Chapter 32: Electromagnetic Waves

Depending on the frequency (wavelength) the electromagnetic radiation is called as:

ultraviolet light, visible light, infrared light, radio waves, x rays.

Maxwells Equations form the theoretical basis enabling us to analyze electromagnetic radiation.

↳ "Electromagnetic Waves"

○ Electromagnetic waves do not require a material medium, unlike sound waves, light can propagate in empty space.

Maxwell's Equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \quad \text{(Gauss' Law)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{(Gauss' Law for magnetism)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d}{dt} \Phi_E \right)_{encl} \quad \text{(Modified Ampère's Law)}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_B \quad \text{(Faraday's Law)}$$

↓

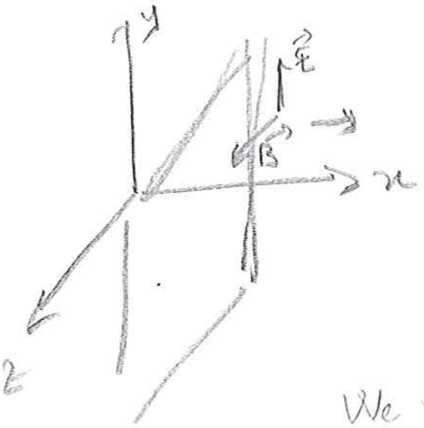
wave eq.

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

32.2 : Plane Electromagnetic Waves:

We now analyze the Maxwell's Equations and show the wave property.

Consider the simplest electromagnetic wave, plane wave.

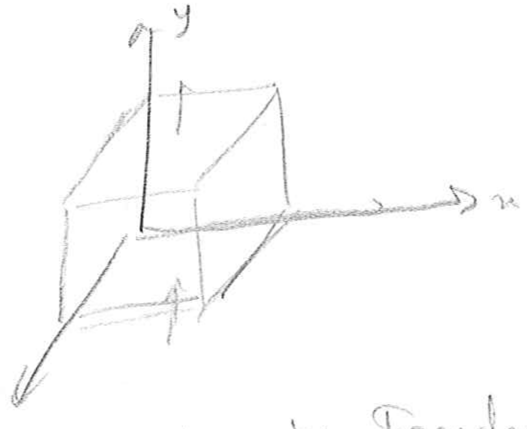


Consider the $\vec{E}$ field is directed along the +y direction while $\vec{B}$ field along the +z direction. And the electromagnetic wave propagates along the x direction.

We will find out the speed of propagation.

We will first assume $\vec{E}$ and $\vec{B}$ to have constant magnitudes.

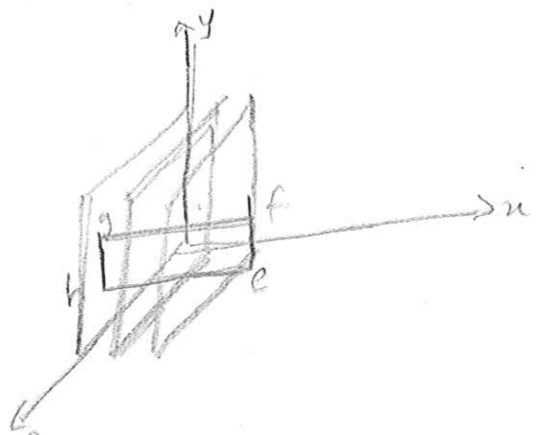
Let's verify that the plane wave indeed satisfies Maxwell's equations:

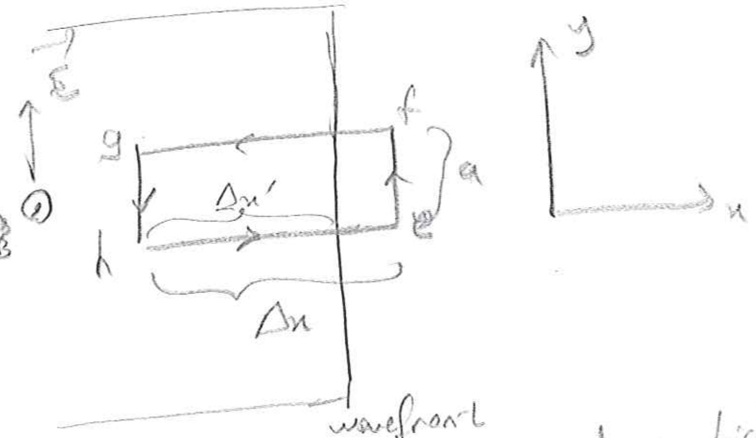


$$\oint \vec{E} \cdot d\vec{A} = \oint \vec{B} \cdot d\vec{A} = 0$$ (even in the region where E and B ≠ 0 near the wavefront)

Because no charge is enclosed.

Now consider the Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$, consider the wavefront





$\oint \vec{E} \cdot d\vec{l} = -Ea$, $\Phi_B = B\,\Delta x' a \Rightarrow \frac{d\Phi_B}{dt} = Ba \frac{d\,\Delta x'}{dt}$

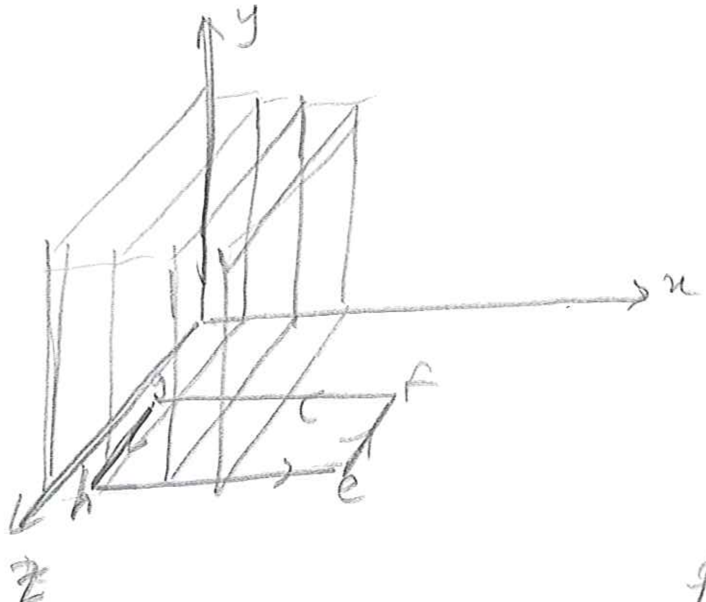
speed at which the wavefront propagates : c

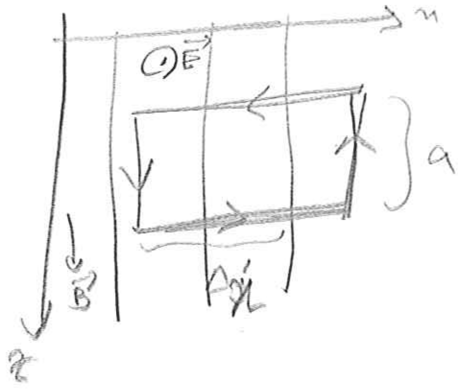
$$\rightarrow -\frac{d\Phi_B}{dt} = -Bac = -Ea \rightarrow \boxed{E = cB}$$

For plane electromagnetic wave in vacuum.

Now the modified Ampère's Law;

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$





$$\oint \vec{B}\cdot d\vec{l} = Ba$$

$$\Phi_E = E\Delta x\, a$$

$$\rightarrow \frac{d\Phi_E}{dt} = Eac$$ ← propagation of the wavefront

$$\rightarrow \oint \vec{B}\cdot d\vec{l} = B\cancel{a} = \mu_0 \epsilon_0 E \cancel{a} c$$

$$\rightarrow \boxed{E = \frac{1}{\mu_0 \epsilon_0 c} B} \rightarrow c = \frac{1}{\mu_0 \epsilon_0 c}$$

$$\rightarrow c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$ Speed of light

$c = 3.02\times10^8$ m/s.

32.3 Sinusoidal Electromagnetic Waves:

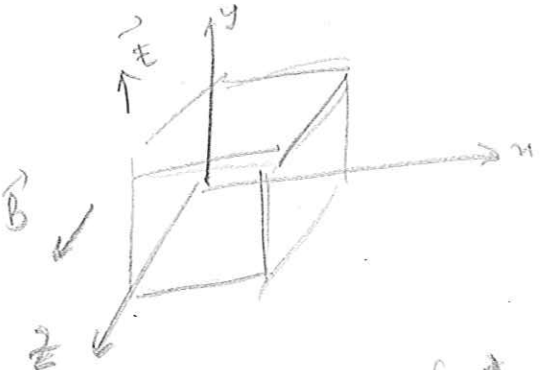
The analysis performed for a plane wave with no frequency is also valid for a sinusoidal electromagnetic wave of the form:

$$\left.\begin{aligned}\vec{E}(x,t) &= \hat{j}\, E_{max} \cos(kx-\omega t) \\ \vec{B}(x,t) &= \hat{k}\, B_{max} \cos(kx-\omega t)\end{aligned}\right\}$$

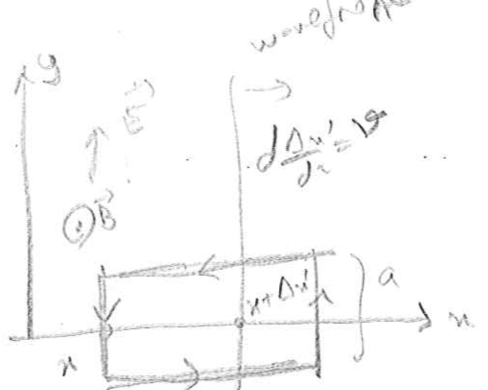
$f(x,t) = A\cos(kx-\omega t)$ in general describes a wave, with angular frequency ω and wavenumber $k = \frac{2\pi}{\lambda}$. And $\frac{\omega}{k} = v$ gives the speed of wave propagation.

Let's check if the sinusoidal waves satisfy the Maxwell's Equations.

$$\oint \vec{E}\cdot d\vec{A} = \oint \vec{B}\cdot d\vec{A} = 0 \quad \checkmark$$



$$\oint \vec{E}\cdot d\vec{l} \stackrel{?}{=} -\frac{d\Phi_B}{dt}$$



$$\oint \vec{E}\cdot d\vec{l} = -E_{max}\cos(kx-\omega t)\, a$$

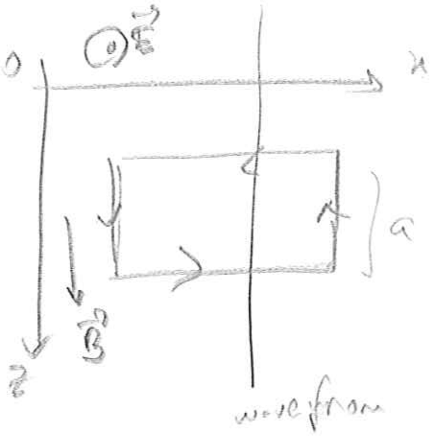
$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int_x^{x+\Delta x'} B_{max}\cos(kx-\omega t)\, a\, dx$$

$$= B_{max}\, a \left\{ \frac{1}{k} \left(\sin(kx + k\Delta x' - \omega t) - \sin(kx-\omega t) \right) \right\}$$

$$\Rightarrow -\frac{d\Phi_B}{dt} = -\frac{B_{max} a}{k}\left\{ (kc-\omega)\cos(kx+k\Delta x'-\omega t) + \omega\cos(kx-\omega t) \right\}$$

$$= -E_{max}\, a \cos(kx-\omega t), \text{ for all } t$$

$$\Leftrightarrow kc = \omega \text{ and } \frac{B_{max}\cancel{a}\,\omega}{k} = E_{max}\cancel{a} \Rightarrow E_{max} = \frac{\omega}{k} B_{max} = c B_{max}$$



$$\oint \vec{B} \cdot d\vec{l} \stackrel{?}{=} \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = B_{max}\, a \cos(kx - wt)$$

$$\Phi_E = \int_{x_0}^{x_0+\Delta x'} E_{max} \cos(kx - wt)\, a\, dx = \frac{E_{max}\, a}{k} \left(\sin(kx + k\Delta x' - wt) - \sin(kx - wt) \right)$$

$$\mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \frac{\mu_0 \epsilon_0 E_{max} a}{k} \left((kc - w) \cos(kx + k\Delta x' - wt) + w \cos(kx - wt) \right)$$

$$= B_{max}\, a \cos(kx - wt) \quad \text{for all } t$$

$$\Leftrightarrow \frac{E_{max} \cancel{a} \mu_0 \epsilon_0 w}{k} = B_{max} \cancel{a}, \quad kc = w$$

$$\Rightarrow E_{max} = \frac{k}{\mu_0 \epsilon_0 w} B_{max} = c\, B_{max} \,//$$

→ <u>In Matter</u>:

We substitute ϵ_0 with $\epsilon = K\epsilon_0$
μ_0 with $\mu = K_m \epsilon$

$$\Rightarrow v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{1}{\sqrt{K K_m}} \cdot \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{c}{\sqrt{K K_m}} \,//$$

speed of electromagnetic waves in a dielectric.

$\sqrt{K K_m} \approx \sqrt{K} = n$ is called as the index of refraction.
(for nonmagnetic materials)

Example 32,2 ;

a) Yellow light has a frequency of $5,09 \times 10^{14}$ Hz.
wavelength in vacuum?
speed of wave propagation in diamond? $(K = 5,84,\ K_m = 1)$
wavelength in diamond?

b) A radio wave with a frequency of 90MHz passes from vacuum into an insulating ferrite.
wavelength in vacuum?
speed of wave propagation in ferrite? $(K = 10,\ K_m = 1000)$
wavelength in ferrite?

a) $\frac{\omega}{k} = c,\ \frac{f}{1/\lambda} = c,\ f\lambda = c \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^{8}}{5,09\times10^{14}} = 5,89\times10^{-7}\,m = 589\,nm$

$$v = \frac{c}{n} = \frac{3\times10^{8}}{\sqrt{5,84}} = 1,24\times10^{8}\,m/s$$

$$\lambda = \frac{\lambda_0}{n} = \frac{5,89\times10^{-7}}{\sqrt{5,84}} = 2,44\times10^{-7}\,m = 244\,nm$$ ← frequency remains the same.

b) $\lambda = \frac{c}{f} = \frac{3\times10^{8}}{90\times10^{6}} = 3,33\,m$

$$v = \frac{3\times10^{8}}{\sqrt{10\times1000}} = 3\times10^{6}\,m/s \quad //$$

$$\lambda = \frac{c}{f\sqrt{10\times1000}} = \frac{3\times10^{8}}{90\times10^{6}\times10^{2}} = \frac{1}{30} = 3,33\times10^{-2}\,m = 3,33\,cm \quad //$$

32.4 Energy and Momentum in Electromagnetic Waves

Energy is stored in electromagnetic waves.

Energy density stored in electromagnetic waves is given by:

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

In vacuum the magnitudes of E and B are related as:

$$B = \frac{E}{c} = \sqrt{\epsilon_0 \mu_0}\, E$$

$$\Rightarrow u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\left(\sqrt{\epsilon_0\mu_0}\, E\right)^2 = \epsilon_0 E^2$$

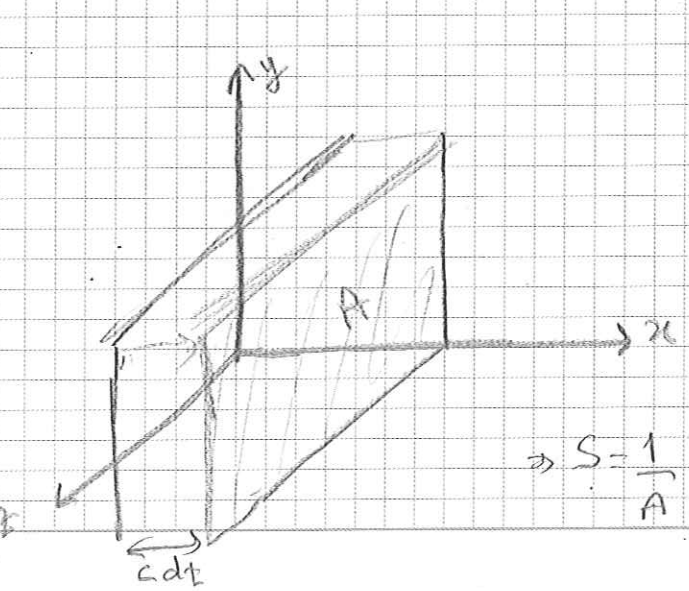
⇒ in vacuum the energy density associated with E field is equal to the energy density of the $\vec{B}$-field.

Note that in general u is a function of space and time.

$$u(x,t) = \frac{1}{2}\epsilon_0 E_{max}^2 \cos^2(kx - \omega t) + \frac{1}{2\mu_0} B_{max}^2 \cos^2(kx - \omega t)$$

Poynting Vector:

The rate of energy transferred by an electromagnetic wave per unit cross-sectional area is described by the Poynting vector.



Consider the wave front of a plane wave.

In time dt the plane moves by cdt.

$$\Rightarrow dU = u\,dV = u\,c\,dt\,A$$

$$\Rightarrow S = \frac{1}{A}\frac{dU}{dt} = uc = \boxed{\epsilon_0 c E^2} \text{ in vacuum.}$$

Alternatively $S = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2 = \frac{EB}{\mu_0}$ $\left(\frac{W}{m^2}\right)$ SI units.

$\Rightarrow \boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$ is the Poynting vector

It describes both the magnitude and direction of the energy flow rate.

Total power out of any closed surface is:

$$P = \oint \vec{S} \cdot d\vec{A}$$

The magnitude of the average value of $\vec{S}$ at a point is called the intensity.

consider the sinusoidal wave:

$$\vec{E}(x,t) = \hat{j} E_{max} \cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k} B_{max} \cos(kx - \omega t)$$

$$\Rightarrow \vec{S} = \frac{1}{\mu_0} \hat{i} E_{max} B_{max} \cos^2(kx - \omega t) \Rightarrow \langle S_x \rangle = \frac{1}{\mu_0} E_{max} B_{max} \langle \cos^2(kx - \omega t) \rangle$$

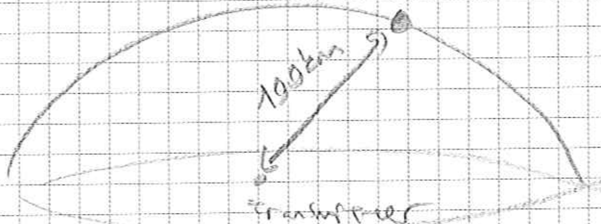
$$\langle \cos^2(kx-\omega t) \rangle = \frac{1}{T} \int_0^T \left(\cos^2(kx - \omega t)\right) dt = \frac{1}{T} \int_0^T \left(\frac{1}{2} + \frac{\cos 2(kx - \omega t)}{2}\right) dt$$

$$= \frac{1}{2} + 0 \Rightarrow \langle S_x \rangle = \frac{E_{max} B_{max}}{2\mu_0}$$

$$\Rightarrow I = S_{av} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}} E_{max}^2 = \frac{1}{2}\epsilon_0 c E_{max}^2$$

Ex 32.4: A radio station on the surface of the earth radiates a sinusoidal wave with an average power of 50 kW. Assume that the transmitter radiates equally in all directions

→ E_{max} and B_{max} detected by a satellite 100 km from the antenna



$$50 \times 10^3 = 2\pi R^2 \times I$$

$$\Rightarrow I = \frac{50 \times 10^3}{2\pi \times (10^5)^2} = 7.96 \times 10^{-7}\ W/m^2$$

$$I = \frac{E_{max}^2}{2\mu_0 c} \Rightarrow E_{max} = 2.45 \times 10^{-2}\ V/m$$

$$B_{max} = \frac{E_{max}}{c} = 8.17 \times 10^{-11}\ T$$

→ Electromagnetic waves also carry momentum.

$$\underbrace{\frac{1}{A}\frac{dp}{dt}}_{\text{radiation pressure}} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$, flow rate of electromagnetic momentum

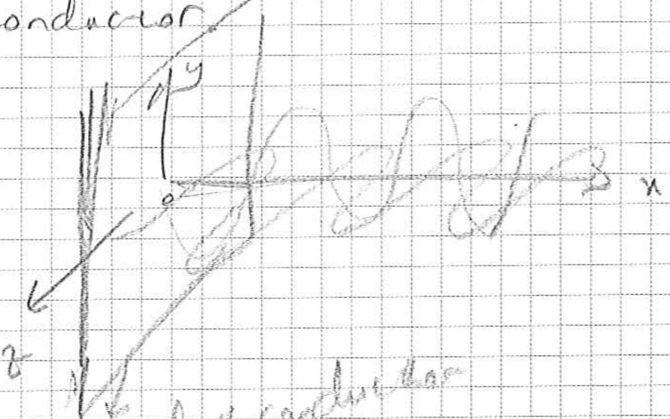
(momentum: p)

$$= \frac{I}{c}$$, gives the average rate of momentum transfer per unit area

oscillating in y direction

32.5. Standing Electromagnetic Waves

Electromagnetic waves can be reflected at the surface of a conductor.



perfect conductor

If an E field propagating in the x direction is incident to a perfect conductor located on the yz plane it will be reflected such that the total electric field is 0 on the surface.

$$\Rightarrow E_y(x,t) = E_{max}\left(\cos(kx+\omega t) - \cos(kx-\omega t)\right) = -2E_{max}\sin kx \sin \omega t$$

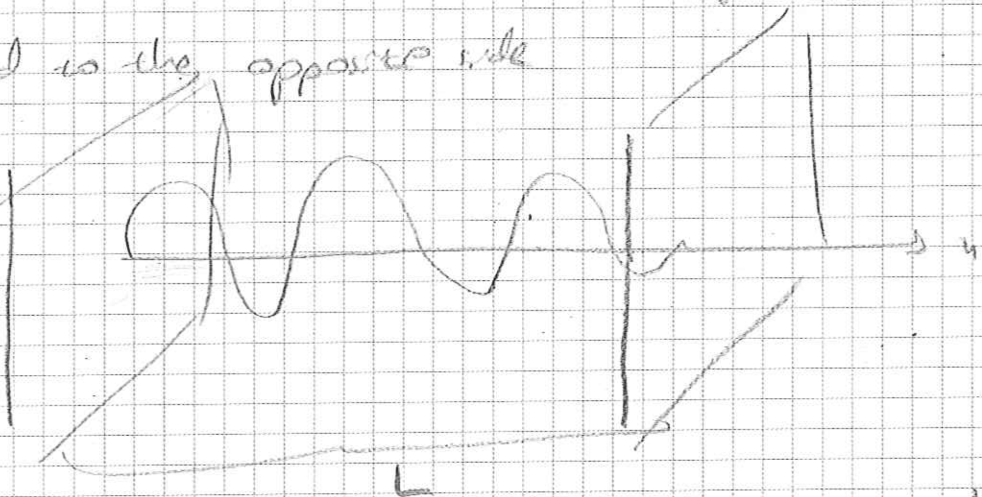
$$B_z(x,t) = B_{max}\left(-\cos(kx+\omega t) - \cos(kx-\omega t)\right) = -2B_{max}\cos(kx)\cos(\omega t)$$ standing wave

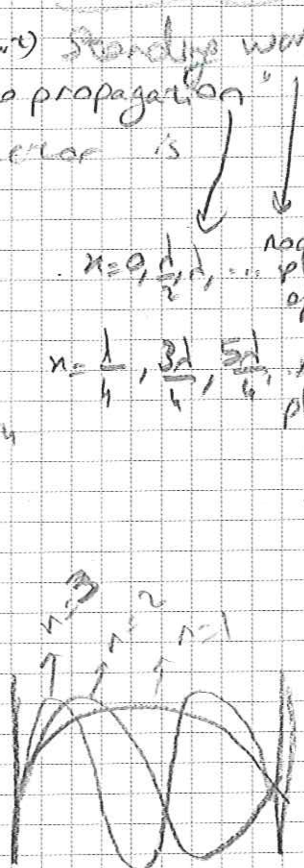
"no propagation"

$x = 0, \frac{\lambda}{2}, \lambda, \ldots$ nodal planes of $\vec{E}$

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ nodal planes of $\vec{B}$

Now consider that a second perfect conductor is placed to the opposite side





$\Rightarrow E_y(0,t) = 0$

$E_y(L,t) = 0 \Rightarrow -2E_{max}\sin(kL)\sin\omega t = 0$

$\Rightarrow \frac{2\pi}{\lambda} L = n\pi, \quad n = 0, 1, 2, \ldots$

$\Rightarrow \boxed{\lambda_n = \frac{2L}{n}, \quad n = 1, 2, \ldots}$

$n=0$ is the DC solution.

The corresponding frequencies are;

$$f_n = \frac{c}{\lambda_n} = n\,\frac{c}{2L}, \quad n = 1, 2, 3, \ldots$$

⇒ In between the conducting plates EM waves at these frequencies only will be found.

This structure is called as a cavity (resonator)

It finds use in microwave areas ($\lambda = 12.2$ cm) and in lasers

Ex 32.7: EM standing waves are set up in a cavity with two parallel, perfect conducting walls separated by 1,5 cm.

a) Calculate the longest wavelength and the lowest freq. of EM waves?

$$\lambda_1 = 2L = 3\,cm$$

$$f_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3\times10^{-2}} = 10^{10}\,Hz \;//$$