

Closed book. No calculators are to be used for this quiz.

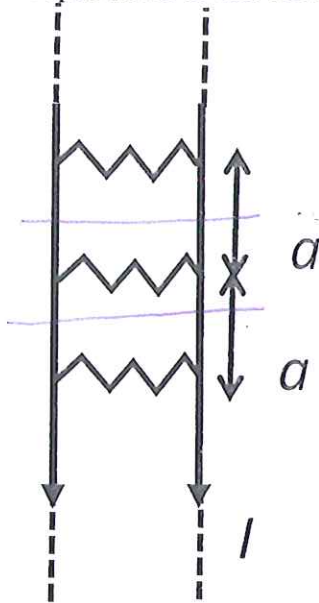
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Two parallel infinite thin wires both carry a current I in the same direction. They are connected by infinitely many insulating springs of spring constants k at equal distances of a . Springs have unstretched length l_0 . What is the equilibrium separation of the wires?



Let l_{eq} be equilibrium separation.

$$\Rightarrow B = \frac{\mu_0 I}{2\pi l_{eq}} \text{ at the wire}$$

We consider the segment of wire of length a , so that spring is at the middle.

$$F_{\text{magnetic}} = F_{\text{spring}}$$

$$F_{\text{magnetic}} = I B a = \frac{\mu_0 I^2}{2\pi l_{eq}} a$$

$$F_{\text{spring}} = k(l_0 - l_{eq}) = \frac{\mu_0 I^2}{2\pi l_{eq}} a$$

$$\Rightarrow l_{eq} = \frac{1}{2} \left(l_0 - \sqrt{l_0^2 - \frac{2\mu_0 I^2 a}{\pi k}} \right)$$

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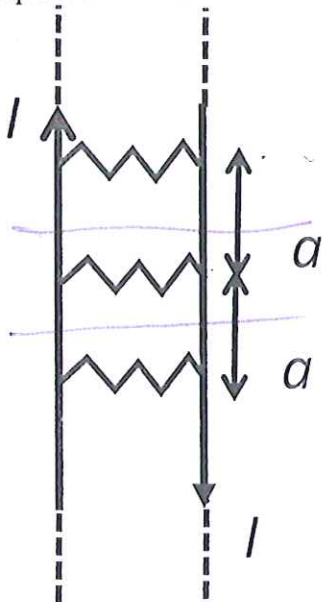
Quiz duration: 10 minutes

Name:

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Two parallel infinite thin wires carry current I in opposite directions. They are connected by infinitely many insulating springs of spring constants k at equal distances of a . Springs have unstretched length l_0 . What is the equilibrium separation of the wires?



Let l_{eq} be equilibrium separation
 \Rightarrow Magnetic field of wire on another

wire: $B = \frac{\mu_0 I}{2\pi l_{eq}}$

We consider the segment of wire of length a , such that spring acts from the middle point:

$$F_{\text{magnetic}} = F_{\text{spring}}$$

$$F_{\text{magnetic}} = I B a = \frac{\mu_0 I^2}{2\pi l_{eq}} a$$

$$F_{\text{spring}} = k(l_{eq} - l_0) = \frac{\mu_0 I^2}{2\pi l_{eq}} a$$

$$\Rightarrow l_{eq} = \frac{1}{2} \left(l_0 + \sqrt{l_0^2 + \frac{2\mu_0 I^2 a}{\pi k}} \right)$$

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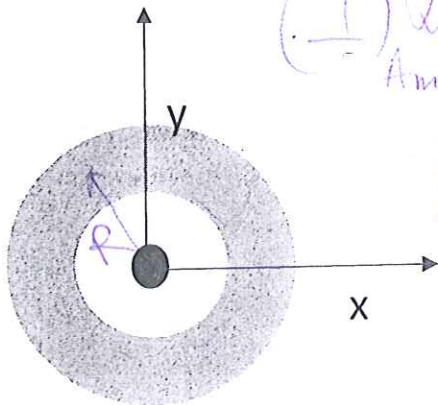
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A thin infinite wire is on the z axis and carries a current in the +z direction. Around the wire is a toroidal solenoid whose axis is also the z-axis (cross section in the figure). The solenoid has N turns and has a current I. What is the current on the thin wire so that there is no magnetic field inside the torus? Derive any magnetic field formulae you use from the relevant laws, do NOT just use them from memory.



(I) Wire field derivation:
Ampere's Law:

$$\mu_0 I_{\text{wire}} = 2\pi R B_{\text{wire}}$$

$$B_{\text{wire}} = \frac{\mu_0}{2\pi R} I_{\text{wire}}$$



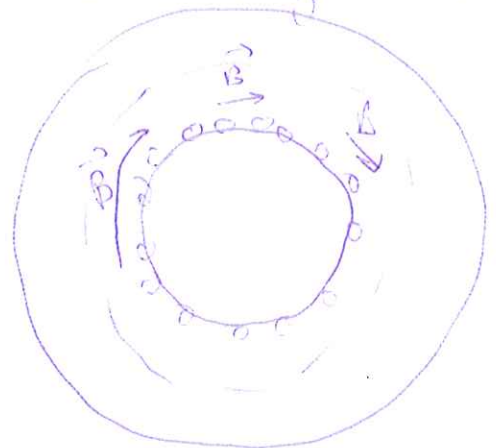
(II) Assume constant solenoid field and magnetic field outside solenoid is zero:

Ampere's Law:

loop is circle with radius R:

$$N\mu_0 I = 2\pi R B_{\text{solenoid}}$$

$$B_{\text{solenoid}} = \frac{\mu_0 N I}{2\pi R}$$



We need: $B_{\text{solenoid}} + B_{\text{wire}} = 0$

$$\Rightarrow \frac{\mu_0 N I}{2\pi R} + \frac{\mu_0 I_{\text{wire}}}{2\pi R} = 0$$

$$\Rightarrow I_{\text{wire}} = -NI$$

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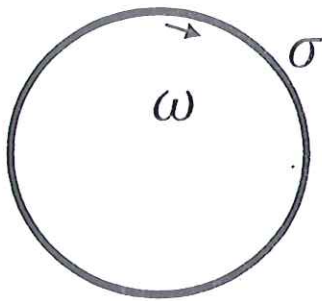
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An infinite, hollow cylindrical shell of radius a carries surface electric charge density σ (cross section in the figure). It is rotating along its axis with angular speed ω . This setup behaves similar to a solenoid. Find the magnetic field inside and outside the cylinder.



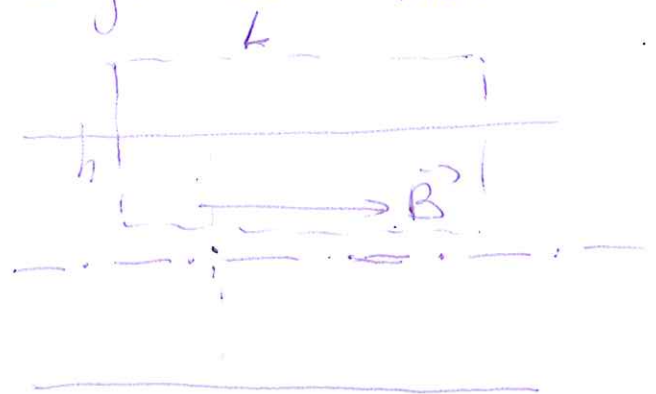
Since it behaves similar to a solenoid, we assume constant uniform magnetic field parallel to the axis inside the cylinder, and zero outside magnetic field.

Cross section passing through the axis:

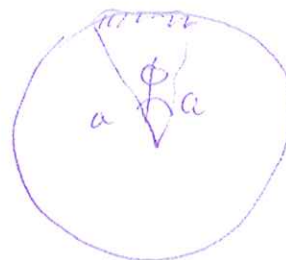
consider a loop of height h and length L (rectangle)

$$\oint \vec{B} \cdot d\vec{l} = B \cdot L = \mu_0 I$$

where $I = \frac{dq}{dt}$



Consider a sector with angle ϕ .
Its charge: $q = \sigma \cdot a \phi L$



$$\Rightarrow I = \frac{dq}{dt} = \sigma a L \frac{d\phi}{dt} = \sigma a L \omega$$

$$\Rightarrow B L = \mu_0 I = \mu_0 \sigma a \omega L \Rightarrow \boxed{B = \mu_0 \sigma a \omega}$$