PHYS 102: General Physics 2 KOÇ UNIVERSITY Spring Semester 2017

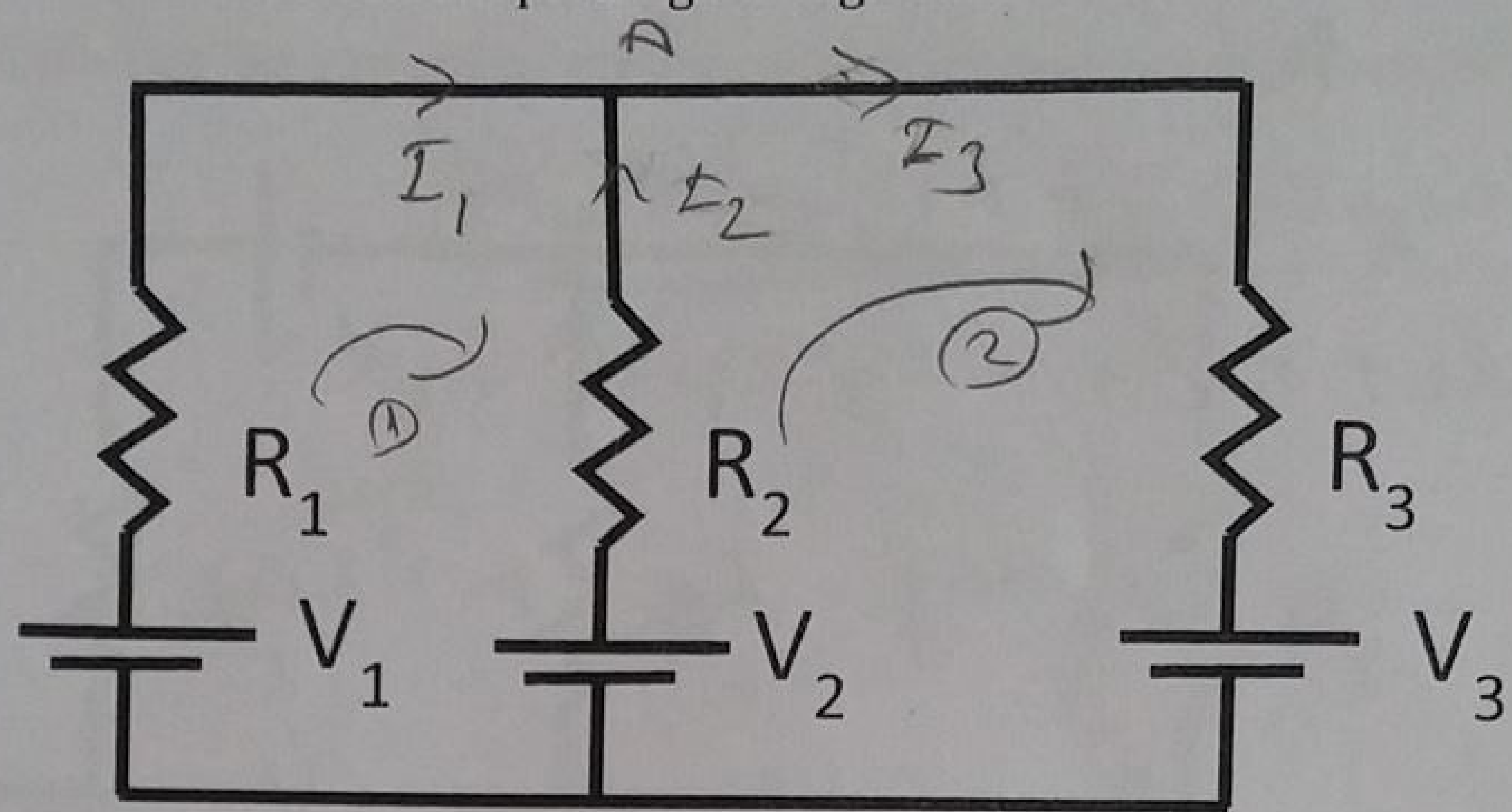
College of Sciences

Section 1 Quiz 5 20 March 2017

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Calculate the current passing through each resistor in the circuit.



at node A : $I_1 + I_2 = I_3$ (i)

1st loop: $-V_1 + I_1 R_1 - I_2 R_2 + V_2 = 0$ (ii)

2nd loop: $-V_2 + I_2 R_2 + I_3 R_3 + V_3 = 0$ (iii)

Putting (i) in (ii)

$V_2 - V_1 + (I_3 - I_2)R_1 - I_2 R_2 = 0$ / $\times R_3$

$V_3 - V_2 + I_2 R_2 + I_3 R_3 = 0$ / $\times -R_1$

$$I_2 = \frac{V_2(R_1 + R_3) - V_1 R_3 - V_3 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

Putting I_2 in (ii), we obtain I_1

$$I_1 = \frac{V_1 - V_2}{R_1} + \frac{R_2}{R_1} I_2$$

$$= \frac{V_1 - V_2}{R_1} + \frac{R_2}{R_1} \frac{V_2(R_1+R_3) - V_1R_3 - V_3R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$

| Re-arranging I_1 |

$$I_1 = \frac{V_1(R_2+R_3) - V_2R_3 - V_3R_2}{R_1R_2 + R_2R_3 + R_3R_1}$$

In a similar way, putting I_2 in (iii), we obtain

$$I_3 = \frac{V_3(R_1+R_2) - V_1R_2 - V_2R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$

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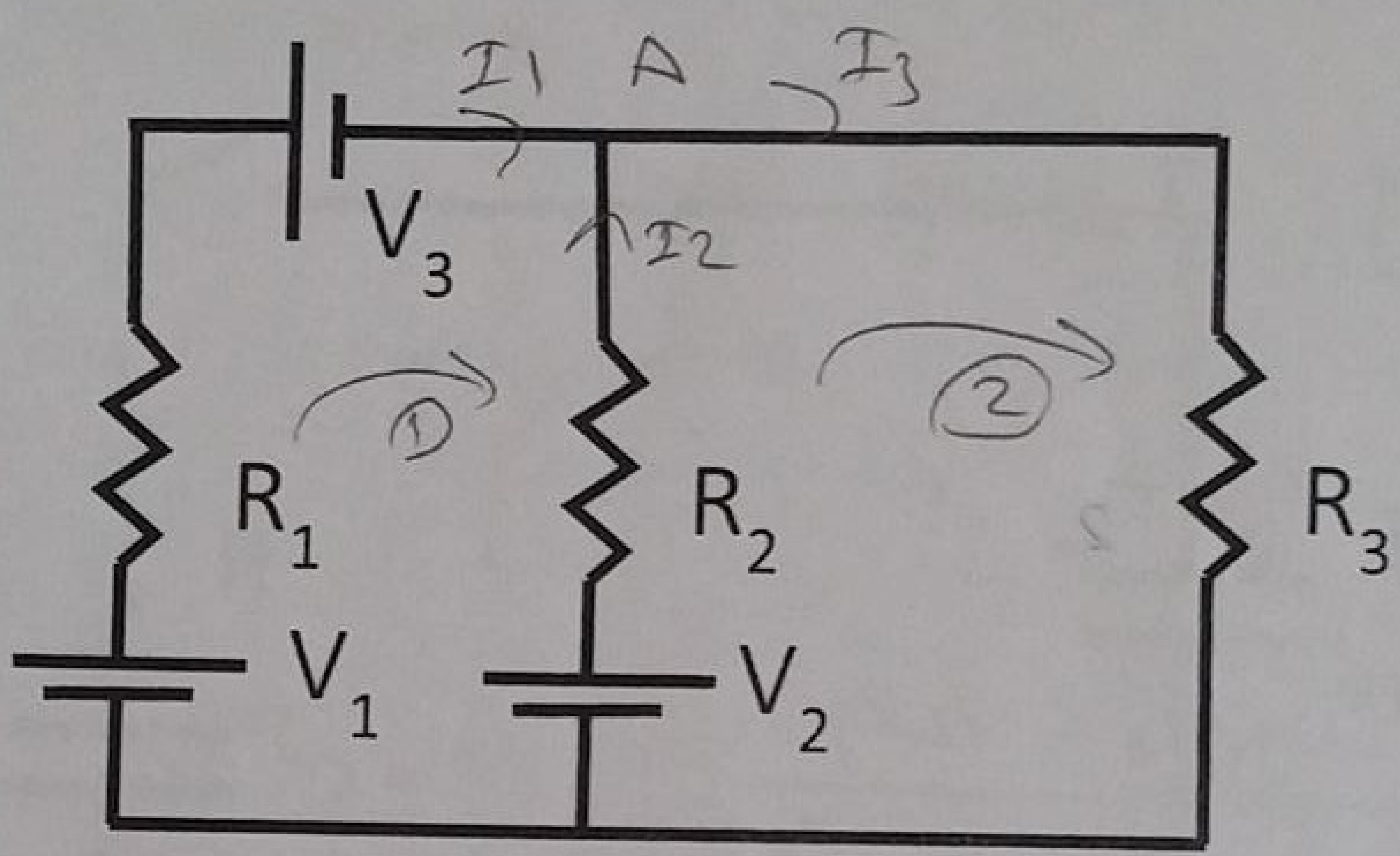
College of Sciences

Section 2 **Quiz 5** **20 March 2017**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Calculate the current passing through each resistor in the circuit.



at node A: $I_1 + I_2 = I_3$ (i)

at loop (1): $-V_1 + I_1R_1 + V_3 - I_2R_2 + V_2 = 0$ (ii)

at loop (2): $-V_2 + I_2R_2 + I_3R_3 = 0$ (iii)

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putting (i) in (ii)

$V_2 + V_3 - V_1 + (I_3 - I_2)R_1 - I_2R_2 = 0$ $\quad /\times R_3$

$+ \quad -V_2 + I_2R_2 + I_3R_3 = 0$ $\quad /\times -R_1$

$$I_2 = \frac{-V_2(R_1+R_3) - V_1R_3 + V_3R_3}{R_1R_2 + R_2R_3 + R_1R_3}$$

Putting I_2 in (ii), we obtain I_1

$$I_1 = \frac{V_1 - V_2 - V_3}{R_1} + \frac{R_2}{R_1} I_2$$

$$= \frac{(V_1 - V_3)(R_2 + R_3) - V_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

and putting I_2 in (iii)

$$I_3 = \frac{V_2}{R_3} - \frac{R_2}{R_3} I_2$$

$$I_3 = \frac{-(V_1 - V_3) R_2 - V_2 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

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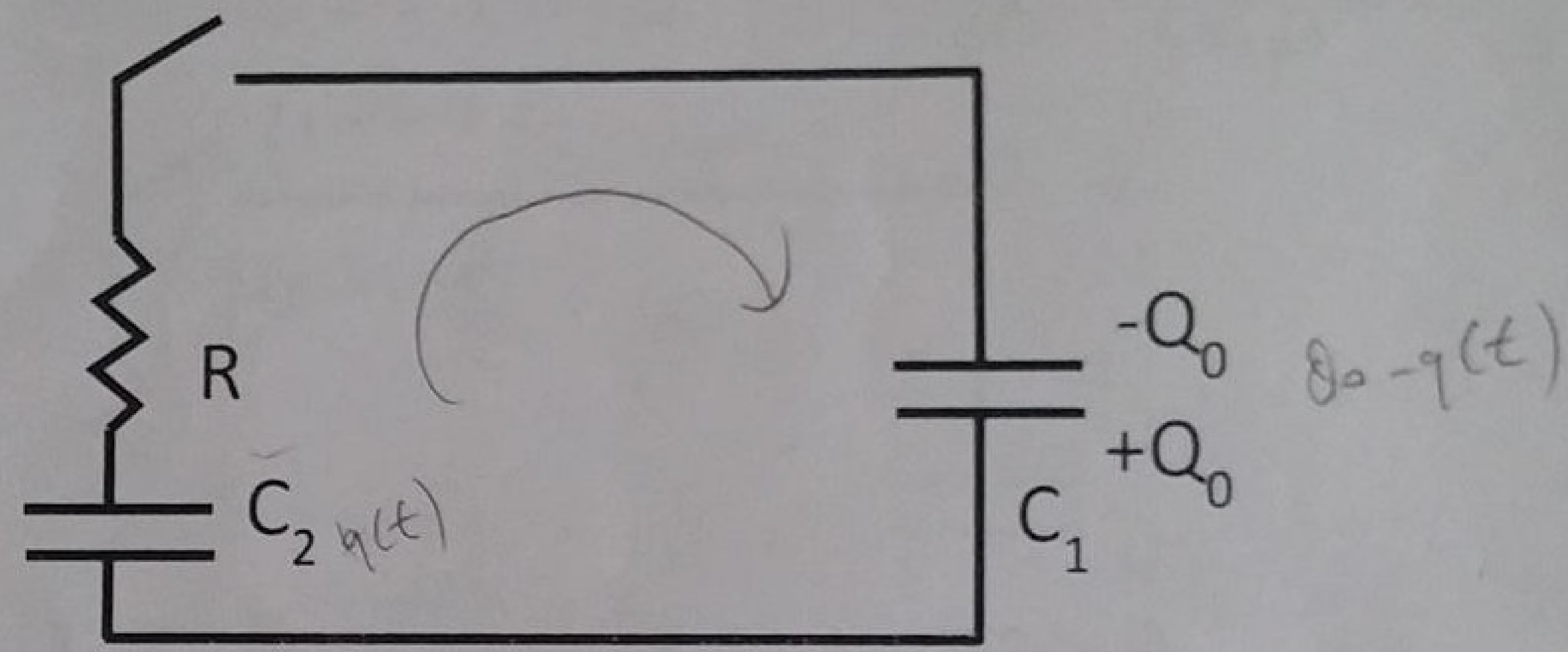
Section 3 | Quiz 5 | 20 March 2017

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A capacitor of capacitance C_1 carries charge Q_0 with polarization as given in the figure. It is connected to a resistor and another capacitor of capacitance C_2 with no initial charge. Find the charge on each capacitor as a function of time if we close the switch at t=0.



$$\left.\begin{aligned} C_2 &\rightarrow Q(t) \\ C_1 &\rightarrow Q_0 - Q(t) \end{aligned}\right\} \text{ charges as a function of time}$$

$$C_2: \quad \frac{Q_0 - Q(t)}{C_1} - \frac{Q(t)}{C_2} - IR = 0, \qquad I = \frac{dQ}{dt}$$

$$\frac{dQ}{dt} + \frac{Q}{RC_2} - \frac{Q_0 - Q}{RC_1} = 0$$

$$\frac{dQ}{dt} + \frac{Q}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) - \frac{Q_0}{RC_1} = 0, \qquad C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

$$\frac{dQ}{dt} + \frac{Q}{RC_{eq}} - \frac{Q_0}{RC_1} = 0, \qquad \frac{Q_0}{C_1} = \mathcal{E}$$

$$\frac{dQ}{dt} + \frac{Q}{RC_{eq}} - \frac{\mathcal{E}}{R} = 0$$

$$\frac{dQ}{dt} = -\frac{1}{RC_{eq}}(Q - \mathcal{E}C_{eq}) \Rightarrow \int_0^Q \frac{dQ'}{Q' - \mathcal{E}C_{eq}} = \int_0^t \frac{-dt'}{RC_{eq}}$$

$$\ln\left(\frac{Q - \mathcal{E}C_{eq}}{-\mathcal{E}C_{eq}}\right) = -\frac{t}{RC}$$

$$Q_2(t) = \mathcal{E}C_{eq}\left(1 - e^{-t/RC_{eq}}\right)$$

$$Q_2(t) = \frac{Q_0 C_{eq}}{C_1}\left(1 - e^{-t/RC_{eq}}\right)$$

for C_1: $Q_1(t) = Q_0 - Q_2(t)$

$$= Q_0 - \frac{Q_0 C_{eq}}{C_1}\left(1 - e^{-t/RC_{eq}}\right)$$

$$= Q_0\left(1 - \frac{C_{eq}}{C_1}\left(1 - e^{-t/RC_{eq}}\right)\right)$$

$$I(t) = \frac{dQ}{dt} = \frac{Q_0 C_{eq}}{C_1}\left(\frac{1}{RC_{eq}}\right) e^{-t/RC_{eq}}$$

$$I(t) = \frac{Q_0}{RC_1} e^{-t/RC_{eq}}$$

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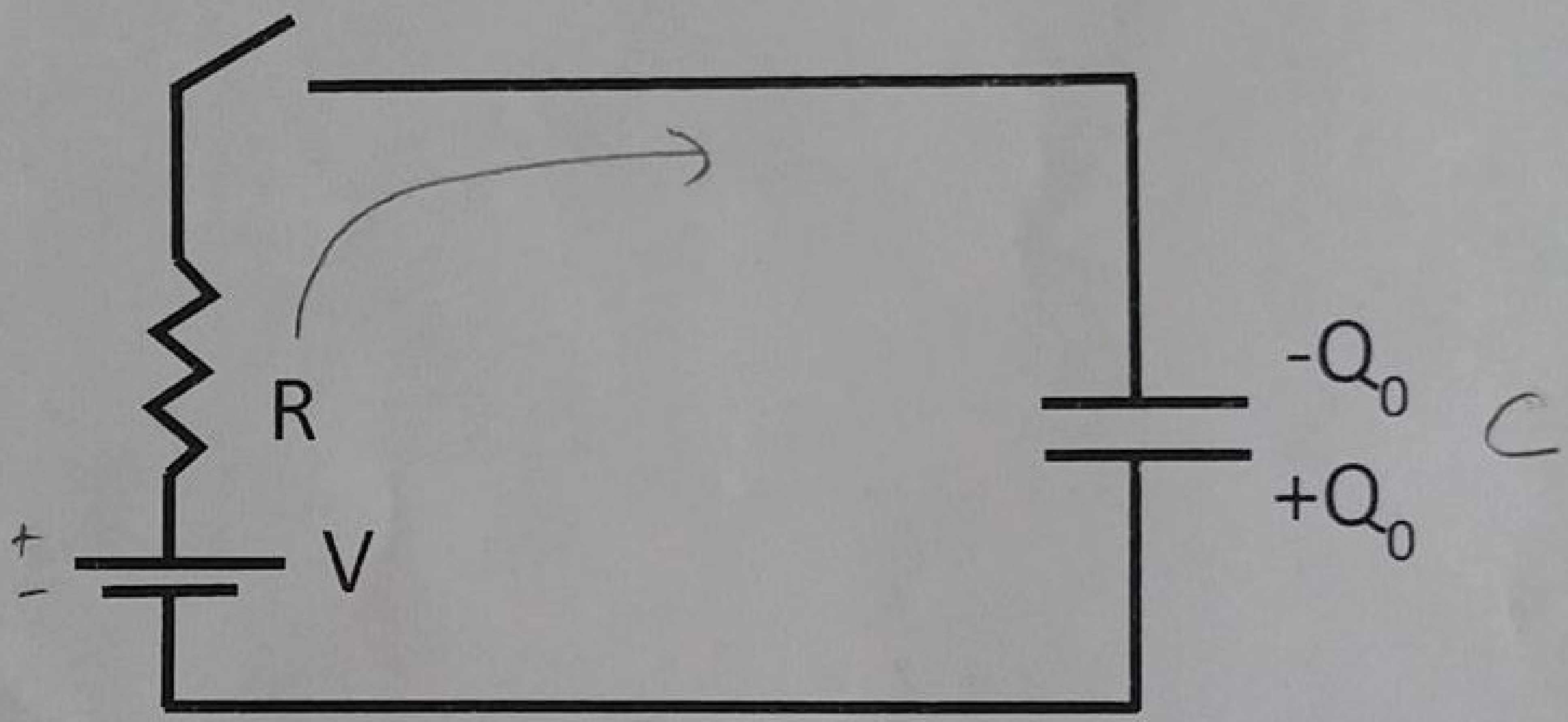
Section 4 Quiz 5 21 March 2017

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A capacitor of capacitance C carries charge Q_0 with polarization as given in the figure. It is connected to a battery and a resistor, and the switch is closed at t=0. Find the charge on the capacitor and the current in the circuit as functions of time.



$$-V + IR + \frac{Q}{C} = 0 \qquad I = -\frac{dQ}{dt}$$

$$-V \mp \frac{dQ}{dt}R + \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} + \frac{1}{RC}(Q + VC) = 0 \Rightarrow \frac{dQ}{dt} = -\frac{1}{RC}(Q + VC)$$

$$\int_{Q_0}^{Q} \frac{dQ'}{Q+VC} = \int_0^t -\frac{dt'}{RC} \Rightarrow \ln\left(\frac{Q+VC}{Q_0+VC}\right) = \frac{-t}{RC}$$

$$Q + VC = (Q_0 + VC)e^{-t/RC}$$

$$Q(t) = Q_0 e^{-t/RC} - VC(1 - e^{-t/RC})$$

$$I(t) = \frac{dQ(t)}{dt}$$

$$= -\frac{Q_0}{RC} e^{-t/RC} - \frac{VC}{RC} e^{-t/RC}$$

$$I(t) = \left(\frac{-VC - Q_0}{RC}\right) e^{-t/RC}$$