

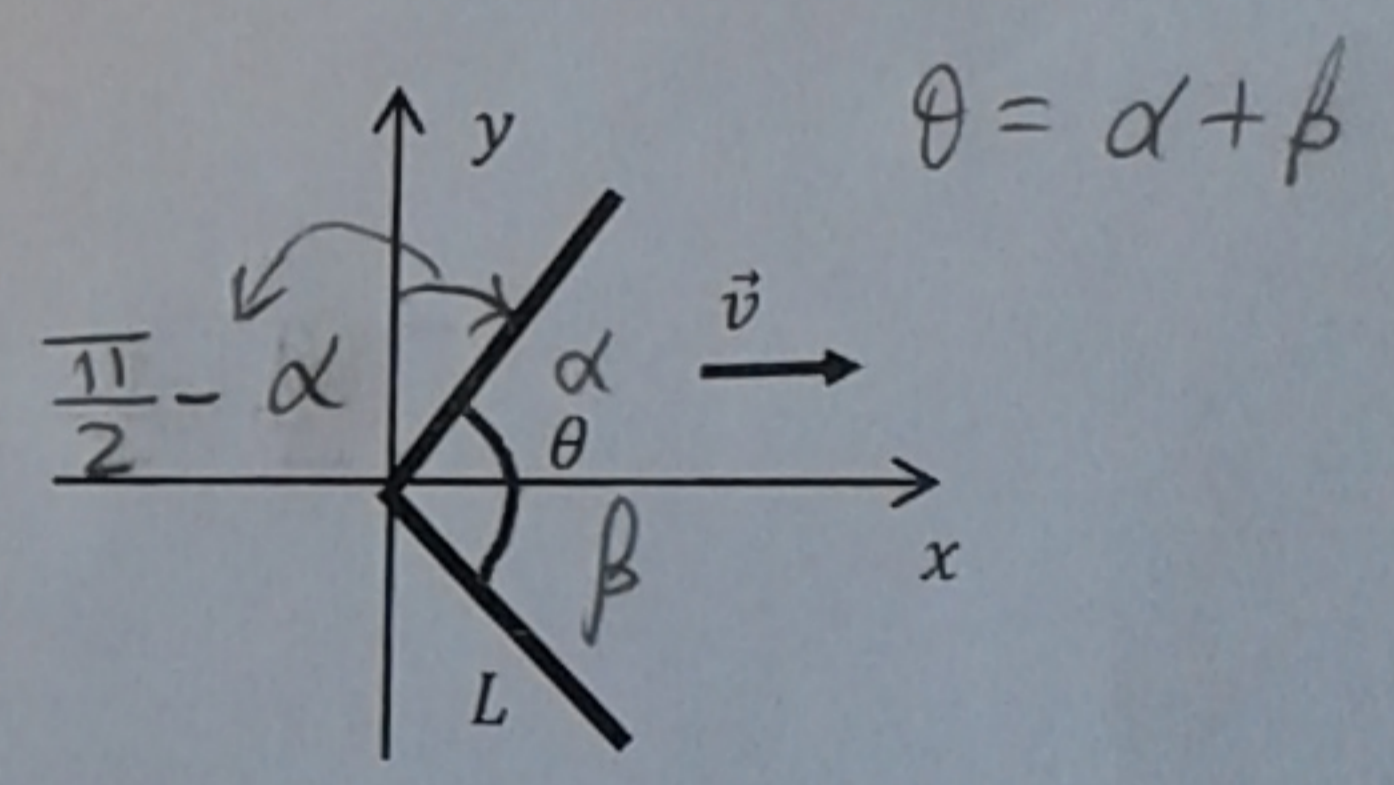
Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

A V-shaped bar with each arm of length L and tip angle θ in the x - y plane is moving with velocity $\vec{v} = v_0 \hat{x}$ along the x -axis in a uniform magnetic field $\vec{B} = -B \hat{z}$. Find the induced emf between the ends of the V-shape. (Hint: $\epsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$)



First, let's determine $\vec{v} \times \vec{B}$:

$$\vec{v} \times \vec{B} = (v_0 \hat{x}) \times (-B \hat{z}) = v_0 B \hat{y}$$

$$\begin{aligned} \text{Then, } \epsilon_u &= \int_0^L (\vec{v} \times \vec{B}) \cdot d\vec{l}_u = \int_0^L v_0 B \hat{y} \cdot d\vec{l}_u = \int_0^L v_0 B \cos\left(\frac{\pi}{2} - \alpha\right) dl_u = \\ &= \int_0^L v_0 B \sin \alpha dl_u = v_0 B L \sin \alpha \quad \text{at the end of upper arm} \end{aligned}$$

$$\begin{aligned} \epsilon_l &= \int_0^L (\vec{v} \times \vec{B}) \cdot d\vec{l}_l = \int_0^L v_0 B \cos\left(\frac{\pi}{2} + \beta\right) dl_l = \\ &= \int_0^L (-v_0 B \sin \beta) dl_l = -v_0 B L \sin \beta \quad \text{at the end of lower arm} \end{aligned}$$

The induced emf between the ends of the V-shape:

$$\Delta \epsilon = \epsilon_u - \epsilon_l = v_0 B L (\sin \alpha + \sin \beta)$$

where $\theta = \alpha + \beta$

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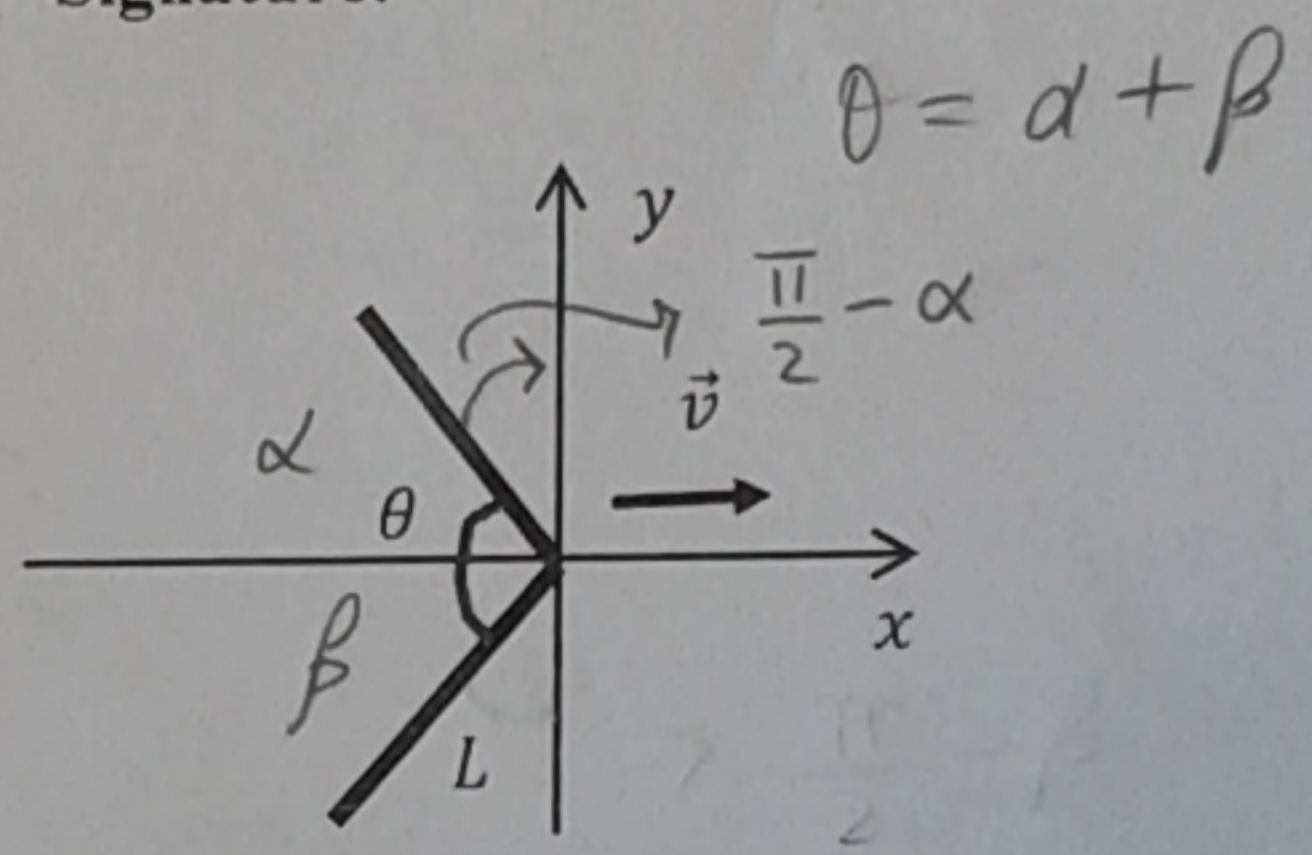
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A V-shaped bar with each arm of length L and tip angle θ in the x - y plane is moving with velocity $\vec{v} = v_0 \hat{x}$ along the x -axis in a uniform magnetic field $\vec{B} = B \hat{z}$. Find the induced emf between the ends of the V-shape.
 (Hint: $\epsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$)



First, $\vec{v} \times \vec{B} = (v_0 \hat{x}) \times (B \hat{z}) = -v_0 B \hat{y}$

Then, $\epsilon_u = \int_0^L (\vec{v} \times \vec{B}) \cdot d\vec{l}_u = \int_0^L (-v_0 B \hat{y}) \cdot d\vec{l}_u = \int_0^L (-v_0 B) \cos(\frac{\pi}{2} - \alpha) dl_u$
 $= \int_0^L (-v_0 B) (\sin \alpha) dl_u = -v_0 B L \sin \alpha$
 at the end of upper arm

$\epsilon_l = \int_0^L (-v_0 B) \cos(\frac{\pi}{2} + \beta) dl_l = \int_0^L (-v_0 B) (-\sin \beta) dl_l$
 $= v_0 B L \sin \beta$
 " " " lower arm

$\implies \Delta \epsilon = \epsilon_u - \epsilon_l = -v_0 B L (\sin \alpha + \sin \beta)$

where $\theta = \alpha + \beta$

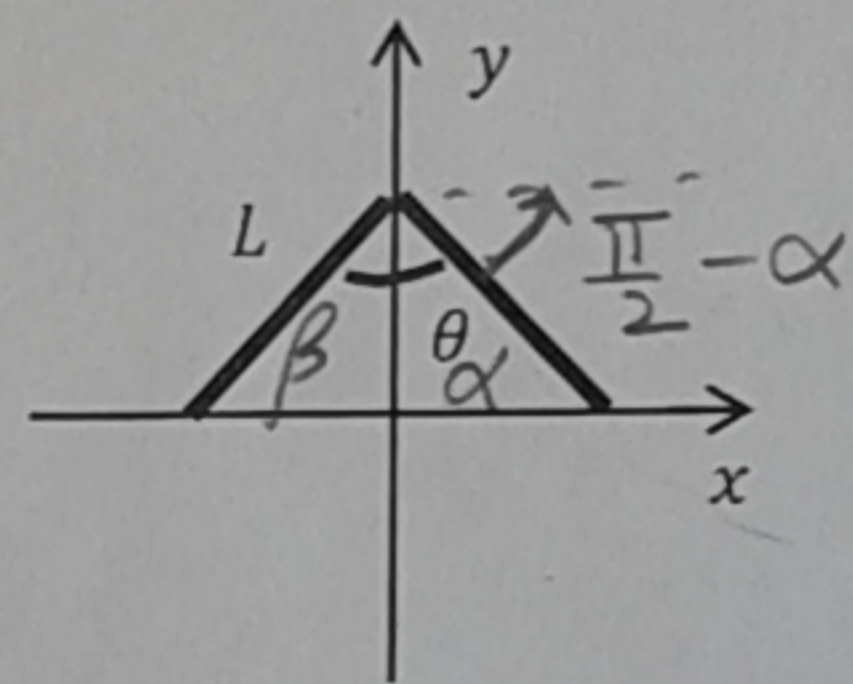
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$\theta = \alpha + \beta$

First, $\vec{v} \times \vec{B} = (v_0 \hat{z}) \times (-B \hat{y}) = v_0 B \hat{x}$

Then, $\epsilon_u = \int_0^L (\vec{v} \times \vec{B}) \cdot d\vec{l}_u = \int_0^L v_0 B \cos(\frac{\pi}{2} - \alpha) dl_u =$
 $= v_0 B L \sin \alpha$ at the end of upper arm

$\epsilon_l = \int_0^L v_0 B \cos(\frac{\pi}{2} + \beta) dl_l = \int_0^L v_0 B (-\sin \beta) dl_l =$
 $= -v_0 B L \sin \beta$ " " " " lower arm

$\Rightarrow \Delta \epsilon = \epsilon_u - \epsilon_l = v_0 B L (\sin \alpha + \sin \beta)$

where $\theta = \alpha + \beta$