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PHYS 102: General Physics 2 KOÇ UNIVERSITY Spring Semestre 2008

College of Arts and Sciences

Section 2 Quiz 8 17 April 2008

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Consider a circular conductor with a radius a that carries a current I. What is the direction and magnitude of the magnetic field at a point P on the axis of the loop, at a distance x from the center.

Hints: Biot and Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\cdot\frac{d\vec{\ell}\times\vec{r}}{r^3}$

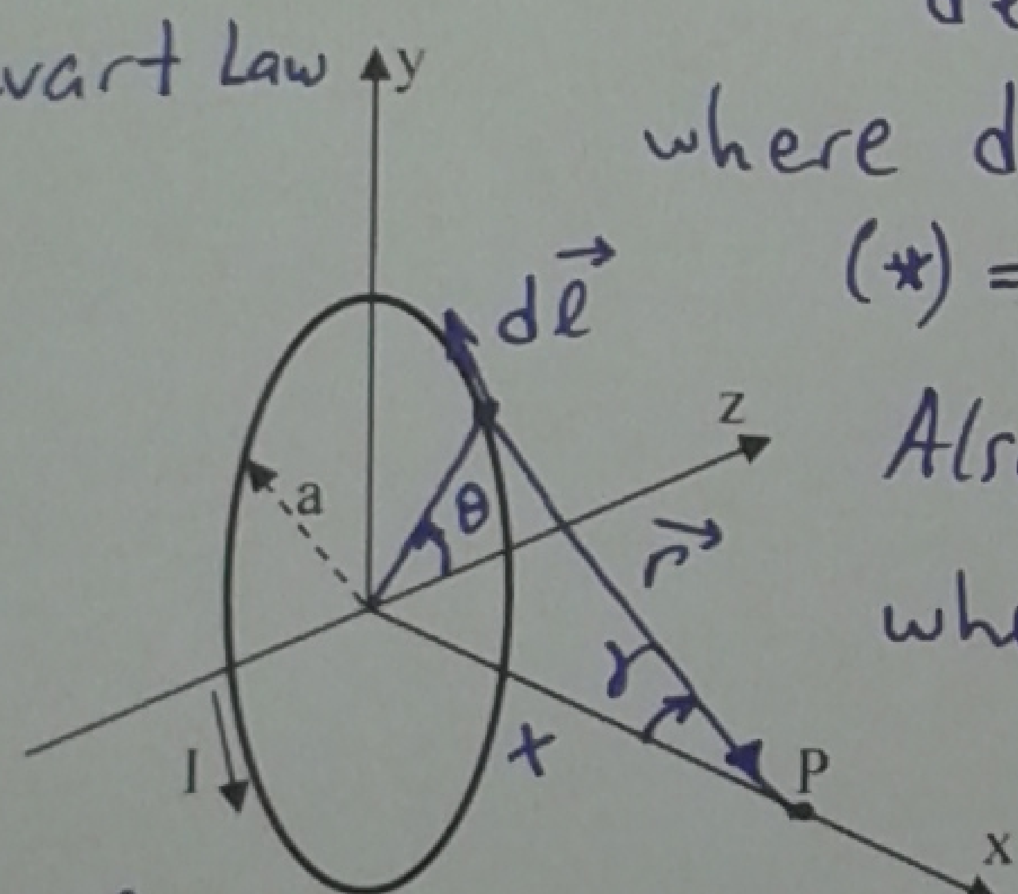
Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

$d\vec{\ell} = -d\ell\sin\theta\,\hat{z} + d\ell\cos\theta\,\hat{y} = (*)$

Use the Biot-Savart Law

where $d\ell = a\,d\theta$

$(*) = a(-\sin\theta\,\hat{z} + \cos\theta\,\hat{y})\,d\theta$



Also, $\vec{r} = a\cos\theta\,\hat{z} + a\sin\theta\,\hat{y} + x\,\hat{x}$

where $a = r\sin\gamma$ and $x = r\cos\gamma$

$\Rightarrow r = \sqrt{x^2+a^2}$

$$\Rightarrow \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{\big(a(-\sin\theta\,\hat{z} + \cos\theta\,\hat{y})\big)\times\big(a(\cos\theta\,\hat{z} + \sin\theta\,\hat{y})\big)}{(x^2+a^2)^{3/2}}\,d\theta$$

Solution:

Direction of $\vec{B}$: +x direction

Magnitude of $\vec{B}$: $B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$

✓: $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi}\cdot\frac{a^2}{(x^2+a^2)^{3/2}}\int_0^{2\pi}\Big[\big(\sin^2\theta\,\hat{x} - \frac{x}{a}\sin\theta\,\hat{y}\big) + \big(\cos^2\theta\,\hat{x} - \frac{x}{a}\cos\theta\,\hat{z}\big)\Big]d\theta$$

$$= \frac{\mu_0 I a^2}{4\pi(x^2+a^2)^{3/2}}\Big[\int_0^{2\pi}\hat{x}\,d\theta + \underbrace{\int_0^{2\pi}\cancel{\left(-\frac{x}{a}\right)\sin\theta\,\hat{y}\,d\theta}}_{=0} + \underbrace{\int_0^{2\pi}\cancel{\left(-\frac{x}{a}\right)\cos\theta\,d\theta}}_{=0}\Big] =$$

$$= \frac{\mu_0 I a^2}{{}_2\cancel{4\pi}(x^2+a^2)^{3/2}}\cdot(\cancel{2\pi}\,\hat{x}) = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}\,\hat{x} \quad //$$

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Section 5 | **Quiz 8** | **17 April 2008**

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Quiz duration: 10 minutes

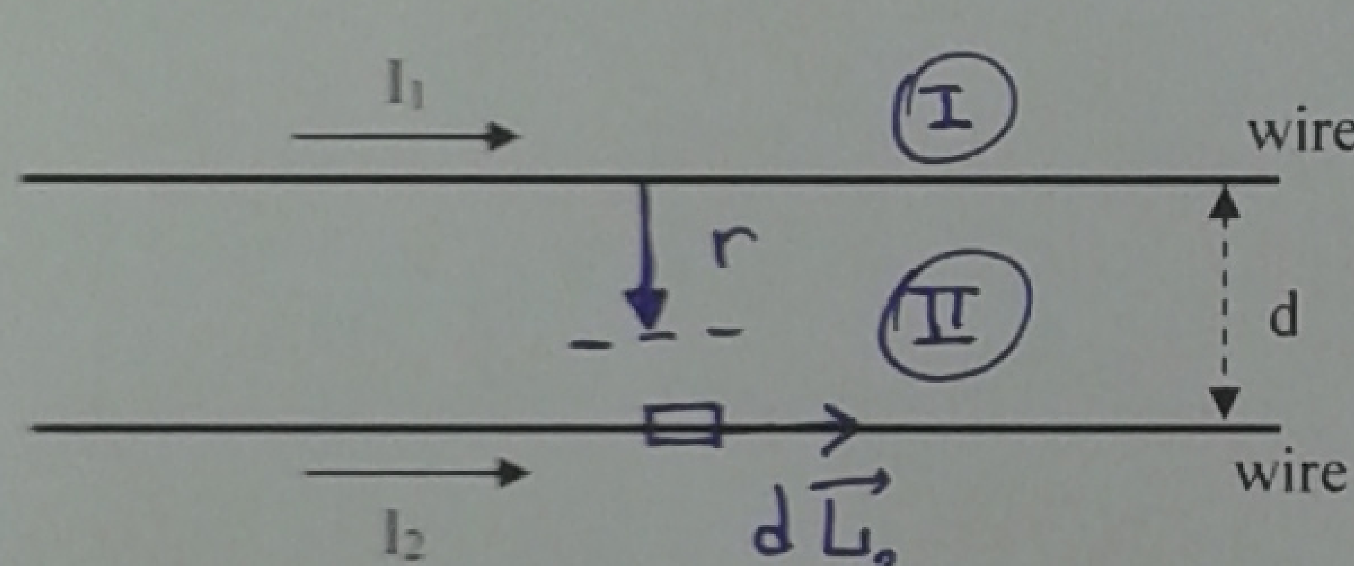
Name: **Student ID:** **Signature:**

Two long, parallel wires carrying currents I_1 and I_2 are separated by a distance of d, as shown below. Find out an expression for the magnitude of the force exerted by the wire 1 on a portion of the wire 2 with a length L. Indicate whether the force is attractive or repulsive.

Simply use the Ampere's Law

Hints: Biot and Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$

Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow B\cdot(2\pi r) = \mu_0 I_{encl}$



$\Rightarrow B_1 = \frac{\mu_0 I_1}{2\pi r}$

(magnetic field at some distance "r" created by I_1)

By the right-hand rule, the direction of $\vec{B_1}$ in the region II is into the plane: $\otimes \vec{B_1}$

Solution:

Magnitude: $F = \frac{\mu_0 I_1 I_2}{2\pi d}L$

Direction: Attractive

The Lorentz force: $\vec{F} = I\vec{L}\times\vec{B} \Rightarrow |\vec{F}| = ILB\sin\theta$

Thus, the force exerted by I_1 on I_2: $\sin\theta = 1$ (since $B_1 \perp I_2$)

$$F_{12} = I_2 L B_1\Big|_{r=d} = I_2 L\left(\frac{\mu_0 I_1}{2\pi d}\right) = \frac{\mu_0 I_1 I_2 L}{2\pi d} \,//$$

Again, by the right-hand rule, the direction of $\vec{F_{12}}$ is upward (attractive)

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Section 3 Quiz 8 17 April 2008

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Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A toroidal solenoid with inner radius a and outer radius b, wound with N turns of wire is carrying a current I. Find the magnitude and direction of the magnetic field at a distance r from the center of the toroid for: $r<a$, $a<r<b$, and $r>b$.

Hints: Biot and Savart Law: $d\bar{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l} \times \hat{r}}{r^2}$ Use the Ampere's Law

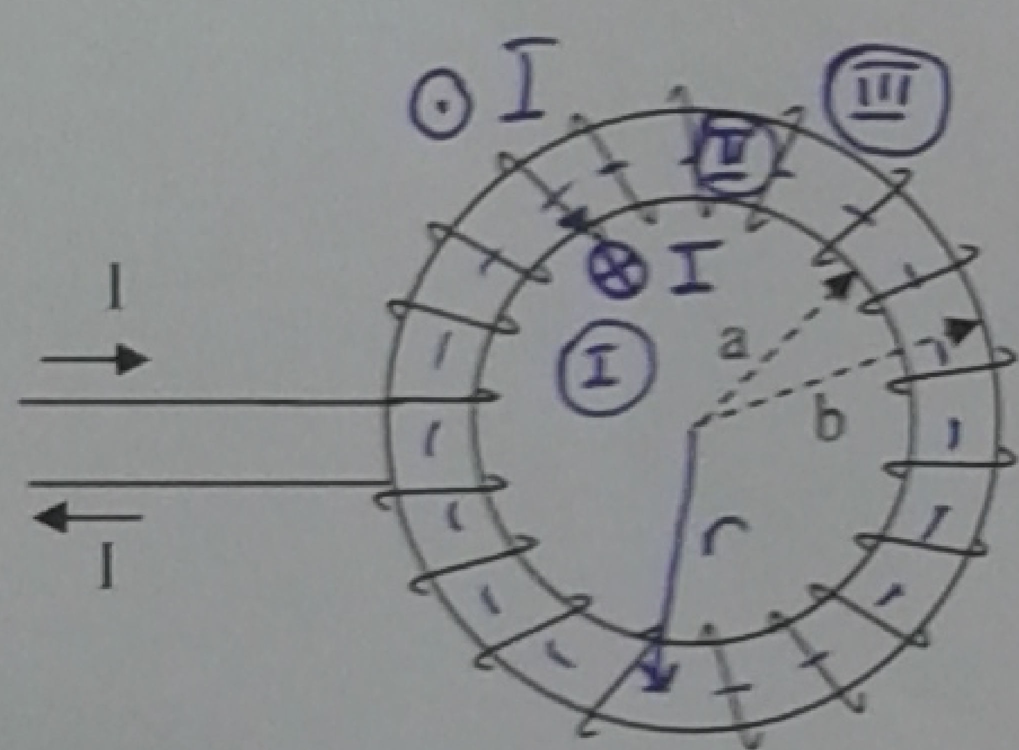
Ampere's Law: $\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{encl}$

For the region I : $r<a$: $I_{encl} = 0$

$\Rightarrow B = 0$

For II : $a<r<b$: $I_{encl} = NI$

$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$



For III : Each current coming out of the plane and going into the plane with the same amount but in opposite direction : $\odot I + \otimes I = I_{encl} = 0$

Solution:

r<a: $B = 0$

r>b: $B = 0$

a<r<b: $B = \frac{\mu_0 NI}{2\pi r}$, direction: tangential given by the right hand's rule

$\Rightarrow$ for $r>b$: $I_{encl} = 0 \Rightarrow B = 0$