{2} + \frac{I^2}{2}\cos 2\omega t$

$\overline{i^2(t)} = \overline{\left(\frac{I^2}{2}\right)} + \underbrace{\overline{\frac{I^2}{2}\cos 2\omega t}}_{=0} = \frac{I^2}{2} + 0 = \frac{I^2}{2}$

$\Rightarrow \boxed{I_{rms} = \frac{I}{\sqrt{2}}}$ root-mean-square value of a sinusoidal current

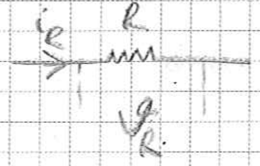
similarly $\boxed{V_{rms} = \frac{V}{\sqrt{2}}}$ //

31.2 Resistance and Reactance

- We will determine how to represent resistors, inductors and capacitors in ac circuits.

$\Rightarrow$ Find the relationships between phasors!

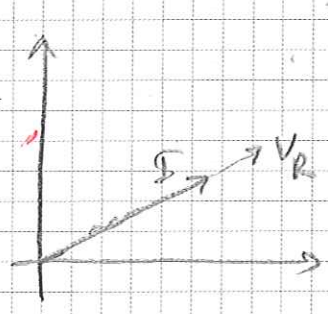
Resistor in an AC circuit



$i_R = I\cos\omega t$

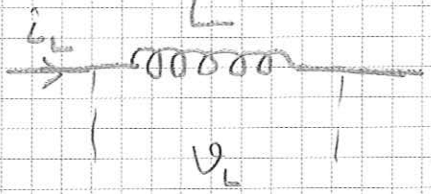
$\Rightarrow v_R = IR\cos\omega t = V_R\cos\omega t$

$\Rightarrow \boxed{V_R = IR}$ phasor relationship.



KOÇ ÜNİVERSİTESİ

Inductor in an AC circuit



$i_L = I\cos\omega t$

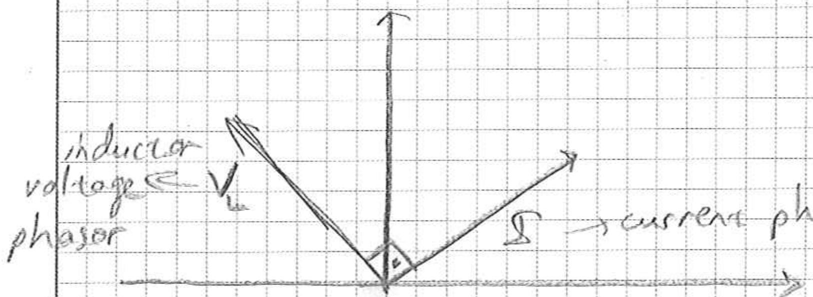
$\mathcal{E} = v_L = L\dfrac{di_L}{dt} = -LI\omega\sin\omega t$

$v_L = -I\omega L\sin\omega t$

$v_L = \underbrace{I\omega L}_{V_L}\cos(\omega t + 90^\circ)$

$\Rightarrow V_L = I X_L \Rightarrow \boxed{X_L = \omega L}$ inductive reactance

amplitude of voltage across an inductor



Ex 31.2: You want the current amplitude in a pure inductor to be 250µA, when voltage amplitude is 3.6V at f = 1.6 MHz

a) $X_L = ?$ $\quad X_L = \omega L = \dfrac{V_L}{I} = \dfrac{3.6\,V}{250\times10^{-6}A} = 14.4\,k\Omega$

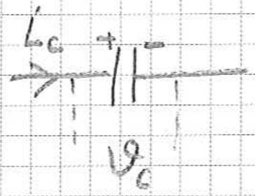
$L = ?$

$L = \dfrac{14.4\times10^{3}\,\Omega}{2\pi\times1.6\times10^{6}\,Hz} = 1.43\,mH$

b) at f = 16 MHz $I = ?$ if V_L is the same

$X_L = 2\pi f L = \dfrac{V_L}{I} \Rightarrow I = \dfrac{V_L}{2\pi f L} \Rightarrow \boxed{I = 25\,\mu A}$

Capacitor in an AC circuit



Choose the positive direction of current to be towards the + plate of the capacitor.
$\Rightarrow$ Capacitor is charging.

$$i_C = \frac{dq}{dt} > 0$$

$$i_C = I\cos\omega t = \frac{dq}{dt}$$

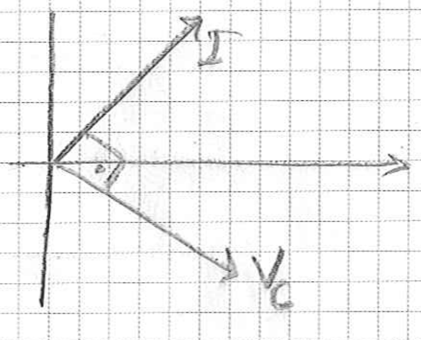
for.

$$\Rightarrow q(t) = \frac{I}{\omega}\sin\omega t \quad , \quad q_0 = 0$$

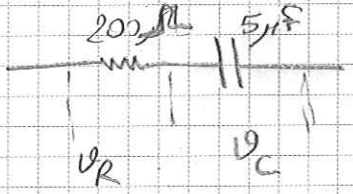
$$q(t) = \frac{I}{\omega}\cos(\omega t - 90^\circ)$$

$$\Rightarrow v_C = \frac{q}{C} = \frac{I}{\omega C}\cos(\omega t - 90^\circ)$$

$$V_C = \frac{I}{\omega C} = I X_C \Rightarrow \boxed{X_C = \frac{1}{\omega C}}$$ capacitive reactance



Ex 31.3: Resistor and capacitor in an AC circuit.



$$v_R = 1.2\,V\cos(2500\,\text{rad/s}\; t)$$

a) $i = ?$ $\quad i = \frac{v_R}{R} = \frac{1.2\,V}{200\,\Omega}\cos(2500\,\text{rad/s}\; t) = 6\,mA\cos(2500\,\text{rad/s}\; t)$

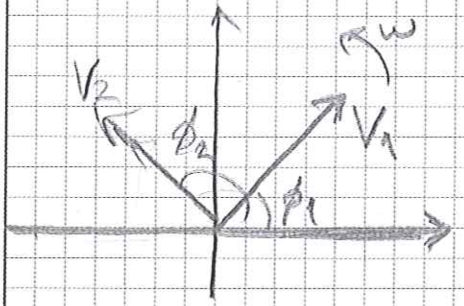
b) $X_C = ?$ $\quad X_C = \frac{1}{\omega C} = \frac{1}{2500\,\text{rad/s} \times 5\times10^{-6}\,F} = 80\,\Omega$

c) $v_C = ?$ $\quad v_C = X_C I \sin\omega t = 80\,\Omega \times 6\times10^{-3}\,A\ \sin(2500\,\text{rad/s}\; t) = 0.48\,V\sin(2500\,\text{rad/s}\; t)$

KOÇ ÜNİVERSİTESİ

125.5

Phasor Analysis



$v(t) = V\cos(\omega t + \phi)$

$v_1(t) = V_1 \cos(\omega t + \phi_1)$, $v_2(t) = V_2 \cos(\omega t + \phi_2)$

Projection of $\vec{V_1}$ to the horizontal axis

$$\left.\begin{array}{l} \vec{V_1} = V_{1x}\,\hat{i} + V_{1y}\,\hat{j} \\ \vec{V_2} = V_{2x}\,\hat{i} + V_{2y}\,\hat{j} \end{array}\right\}$$

$\rightarrow v(t) = v_1(t) + v_2(t)$

$$\vec{V_1} + \vec{V_2} = \underbrace{(V_{1x} + V_{2x})}_{v(t) = v_1(t) + v_2(t)}\hat{i} + (V_{1y} + V_{2y})\,\hat{j}$$

Summation of the projections of V_1 and V_2 to the horizontal axis is the projection of the vectoral sum of V_1 and V_2 to the horizontal axis.

$\Rightarrow$ The phasor of $v_1(t) + v_2(t)$

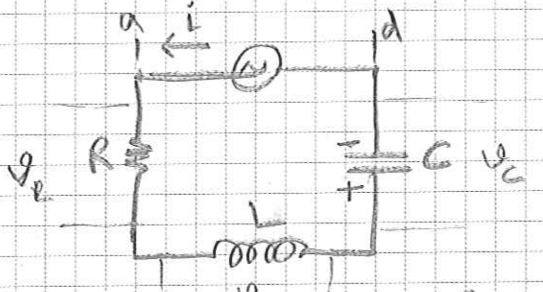
is

$$\vec{V_1} + \vec{V_2}$$

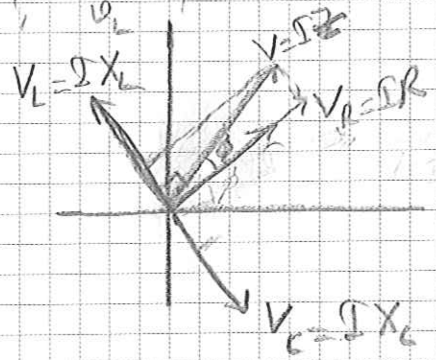
KOÇ ÜNİVERSİTESİ

31.3 L-R-C Series Circuit

Consider the circuit:



We will use a phasor diagram in order to solve for the circuit.



$$v_{ad} = V\cos(\omega t+\phi)$$

$$\Rightarrow \vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$

$$\boxed{V = I\sqrt{R^2 + (X_L - X_C)^2}}$$

We define the impedance Z of an AC circuit as:

$$V = IZ \Rightarrow \boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$ impedance of an L-R-C series circuit

Band pass filter:

$\omega \to \infty \quad Z \to \infty$

$\omega \to 0 \quad Z \to \infty$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Leftarrow \omega_R = \frac{1}{\sqrt{LC}}$$

∴ For a given amplitude V of the source voltage, the amplitude $I = \frac{V}{Z}$ will be different at different frequencies.

The angle ϕ between the voltage and current phasors is given by:

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\cancel{I}(X_L - X_C)}{\cancel{I}R} = \boxed{\frac{\omega L - \frac{1}{\omega C}}{R}}$$

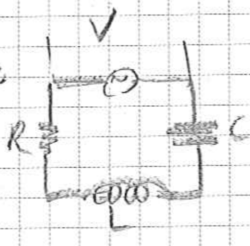
∴ If the current is $i = I\cos\omega t$, $\boxed{v = V\cos(\omega t+\phi) = IZ\cos(\omega t+\phi)}$

rms values are related as; $V_{rms} = \frac{V}{\sqrt{2}}$, $I_{rms} = \frac{I}{\sqrt{2}}$

$= \frac{IZ}{\sqrt{2}}$

$$\boxed{V_{rms} = I_{rms} Z}$$

Ex 31,4:



$R = 300\Omega$, $L = 60mH$, $C = 0.5\mu F$, $V = 50V$, $\omega = 10000 rad/s$

$\rightarrow X_L, X_C, Z, I, \phi, V_R, V_L, V_C = ?$

$X_L = \omega L = 10000 rad/s \times 60 \times 10^{-3} H = 600\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{10000 rad/s \; 0.5 \times 10^{-6} F} = 200\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\Omega)^2 + (400\Omega)^2} = 500\Omega$

$I = \frac{V}{Z} = \frac{50V}{500\Omega} = 0,1A$

$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{400}{300}\right) = 53°$

$V_R = IR = 0,1A \times 300\Omega = 30V$

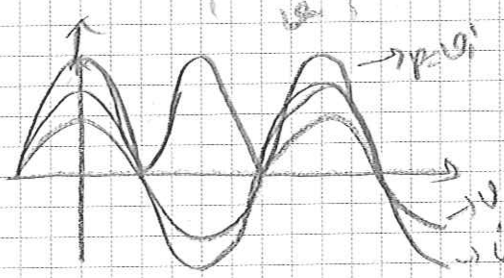
$V_L = IX_L = 0,1A \times 600\Omega = 60V$

$V_C = IX_C = 0,1A \times 200\Omega = 20V$

31.4 Power in Alternating-Current Circuits

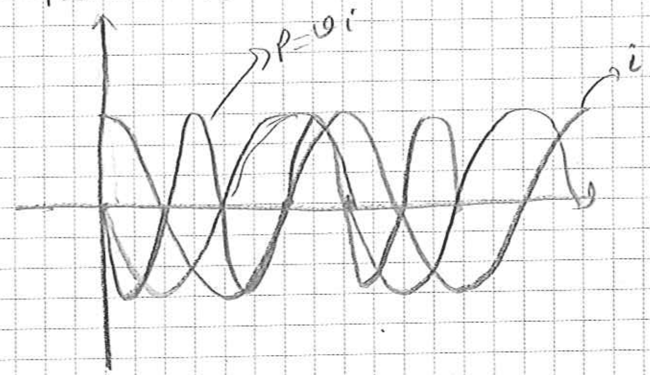
Instantaneous power delivered to a circuit element:

$p = vi$



← For a pure resistor

Pure inductor



for a pure resistor; $p = v i = V I \cos^2 \omega t$

$$\Rightarrow P_{av} = V I \, \overline{\cos^2 \omega t} = V I \left(\frac{1}{2} + \overline{\frac{\cos 2\omega t}{2}} \right)$$

$$= \frac{V I}{2}$$

$$\Rightarrow \boxed{P_{av} = V_{rms} I_{rms}}$$

for an arbitrary circuit element;

$$p = v i = V \cos(\omega t + \phi) I \cos \omega t = V I \cos(\omega t + \phi) \cos(\omega t)$$

$$= V I \left\{ \frac{1}{2} \left\{ \cos(2\omega t + \phi) + \cos(\phi) \right\} \right\}$$

$$\Rightarrow P_{av} = \frac{V I}{2} \left\{ \underbrace{\overline{\cos(2\omega t + \phi)}}_{0} + \overline{\cos(\phi)} \right\} = \frac{V I}{2} \cos\phi = \boxed{V_{rms} I_{rms} \cos\phi}$$

The factor $\cos\phi$ is called the power factor.

for a pure resistance $\phi = 0 \Rightarrow P_{av} = V_{rms} I_{rms}$

" " inductor $\phi = 90^\circ \Rightarrow P_{av} = 0$

capacitor $\phi = -90^\circ \Rightarrow P_{av} = 0$

} no average power delivered to an inductor or a capacitor

Ex 31.6: Power in a hair dryer

An electric hair dryer is rated $\underbrace{1500\,W}_{\text{average power}}$ at $\underbrace{120V}_{\text{rms voltage}}$

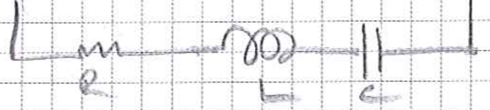
a) $R=?$ $P_{av}=V_{rms}I_{rms}=\frac{V_{rms}^2}{R} \Rightarrow R=\frac{(120V)^2}{1500W}=9,6\Omega$

b) rms current ; $I_{rms}=\frac{V_{rms}}{R}=\frac{120V}{9,6\Omega}=12,5A$

c) Maximum instant. power

$$P_{max}=VI=2V_{rms}I_{rms}=3000W$$

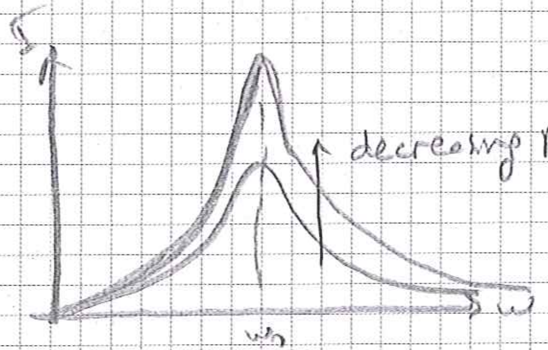
31.5 Resonance in Alternating Current Circuits

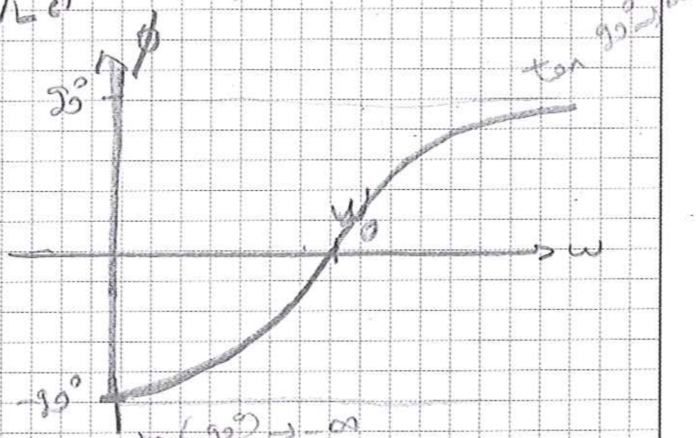


$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\tan\phi=\frac{X_L-X_C}{R}=\frac{\omega L-\frac{1}{\omega C}}{R},$$ angle ϕ between V and I.

Z reaches to a minimum at $\omega_0=\frac{1}{\sqrt{LC}}$



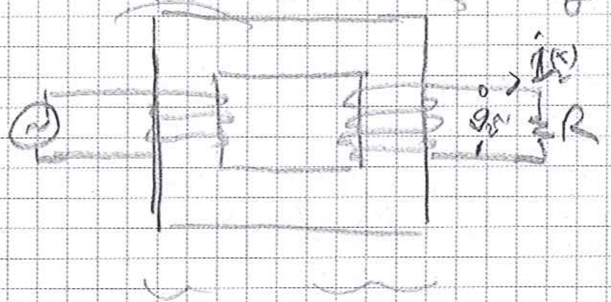


"Resonance"
Tuning is possible by changing L or C.

"Analog Radio and Television receiving circuits"

31.6 Transformers

It is much easier to change voltage levels in AC as compared to DC.



Two coils of different windings.

Core made of a magnetizable material so that the magnetic field lines remain in the core.

When we assume that all the magnetic field lines are confined to the iron core, Φ_B is the same in both coils;

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_B}{dt}, \quad \mathcal{E}_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\Rightarrow \frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2} \Rightarrow \boxed{\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}}$$

Powerwise: Power delivered from the primary:

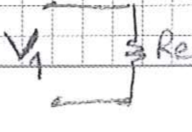
$P_1 = \cancel{V_1 I_1} = \boxed{V_2 I_2}$, there is no resistance in windings.

$$V_2 = V_1 \frac{N_2}{N_1} = I_2 R$$

$$\cancel{V_1 I_1} = \cancel{V_1} \frac{N_2}{N_1} V_1 \frac{N_2}{N_1} \cdot \frac{V_1}{R} \Rightarrow \boxed{\cancel{\frac{V_1}{I_1} = \frac{R}{(N_2/N_1)^2}}}$$

Transformer transforms the resistance

$$= \frac{(V_1)^2}{R} \cdot \left(\frac{N_2}{N_1}\right)^2$$



$$\Rightarrow P = V_1 \frac{V_1}{R_{eq}} = \frac{V_1^2}{R_{eq}} \Rightarrow \boxed{R_{eq} = \frac{R}{(N_2/N_1)^2}}$$